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Analysis of a Call Center with Impatient Customers and Repairable Server

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Abstract

This article considers a call center with impatient customers, repairable server and Interactive Voice Response Units (IVRU). Calls enter the center whenever a trunk line is available; otherwise it is lost. After completing the instructions at the IVRU, the call may leave the center or be routed to an available agent. While waiting for the agent, calls may abandon the queue once the waiting time beyond their patience. We consider that each call abandons the queue independently of each other while waiting for the agent after a random amount of time. The failure rate of the server in the idle and busy periods are both exponential variables with different parameters. We obtain seven performance measures for the stationary distribution of the system. Numerical examples will illustrate the effects of impatience, repair and some other on design parameters.

Key words

IVRU, impatient, repairable, call center, performance measures.

1. Introduction

As used traditionally, a call center can be defined as a place to deal with a large number of incoming and outgoing calls and provide information services [1]. However, with the rapid development of modern computer technology, CTI technology, multimedia technology and the Internet, call centers have also been given a new extension [2]. The modern call centers can provide manual dialogues, data queries, e-mails, faxes, multimedia services and so on [4].

According to the actual situation of the call centers, we can establish appropriate queuing models to analysis performance measures of different call centers. Several researchers [3, 5] studied a call center with retrial and impatient customers. They obtained the stationary distribution of the call center and some other performance measures, but they did not think about a call center with an Interactive Voice Response Units (IVRU). Typically a call center consists of telephone trunk lines, an automatic call distributor (ACD), an IVR and telephone sales agents. Srinivasan [6] consider a call center with IVRU in 2004. Customers often call a special number provided by a call center. Once a trunk line is free, it can be caught by a customer, which is instructed to select among several options provided by the call center via IVRU. Wang [7] considered a call center with impatient customers in 2008. Each customer abandons the call center independently of each other while waiting for agents after a period of time. Chen [8] further obtained performance measures of a call center with impatient customers and optional feedback. Khudyakov [9] approximated the performance measures of a call center with an IVRU in an asymptotic regime. Brezavšček [10] optimize performance measures of a call center by using the stochastic queuing models.

In the actual operations of a call center, it is necessary to consider the repair of a damaged service station, which is more appropriate to the actual situation. In this paper, a call center with IVRU, impatient calls and repairable server is considered. Based on stochastic queuing models, some important performance indexes of the system in steady-state are given, we also show some numerical tables to analyze the changes for certain parameters. The rest of this paper is organized as follows. In section 2, we present the description of our model. In section 3, we obtain some significant performance measures for the steady state of our system. In Section 4, some numerical tables are given; conclusions and future work are summarized finally.

2. Model Description

In our paper, we consider a call center which is made up of telephone trunk lines, a switching machine known as the automatic call distributor (ACD) together with an Interactive Voice Response Units (IVRU) and a server (or agent). Readers can see our model in Figure 1.

(1) Consider a Markovian model for a call center in which calls arrive according to a Poisson process with constant rate λ. The call center consists of N truck lines and one agent. A call arrives when a trunk line is accessible; it will spend some time at the IVRU before it obtains an available agent. We assume that the IVRU processing times are independent and identically

distributed exponential random variables with rate μ_1. After finishing the IVRU process, the call may leave the system and no longer interferes with the center with probability $1-p$; or it may request further service from the agent with probability p. The first queue is formed by the calls in IVRU processor, the second queue is formed by the calls which will be served by the agent. The service rule for the second queue is first come first service. The agent' service time is independent and identically distributed exponential random variables with rate μ_2. In the waiting process, the patient time of calls is exponential distribution with rate θ, that is, the customer may run out of patience and leave the queue before its service begins. We consider the call center have N truck lines, the waiting calls served by an agent, then there are $N-1$ positions in front of the server.

(2) The server may be broken in idle and busy period. The failure rate of the server in the idle periods and busy periods are both exponential variables with parameters v_1 and $v_2(v_1 < v_2)$ respectively. The repair times are independent and identically distributed exponential random variables with rate η.The server after repair is as good as a new one.

(3) All above random variables in our model are mutually independent.

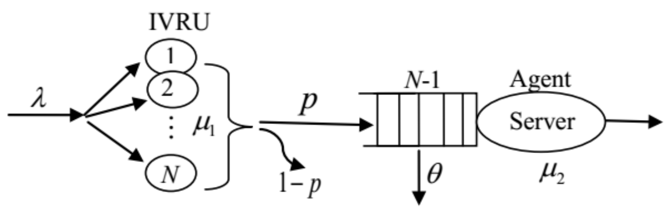

Fig.1. Call Center Model with Impatient Customers and Repairable Server

3. Derivation of the stationary distribution

In order to solve the call center with IVRU, we regard it as a two-stage tandem queuing system. We consider the IVRU service as the first level queue, the IVRU can handle at most N calls at a time; the customers in front of the server (including waiting and being of service) as the second level queue.

