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Medical Diagnosis via Distance Measures on Picture Fuzzy Sets

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Abstract

Most repeatedly uncertainties are encountered in medical diagnosis and consequently it becomes the most imperative and fascinating area of applications of fuzzy set theory. In literature, it is seen that application of type-I fuzzy and intuitionistic fuzzy set (IFS) in medical diagnosis is a commonly use area of research among the researchers. In IFS membership and non-membership degrees are considered but one important concept, i.e., degree of neutrality has not been considered. That's why picture fuzzy sets (PFS) may come into picture. In this present article, an attempt has been made to carry out medical diagnosis using the distance measure on PFSs and finally, exhibit the technique with a suitable case study.

Key words

Medical diagnosis, Uncertainty, Fuzzy set, Picture fuzzy set, Distance measure.

1. Introduction

More often, it is seen that real world problems are tainted with uncertainty due to lack of knowledge, imprecision, vagueness etc. To handle such type of uncertainty L.A. Zadeh [1] developed fuzzy set theory in 1965. Thereafter, not only various direct/indirect extensions of fuzzy set have been made but also effectively applied in most of the problems of real world situation including medical diagnosis. Gehrke et al. (1996) [2] stated that many people believe that assigning an exact number to expert's opinion is too restrictive and the assignment of an interval valued is more realistic. That is, in some real-world situations interval valued fuzzy set (IVFS) is acceptable. Sambuc (1975) [3] first presented the concept of IVFS names as $\phi-$fuzzy set. After that different researchers have been studied this issue and applied in different areas. Some authors investigated and suggested some methods for measuring distance between intervals valued fuzzy sets (IVFS). Burillo and Bustine (1996) [4] discussed distances for IVFSs, Grzegorzewski (2004) [5] proposed new distance measure based on Housdroff space. Zeng and Guo (2008) [6] presented the distance measures of IVFSs and relationship between entropy and similarity measure of IVFS. Park et al., (2008) [7] proposed new distance measures for IVFS by incorporating amplitude of membership concept. Li (2009) [8] studied on distances of IVFS and some of its properties on metric space.

On the other hand, another significant generalization of fuzzy set theory is the theory of intuitionistic fuzzy set (IFS), introduced by Atanassov (1986) [9] ascribing a membership degree and a non-membership degree separately in such a way that sum of the two degrees must not exceed one. It is observed that fuzzy sets are IFSs but converse is not necessarily correct. Distance measure between intuitionistic fuzzy sets is an important concept in fuzzy mathematics because of its wide applications in real world. Many distance measures between IFSs have been proposed. Szmidt and Kacprzyk [10-13] discussed distance measures on IFSs which are generalization of Hamming distance, Euclidean distance. Grzegorzewski (2004) [5] suggested another generalization of those distances based on Housdroff metric. Szmidt and Janusz [14] studied IFSs of two and three term representations in the context of a hausdorff distance. Wang & Xin (2005)[15] and Chen [16] discussed distance measure between IFSs.

Tevetkov et al., [17] proposed distance measures for IFSs considering the counterparts of the earlier distance measures (proposed by Szmidt and Kacprzyk). Papakostas et al.,[18] discussed distance and similarity measures between intuitionistic fuzzy sets, Szmidt [19] studied distances and similarities in IFS.

Later in 2013 Cuong and Kreinovich [20] introduced another extension viz., Picture fuzzy set (PFS) which is a direct extension of fuzzy set and Intuitionistic fuzzy set by incorporating the concept of positive, negative and neutral membership degree of an element. Cuong (2014) [21] discussed some properties of PFSs and suggested distance measures between PFSs. Phong et al., (2014) [22] studied some compositions of picture fuzzy relations, Cuong and Hai (2015) [23] investigated main fuzzy logic operators: negations, conjunctions, disjunctions and implications on picture fuzzy sets and also constructed main operations for fuzzy inference processes in picture fuzzy systems. Cuong et al., (2015) [24] studied properties of an involutive picture negator and some corresponding De Morgan fuzzy triples on picture fuzzy sets, Son (2016) [25] proposed a new distance measure between PFSs and applied in fuzzy clustering, Cuong et al., (2016) [26] investigated the classification of representable picture t-norms and picture t-conorms operators for picture fuzzy sets.

Even though, medical diagnosis is being studied using various theory but yet no study have been made using distance measures on PFSs. It is observed that concept of PFS has more important in medical diagnosis and can effectively be used in medical diagnosis. Because, if some is suffering from stomach and chest problems then it is clear the symptoms temperature, headache has no effect on the diseases. That is, these symptoms have neutral effect on the diseases. Similarly, symptoms stomach pain and chest pain have neutral effect on the diseases viral fever, malaria, typhoid etc. On the other hand, symptom temperature has direct effect on malaria. In this case, degree of neutrality can be considered as around zero. So, concept of neutral membership degree can be considered as an integral part of medical diagnosis which is missing in IFS. Thus, it motivates us to study medical diagnosis via distance measure on PFS.

In this present study, first the distance measures on PFSs proposed by Son (2016) [25] has been modified and an effort has been made to carry out medical diagnosis via the modified distance measure.

2. Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory [1, 27], Intuitionistic fuzzy set (IFS) [9] and picture fuzzy set [20] are reviewed.

2.1 Fuzzy Set

Fuzzy set is a set in which every element has degree of membership of belonging in it. Mathematically, let X be a universal set. Then the fuzzy subset A of X is defined by its membership function

$$\mu_A : X \rightarrow [0,1] \tag{1}$$

which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in A$, where the value of $\mu_A(x)$ at x shows the grade of membership of x in A.

2.2 Intuitionistic Fuzzy Set

A Intuitionistic fuzzy set A on a universe of discourse X is of the form

$$A = \{(x, \mu_A(x), \nu_A(x) : x \in X\}, \tag{2}$$

where $\mu_A(x) \in [0.1]$ is called the "degree of membership of x in A", $V_A(x) \in [0,1]$ is called the "degree of non-membership of x in A", and where $\mu_A(x)$ and $V_A(x)$ satisfy the following condition:

$$0 \le \mu_A(x) + \nu_A(x) \le 1. \tag{3}$$

The amount $\pi_A(x) = 1 - (\mu_A(x) = V_A(x))$ is called hesitancy of x which is reflection of lack of commitment or uncertainty associated with the membership or non-membership or both in A.

2.3 Picture Fuzzy Set

A picture fuzzy set A on a universe discourse X is an object of the form

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x) : x \in X\},$$

where $\mu_A(x) \in [0,1]$ is called the "degree of positive membership of x in A", $\eta_A(x) \in [0,1]$ is called the "degree of neutral membership of x in A", $V_A(x) \in [0,1]$ is called the "degree of negative membership of x in A", and where $\mu_A(x)$, $\eta_A(x)$ and $V_A(x)$ satisfy the following condition:

$$0 \le \mu_A(x) + \eta_A(x) + \nu_A(x) \le 1. \tag{4}$$

The amount $\rho_A(x) = 1 - (\mu_A(x) + \eta_A(x) + V_A(x))$ is called the degree of refusal of x in A which is reflection of lack of commitment or uncertainty associated with the positive membership, neutral membership, negative membership or all in A.

Generally, picture fuzzy set based model may be adequate in situations when we face human opinions involving more answers of type: yes, abstain, no, refusal. For example [25] in a democratic election station, the council issues 500 voting papers for a candidate. The voting results are divided into four groups accompanied with the number of papers namely "vote for" (300), "abstain" (64), "vote against" (115) and "refusal of voting" (21). Group "abstain" means that the voting paper is a whitepaper rejecting both "agree" and "disagree" for the candidate but still takes the vote. Group "refusal of voting" is either invalid voting papers or bypassing the vote.