Let $M(t)$:= The number of calls in the first level queue at time t (The number of calls in the IVR at time t);

$N(t)$:= The number of customers in the second level queue at time t (The number of customers requiring agency service at time t);

$J(t)$:= The state of the server at time t， where $J(t)=0$ means the server is normal (including the server is busy or free), $J(t)=1$ represents the server is broken.

Because all the random variables are exponentially distributed and independent of each other, the state of our system is finite, then $\{M(t), N(t), J(t): t \geq 0\}$ is an irreducible finite state Markov chain. The state space S is defined as $S=\{(m,n,j) \mid 0 \leq m,n \leq N, 0 \leq m+n \leq N, j=0,1\}$.

Because the model considered in this paper is a finite state Markov chain, the stationary distribution of our system is exist. We can obtain the stationary distribution of the system by Matrix-geometric method. For more details of this method, reader can refer to the literature [6, 7 and 11].

Let $\pi_{(m,n,j)}$:= a steady state probability when the system has m calls in the first-level queue and n customers in the second-level queue， the server in state j $(0 \leq m,n \leq N, 0 \leq m+n \leq N,$ $j=0,1)$. We further define:

$$\pi_0 = (\pi_{(0,0,0)}, \pi_{(0,0,1)}, \pi_{(0,1,0)}, \pi_{(0,1,1)}, \pi_{(0,2,0)}, \pi_{(0,2,1)}, \ldots \pi_{(0,N,0)}, \pi_{(0,N,1)}),$$

$$\pi_i = (\pi_{(i,0,0)}, \pi_{(i,0,1)}, \pi_{(i,1,0)}, \pi_{(i,1,1)}, \ldots \pi_{(i,N-i,0)}, \pi_{(i,N-i,1)}); i=1,2,\ldots N,$$

$$\pi = (\pi_0, \pi_1, \pi_2, \ldots \pi_N).$$

The system satisfies the following equation:

$$(\pi_0, \pi_1, \pi_2, \ldots \pi_N)Q = 0, \tag{1}$$

$$\sum_{m=0}^{N}\sum_{n=0}^{N-m} \pi_{(m,n,0)} + \sum_{m=0}^{N}\sum_{n=0}^{N-m} \pi_{(m,n,1)} = 1, \tag{2}$$

where Q matrix in the above equation is the infinitesimal generator as follow.

$$Q = \begin{pmatrix} A_0 & C_0 & & & & & \\ B_1 & A_1 & C_1 & & & & \\ & \ddots & \ddots & \ddots & & & \\ & & B_i & A_i & C_i & & \\ & & & \ddots & \ddots & \ddots & \\ & & & & B_{N-1} & A_{N-1} & C_{N-1} \\ & & & & & B_N & A_N \end{pmatrix}.$$

The Q matrix above is a block tridiagonal matrix, which is based on the first coordinate horizontal transfer of the steady state probability vector. A_i $(i=0,1,\ldots,N)$ is a lower block tridiagonal matrix; B_i $(i=1,2,\ldots,N)$ is a upper block tridiagonal matrix; C_i $(i=0,1,\ldots,N)$ is a diagonal matrix. The details of elements in Q are expressed below.

$$A_i = \begin{pmatrix} A_{00}^i & & & & \\ A_{10}^i & A_{11}^i & & & \\ & A_{21}^i & A_{22}^i & & \\ & & \ddots & \ddots & \\ & & & A_{N-iN-i-1}^i & A_{N-iN-i}^i \end{pmatrix} \qquad i = 0,1,\ldots,N \,.$$

$$A_{kk}^i = \begin{cases} \begin{pmatrix} -i\mu_1 - v - (k \wedge 1)\mu_2 - [(k-1) \vee 0]\theta & v \\ \eta & -i\mu_1 - k\theta - \eta \end{pmatrix}, & i+k = N \\ \begin{pmatrix} -i\mu_1 - v - (k \wedge 1)\mu_2 - [(k-1) \vee 0]\theta - \lambda & v \\ \eta & -i\mu_1 - k\theta - \lambda - \eta \end{pmatrix}, & i+k \neq N \end{cases}$$

$$v = \begin{cases} v_1, & k = 0 \\ v_2, & k \geq 1 \end{cases}; k \wedge 1 = \min(k,1); (k-1) \vee 0 = \max(k-1,0).$$

$$i = 0,1,\ldots N; k = 0,1,\ldots N-i.$$

$$A_{kk-1}^i = \begin{pmatrix} \mu_2 + (k-1)\theta & 0 \\ 0 & k\theta \end{pmatrix}, \qquad i = 0,1,\ldots N; k = 1,2,\ldots N-i;$$

$$B_i = \begin{pmatrix} B_{00}^i & B_{01}^i & & & \\ & B_{11}^i & B_{12}^i & & \\ & & \ddots & \ddots & \\ & & & B_{N-iN-i}^i & B_{N-iN-i+1}^i \end{pmatrix}, \quad i = 1,2,\ldots N$$

$$B_{kk}^i = \begin{pmatrix} i\mu_1(1-p) & 0 \\ 0 & i\mu_1(1-p) \end{pmatrix}, \; i = 1,2,\ldots N, \; k = 0,1,\ldots N-i$$

$$B_{kk+1}^i = \begin{pmatrix} i\mu_1 p & 0 \\ 0 & i\mu_1 p \end{pmatrix}, \; i = 1,2,\ldots N, \; k = 0,1,\ldots N-i,$$

$$C_i = \begin{pmatrix} C_{00}^i & & & & & \\ & C_{11}^i & & & & \\ & & C_{22}^i & & & \\ & & & \ddots & & \\ & & & & C_{N-i-1\,N-i-1}^i & \\ & & & & & 0_2 \end{pmatrix}, i = 0,1,\ldots,N-1.$$

$$C_{kk}^i = \lambda E_2, 0_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, i = 0,1,\ldots N-1; k = 0,1,\ldots N-i-1.$$

Taking into account the parameters in our model are more complex that we need to simplify them and reduce the running time. By using the Structured Gaussian Elimination method [12], we can obtain the stationary distribution probability of the system by solving equations (1) and (2), and get the performance measures of the system below.

1) The idle probability of the system: $P_0 = \pi_{(0,0,0)}$.

2) The idle probability of all telephone trunk lines: $P_{IL} = \pi_{(0,0,0)} + \pi_{(0,0,1)}$.

3) The busy probability of all telephone trunk lines:

$$P_{BL} = \sum_{m+n=N} \sum_{j=0}^{1} \pi(m,n,j).$$

4）The partial idle probability of telephone trunk lines: $P_{PI} = 1 - \pi_{(0,0,0)} - \pi_{(0,0,1)} - P_B$.

5）The repair probability of the server: $P_R = \sum_{n=0}^{N} \sum_{m=0}^{N-n} \pi_{(m,n,1)}$.

6）The leaving probability in the second-level queue:

$$P_{LS} = \sum_{n=2}^{N} \sum_{m=0}^{N-n} \frac{(n-1)\theta}{\mu_2 + (n-1)\theta} \pi_{(m,n,0)} + \sum_{n=1}^{N} \sum_{m=0}^{N-n} \frac{n\theta}{\eta + n\theta} \pi_{(m,n,1)}.$$

7）The number of waiting customers in the second queue:

$$E(N_{Agent}) = \sum_{n=2}^{N} \sum_{m=0}^{N-n} (n-1)\pi_{(m,n,0)} + \sum_{n=1}^{N} \sum_{m=0}^{N-n} n\pi_{(m,n,1)}.$$

4. Numerical examples

In this section, we will investigate the impact of various parameters on the performance measures that have already been obtained in Section 3. More specifically, we mainly demonstrate how the arrival rate λ, the impatient rate θ, the failure rate v_2 in busy period and the repair rate η affect the performance measures of the system by four numerical examples below.

Case 1: We assume $\mu_1^{-1} = 1/2\,\text{min}$, $\mu_2^{-1} = 1\text{min}$, $\eta^{-1} = 1/2\,\text{min}$, $\theta^{-1} = 1/3\,\text{min}$, $v_1^{-1} = 10\,\text{min}$, $v_2^{-1} = 5\,\text{min}$, $p = 0.5$, $N = 3$. We study the effect of the arrival rate to seven performance measures of the system. The numerical values are shown in Table 1.

Table 1. The influence of arrival rate on seven performance measures

λ	P_0	P_{IL}	P_{BL}	P_{PI}	P_R	P_{LS}	$E(N_{Agent})$
1	0.37862	0.40577	0.04405	0.55018	0.06258	0.03614	0.05701
4	0.04951	0.05568	0.37980	0.56452	0.07656	0.13806	0.23490
8	0.00978	0.01141	0.61533	0.37326	0.08017	0.18488	0.32718
10	0.00548	0.00646	0.67893	0.31461	0.08085	0.19572	0.34999

From table 1, we see that with the increasing of the arrival rate λ, the probability of the service station in repair P_R, the waiting customers of second queue $E(N_{Agent})$ and the busy probability of all telephone trunk lines P_{BL} are increasing, but the idle probability P_0 is decreasing. These phenomena are very much in line with our expectations.