2.4 Picture Fuzzy Relation

A picture fuzzy relation R is a picture fuzzy subset of $X \times Y$ by

$$R = \{((x,y), \mu_A(x), \eta_A(x), \nu_A(x)) : x \in X, y \in Y\} \tag{5}$$

where $\mu_A: X \times Y \rightarrow [0,1]$, $\eta_A: X \times Y \rightarrow [0,1]$ and $V_A: X \times Y \rightarrow [0,1]$ satisfy the condition $0 \leq \mu_A(x,y) + \eta_A(x,y) + V_A(x,y) \leq 1$ for each $(x,y) \in X \times Y$.

2.5 Picture Distance Measure

A function $d(A,B)$ with $A, B \in PFS(X)$ is called picture distance measure if it satisfies (Son, 2016):

(i) $0 \leq d(A,B) \leq 1$,

(ii) $d(A,B) = d(B,A)$,

(iii) $d(A,B) = 0 \Leftrightarrow A = B$

(iv) $\mu_{AB} \times d(A,B) + \mu_{AC} \times d(A,C) \geq \mu_{BC} \times d(B,C) \forall A,B,C \in PFS(X)$ (6)

where the symbol "×" is the arithmetic product μ_{AB}, μ_{BC}, and μ_{AC} are composition operations of $A, B, C \in PFS(X)$. As an example, the following min-max composition formulae are used to calculate the triple $(\mu_{AB}, \mu_{BC}, \mu_{AC})$, from the membership functions of $A, B, C \in PFS(X)$.

$$\mu_{AB} = \min_i \left\{ \max \left\{ \mu_A(x_i), \mu_B(x_i) \right\} \right\}$$

$$\mu_{BC} = \min_i \left\{ \max \left\{ \mu_B(x_i), \mu_C(x_i) \right\} \right\}$$

$$\mu_{AC} = \min_i \left\{ \max \left\{ \mu_A(x_i), \mu_C(x_i) \right\} \right\} \tag{7}$$

2.6 Distance Measure on PFSs

In this section, we modify the distance measure proposed by Son (2016).

The modified picture distance measure between $A, B, C \in PFS(X)$ is defined by

$$d(A,B) = \frac{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\square\mu_i^p + \square\eta_i^p + \square\gamma_i^p}{3} + \max\left(\square\mu_i^p, \square\eta_i^p, \square\gamma_i^p\right)\right)^{1/p}}{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\square\mu_i^p + \square\eta_i^p + \square\gamma_i^p}{3} + \max\left(\square\mu_i^p, \square\eta_i^p, \square\gamma_i^p\right)\right)^{1/p} + \frac{1}{N}\sum_{i=1}^{N}\left(\max\{\varphi_i^A, \varphi_i^B\} + \left|\varphi_i^A - \varphi_i^B\right|^p\right)^{1/p} + 1} \tag{8}$$

where

$$\Delta\mu_i = \left|\mu_A(x_i) - \mu_B(x_i)\right|, (i = 1, \ldots, N)$$
$$\Delta\eta_i = \left|\eta_A(x_i) - \eta_B(x_i)\right|, (i = 1, \ldots, N)$$
$$\Delta\gamma_i = \left|\gamma_A(x_i) - \gamma_B(x_i)\right|, (i = 1, \ldots, N)$$
$$\phi_i^A = \left|\mu_A(x_i) + \eta_A(x_i) + \lambda_A(x_i)\right|, (i = 1, \ldots, N)$$
$$\phi_i^B = \left|\mu_B(x_i) + \eta_B(x_i) + \lambda_B(x_i)\right|, (i = 1, \ldots, N) \quad (9)$$

2.7 Proof

Conditions (i)-(iii) are obvious. We need to prove the condition (iv). Son (2016) approach has been adopted to prove the property.

In the extent of this proof, we will show that exists discrete values for the triple $(\mu_{AB}, \mu_{BC}, \mu_{AC})$.

Consider p=1. For $A, B, C \in PFS(X)$, let us denote:

$$AB_1 = \sum_{i=1}^{N} \frac{\Delta\mu_i + \Delta\eta_i + \Delta\gamma_i}{3},$$

$$AB_2 = \max\{\Delta\mu_i, \Delta\eta_i, \Delta\gamma_i\},$$

$$AB_3 = \max_i\{\phi_i^A, \phi_i^B\},$$

$$AB_4 = \sum_{i=1}^{N} \left|\phi_i^A - \phi_i^B\right| \quad (10)$$

The following inequality is needed to prove:

$$\frac{AB1 + AB2}{AB1 + AB2 + AB3 + AB4 + 1} + \frac{AC1 + AC2}{AC1 + AC2 + AC3 + AC4 + 1} \geq \frac{BC1 + BC2}{3(BC_1 + BC2 + BC3 + BC4 + 1)} \quad (11)$$

The facts below come from the definition of PFS.

$$\left|\mu_A(x_i) - \mu_B(x_i)\right| + \left|\mu_A(x_i) - \mu_C(x_i)\right| \geq \left|\mu_B(x_i) - \mu_B(x_i)\right|,$$
$$\left|\eta_A(x_i) - \eta_B(x_i)\right| + \left|\eta_A(x_i) - \eta_C(x_i)\right| \geq \left|\eta_B(x_i) - \eta_B(x_i)\right|,$$
$$\left|\gamma_A(x_i) - \gamma_B(x_i)\right| + \left|\gamma_A(x_i) - \gamma_C(x_i)\right| \geq \left|\gamma_B(x_i) - \gamma_B(x_i)\right|. \quad (12)$$

It follows that,

$$AB1 + AC1 \geq BC1$$
$$AB2 + AC2 \geq BC2 \qquad (13)$$

Assume:

$$\max\{|\mu_B(x) - \mu_C(x)|, |\eta_B(x) - \eta_C(x)|, |\gamma_B(x) - \gamma_C(x)|\}$$
$$= |\mu_B(x) - \mu_C(x)| \qquad (14)$$

Then,

$$|\mu_B(x) - \mu_C(x)| \leq |\mu_A(x) - \mu_B(x)| + |\mu_A(x) - \mu_C(x)|$$
$$\leq \max\{|\mu_A(x) - \mu_B(x)|, |\eta_A(x) - \eta_B(x)|, |\gamma_A(x) - \gamma_B(x)|\} + \max\{|\mu_A(x) - \mu_C(x)|, |\eta_A(x) - \eta_C(x)|, |\gamma_A(x) - \gamma_C(x)|\}$$
$$\frac{BC1 + BC2}{BC1 + BC2 + BC3 + BC4 + 1} \leq \frac{AB1 + AB2}{AB1 + AB2 + AC1 + AC2 + BC3 + BC4 + 1} \quad (1) \qquad (15)$$

If one of the facts below happen,

$$\max_i\{\phi_i^A, \phi_i^B, \phi_i^C\} = \phi_j^B,$$
$$\max_i\{\phi_i^A, \phi_i^B, \phi_i^C\} = \phi_j^B,$$
$$Then\ BC3 \geq AB3 \qquad (16)$$

Again

$$\max_i\{\phi_i^A, \phi_i^B, \phi_i^C\} = \phi_j^A,$$
$$Then\ \max_i\{\phi_i^A, \phi_i^B\} - \max_i\{\phi_i^B, \phi_i^C\} \leq \max_i\{\phi_i^A - \phi_i^B, \phi_i^B - \phi_i^C\} \leq \max_i\{\phi_i^A - \phi_i^B + \phi_i^B - \phi_i^C\}$$
$$= \max_i\{\phi_i^A - \phi_i^C\} \leq 3A_{C2}$$

if (17)

Analogously, we get

$BC2 \geq AB4$

$or\ 3AC2 + BC4 \geq AB4$

Thus,

$$3(AB_1 + AB2 + AC1 + AC2 + BC3 + BC4 + 1) \geq AB1 + AB2 + AB3 + AB4 + 1 \quad (2)$$

$$3(AB_1 + AB2 + AC1 + AC2 + BC3 + BC4 + 1) \geq AC1 + AC2 + AC3 + AC4 + 1 \quad (3) \qquad (18)$$

Combining (1, 2, 3), the inequality is proven. Thus, we have d is a picture distance measure.

2.8 Medical Diagnosis

The area of medicine and decision making are the most fruitful and interesting area of applications of fuzzy set theory. In real life situations, due to the imprecise nature of medical documents and uncertain information gathered for decision making requires the use of fuzzy. After the development of fuzzy set, various extensions have been seen and successfully applied in medical diagnosis. Sanchez (1976, 1979) [30,31] first formulated the diagnosis models involving fuzzy matrices representing the medical knowledge between symptoms and diseases. Yao and Yao (2001) [32] discussed decision making for medical diagnosis based on fuzzy numbers and compositional rule of inference. Elizabeth and Sujatha (2013) [33] presented a procedure for medical diagnosis and fuzzy decision making.

Chetia and Das (2010) [34] extended sanchez’s approach for medical diagnosis using interval valued fuzzy soft sets and exhibited the technique with hypothetical case study. Ahn et al., (2011) [35] presented a fuzzy diagnosis method based on the interval valued interview chart and the interval valued Intuitionistic fuzzy weighted arithmetic average operator and studied the occurrence information symptoms as the weights. Elizabeth and Sujatha (2014) [36] discussed medical diagnosis based on IVFN matrices. Meenakshi and Kaliraja (2011) [37] extended Sanchez’s approach for medical diagnosis using interval valued fuzzy matrix. Choi et al., (2012) [38] proposed a fuzzy diagnosis method based on interval valued intuitonistic fuzzy sets.

De et al., (2001) [39] studied Sanchez’s approach for medical diagnosis and extended the concept with the notion of IFS. Szmidt and Kacprzyk (2001)[40] studied medical diagnosis using their proposed distance measures. Own [41] studied advantages of type-2 fuzzy and switching relation between type-2 fuzzy sets and IFSs defined axiomatically and finally, switching results are applied in medical diagnosis. Samuel and Choi et al., [38] proposed a fuzzy diagnosis method based on interval valued intuitionistic fuzzy sets, Samuel and Balamurngan [42-44] studied medical diagnosis for IFSs and introduced a new concept of IFS with n parameters and applied in medical diagnosis. Davvaz and Sadrabadi [45] verified the application of IFSs in medical. Manimaran et al (2016) [46] presented a review of fuzzy environmental study in medical diagnosis system. Muthumeenakshi (2016) [47] applied pentagonal valued hesitant fuzzy set in medical diagnosis

However, in literature there is no study of application of distances on PFSs in medical diagnosis has been found. It is observed in medical diagnosis that some symptoms may have neutral effect on diseases. Eg., degree of neutrality can be considered for the symptoms temperature, headache on the diseases stomach and chest problems. Similarly, symptoms stomach pain and chest pain have neutral effect on the diseases viral fever, malaria, typhoid etc. Therefore, it is convenient to consider degree of neutrality i.e., application of PFSs in medical diagnosis. In this regard, in this article an effort has been made to study medical diagnosis using PFS based on the distance measure.

2.9 Methodology

In this section, we present an application of PFS in Sanchez's approach for medical diagnosis. Let *S*, *D* and *P* be the sets of symptoms, diseases and patients respectively.

Analogous to Sanchez's notation of "Medical knowledge" as a picture fuzzy relation *R* from *S* to *D* which reveals the degree of association, degree of non-association and degree of neutrality between the symptoms and diseases. Similarly, another fuzzy relation *Q* from *P* to *S* is defined.

Procedure is given below:

1. Determination of symptoms and formulation of medical knowledge based on picture fuzzy relation from *S* to *D* and another picture fuzzy relation *Q* from *P* to *S*.

2. Compute the distances between patients and diseases using the distance measure.

3. Shortest distance between patient and disease will indicate that the patient is likely to have the disease.

4. Case Study

In this section, a hypothetical case study has been carried out to perform medical diagnosis using the concept of picture fuzzy sets based on the proposed distance measures. Here, it is proposed to take into account the four parameters characterization of picture fuzzy sets: the positive membership degree (μ), neutral membership degree (η), the negative membership degree (*V*).

Tab.1. Symptoms-diseases Picture Fuzzy Relation

R	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4,0,0,0)	(0.7,0,0)	(0.3,0.4,0.3)	(0.1,0.3,0.5)	(0.1,0.3,0.5)
Headache	(0.3,0.2,0.4)	(0.2,0.4,0.35)	(0.6,0.2,0.1)	(0.2,0.4,0.3)	(0,0.5,0.35)
Stomach Pain	(0.10.35,0.5)	(0,0.4,0.5)	(0.2,0.3,0.4)	(0.8,0,0)	(0.2,0.3,0.5)

Cough	(0.4,0.3,0.2)	(0.7,0.1,0)	(0.2,0.35,0.3)	(0.2,0.4,0.3)	(0.2,0.35,0.4)
Chest Pain	(0.1,0.25,0.5)	(0.1,0.3,0.5)	(0.1,0.2,0.6)	(0.2,0.35, 0.3)	(0.8,0,0.1)

Let $P = \{A, B, C, D\}$ be the set of patients, S = {*temperature, headache, stomach pain, cough, chest pain*} be the set of symptoms, D = {*viral fever, Malaria, typhoid, stomach problem, chest problem*} be the set of diseases. Our intention is to carry out the right decision for each patient P_i, i=1,2,3,4 from the set of symptoms s_j, j=1,2,…,5 for each disease d_k, d=1,2,…,5.

The symptom-disease picture fuzzy relation R and patient-symptom picture fuzzy relation Q are given in Table 1 and Table 2 respectively.

Tab.2. Patients-symptoms Picture Fuzzy Relation

Q	Temperature	Headache	Stomach Pain	Cough	Chest Pain
A	(0.8,0,0.1)	(0.6,0.3,0.1)	(0.2,0.4,0.4)	(0.6,0.15,0.1)	(0.1,0.4,0.4)
B	(0,0.5,0.4)	(0.4,0.25,0.3)	(0.6,0.2,0.1)	(0.1,0.3,0.6)	(0.1,0.35,0.4)
C	(0.8,0,0.1)	(0.8,0,0.1)	(0,0.4,0.5)	(0.2,0.3,0.4)	(0,0.4,0.4)
D	(0.6,0.2,0.1)	(0.5,0.25,0.25)	(0.3,0.3,0.2)	(0.7,0,0.25)	(0.3,0.4,0.2)

Using the distance measure proposed by Son (2016) as composition, patients-diseases relation can be obtained where number of patient N=4 and depicted in table 3-7.

Tab.3. Patients-symptoms Picture Fuzzy Relation for p=1

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.0734	0.0599	0.0838	0.1386	0.1514
B	0.1061	0.1409	0.0878	0.0679	0.1185
C	0.0868	0.0903	0.0829	0.1546	0.1534
D	0.0866	0.0848	0.1032	0.1240	0.1384

Tab.4. Patients-symptoms Picture Fuzzy Relation for p=2

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.0986	0.0850	0.1186	0.1753	0.1911
B	0.1425	0.1837	0.1105	0.0841	0.1542
C	0.1184	0.1369	0.1104	0.1976	0.2061
D	0.1054	0.1042	0.1334	0.1551	0.1686

Tab.5. Patients-symptoms Picture Fuzzy Relation for p=3

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1109	0.1007	0.1355	0.1906	0.2069
B	0.1572	0.2008	0.1195	0.0907	0.1726
C	0.1349	0.1591	0.1266	0.2163	0.2274
D	0.11126	0.111125	0.1458	0.1668	0.1775

Tab.6. Patients-symptoms Picture Fuzzy Relation for p=4

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1195	0.1123	0.1464	0.2005	0.2164
B	0.1654	0.2104	0.1253	0.0953	0.1860
C	0.1459	0.1723	0.1389	0.2276	0.2394
D	0.1145	0.1150	0.1537	0.1734	0.1821

Tab.7. Patients-symptoms Picture Fuzzy Relation for p=5

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1259	0.1209	0.1541	0.2078	0.2228
B	0.1706	0.2167	0.1298	0.0990	0.1964
C	0.1537	0.1810	0.1483	0.2352	0.2471
D	0.1168	0.1175	0.1592	0.1778	0.1851

Tab.8. Patients-symptoms Picture Fuzzy Relation for p=6

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1308	0.1274	0.1598	0.2136	0.2276
B	0.1743	0.2212	0.1334	0.1022	0.2046
C	0.1595	0.1873	0.1555	0.2409	0.2526
D	0.1186	0.1193	0.1635	0.1809	0.1873

Tab.9. Patients-symptoms Picture Fuzzy Relation for p=7

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1347	0.1324	0.1642	0.2182	0.2313
B	0.1771	0.2246	0.1365	0.1049	0.2112
C	0.1640	0.1920	0.1611	0.2453	0.2566
D	0.1200	0.1207	0.1669	0.1833	0.1889

Tab.10. Patients-symptoms Picture Fuzzy Relation for p=8

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1378	0.1363	0.1677	0.2220	0.2342
B	0.1794	0.2273	0.1391	0.1072	0.2165
C	0.1676	0.1958	0.1654	0.2488	0.2597
D	0.1211	0.1217	0.1697	0.1852	0.1901

Tab.11. Patients-symptoms Picture Fuzzy Relation for p=9

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1405	0.1394	0.1705	0.2252	0.2365
B	0.1812	0.2296	0.1413	0.1092	0.2208
C	0.1706	0.1989	0.1689	0.2518	0.2621
D	0.1220	0.1226	0.1720	0.1867	0.1912

Tab.12. Patients-symptoms Picture Fuzzy Relation for p=10

$d(p_i, d_k)$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.1427	0.1420	0.1730	0.2279	0.2385
B	0.1827	0.2315	0.1433	0.1109	0.2244
C	0.1730	0.2015	0.1718	0.2543	0.2642
D	0.1228	0.1232	0.1740	0.1880	0.1920

5. Results and Discussion

Application of IFS is most popular in medical diagnosis among the researches. But, in IFS one significant perception of degree of neutrality is missing which is frequently appears in medical diagnosis. For example, if some is suffering from stomach and chest problems then it is clear the symptoms temperature, headache have no effect on the diseases. That is, these symptoms have neutral effect on the diseases. Similarly, symptoms stomach pain and chest pain have neutral effect on the diseases viral fever, malaria, typhoid etc. On the other hand, symptom temperature has direct effect on malaria. In this case, degree of neutrality can be considered as around zero. So, concept of neutral membership degree is an integral part of medical diagnosis which is missing in IFS. Thus, it motivates us importance of study medical diagnosis via PFS. In this regard, medical diagnosis has been carried out. In this case study we considered *P* = {*A, B, C, D*} be the set of patients, *S* = {*temperature, headache, stomach pain, cough, chest pain*} be the set of symptoms, *D* = {*viral fever, Malaria, typhoid, stomach problem, chest problem*} be the set of diseases. Modified distance measure has been use to obtain patients-diseases relationship and those have been depicted in figure 3-12. As shortest distance between patient and disease indicates that the patient is likely to have the disease. Therefore, from the table 3-12, it can be observed that *A* suffers from Malaria, *B* suffers from stomach problem, *C* suffers from typhoid. But for *D* we obtained two results i.e., in table 3-5, it is seen that *D* is suffering from malaria while table 6-12 indicate that *D* suffers from viral fever.

Conclusion

Various approaches have been have studied in medical diagnosis. However, one of the important concept i.e., of neutrality membership degree was missing in all the theories and which motivated us to study medical diagnosis using IFS. In this paper, we modified distance measures between PFSs initially proposed by Son (2016) and applied in medical diagnosis. This distance measure made it possible to introduce weights of all symptoms and accordingly patients have been diagnosed directly. As medical diagnosis is not an easy task and our aim is not to replace physicians/doctors but to assist them. In our study, for the patient *D* we obtained two results, so collaboration with physicians and researchers may deal with properly such situations.

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