Case 2: We assume $\mu_1^{-1} = 1/2\,\text{min}$, $\mu_2^{-1} = 1\,\text{min}$, $\eta^{-1} = 1/2\,\text{min}$, $\theta^{-1} = 1/3\,\text{min}$, $v_1^{-1} = 10\,\text{min}$, $v_2^{-1} = 5\,\text{min}$, $p = 0.5$, $N = 3$. We get the effect of the patience rate to seven performance measures of the system. The numerical values are shown in Table 2.

Table 2. The influence of patience parameter on seven performance measures

θ	P_0	P_{IL}	P_{BL}	P_{PI}	P_R	P_{LS}	$E(N_{Agent})$
1	0.01429	0.01493	0.61727	0.34389	0.08050	0.15600	0.56812
2	0.01630	0.01757	0.58111	0.40132	0.07896	0.17086	0.42848
6	0.01962	0.02251	0.53027	0.44722	0.07641	0.13110	0.22093
10	0.02084	0.02446	0.51376	0.46177	0.07547	0.09916	0.14953

From table 2, with the increasing of the patience parameter θ, we find that P_0 is increasing, while P_R, $E(N_{Agent})$ and P_{BL} are all decreasing. The leaving probability in the second-level queue P_{LS} is increase at first, then decrease, this is due to the increase of impatience parameter, the customer join the system is reduced, resulting in less waiting time, further attract more customers to the system.

Case 3: We assume $\mu_1^{-1} = 1/3\,\text{min}$, $\mu_2^{-1} = 1\,\text{min}$, $\theta^{-1} = 1/2\,\text{min}$, $\lambda^{-1} = 1/6\,\text{min}$, $v_1^{-1} = 10\,\text{min}$, $v_2^{-1} = 5\,\text{min}$, $p = 0.4$, $N = 3$. We get the effect of the repair rate to seven performance measures of the system. The numerical values are shown in Table 3.

Table 3. The influence of repair rate on seven performance measures

η	P_0	P_{IL}	P_{BL}	P_{PI}	P_R	P_{LS}	$E(N_{Agent})$
0.1	0.01555	0.05775	0.39308	0.54917	0.63346	0.41678	0.80168
6	0.03982	0.04097	0.43309	0.52595	0.02818	0.18170	0.33145
10	0.03999	0.04060	0.43399	0.52541	0.01712	0.18086	0.32129
20	0.04004	0.04030	0.43473	0.52498	0.00865	0.18205	0.31304

With the increase of the parameter η, i.e., when the service repair time is shorter, all of the lines P_0 and the probability of the system idle probability of all busy P_{BL} increases, the repair probability P_R and waiting customers in the second queue $E(N_{Agent})$ are decreasing, the leaving probability in the second-level queue P_{LS} decreases first and then increases slightly.

Case 4: We assume $\mu_1^{-1} = 1/3\,\text{min}$, $\mu_2^{-1} = 1/2\,\text{min}$, $\eta_2^{-1} = 1/2\,\text{min}$, $\theta^{-1} = 1/3\,\text{min}$, $\lambda^{-1} = 0.1\,\text{min}$, $v_1^{-1} = \infty$, $p = 0.5$, $N = 3$. We obtain the effect of parameter v_2 to seven performance measures of the system. The numerical values are shown in Table 4.

Table 4. The influence of parameter v_2 to seven performance measures

v_2	P_0	P_{IL}	P_{BL}	P_{PI}	P_R	P_{LS}	$E(N_{Agent})$
0.1	0.01835	0.01908	0.56198	0.41894	0.03350	0.15337	0.30063
0.4	0.01755	0.01956	0.55960	0.42085	0.09307	0.16958	0.36856
0.8	0.01652	0.02002	0.55726	0.42272	0.15658	0.18569	0.43598
1	0.01448	0.02139	0.55042	0.42819	0.32235	0.23230	0.63134

From table 4, we see that with the increasing of the parameter v_2, the idle probability P_0 and the busy probability P_{BL} are decreasing; the repair probability P_R, the waiting customers of second queue $E(N_{Agent})$ and the leaving probability in the second-level queue P_{LS} are all increase.

Conclusion

In this paper, a call center with impatient customers, repairable server and the role of Interacting Voice Response Units (IVRU) mechanism is investigated. By using the Structured Gaussian Elimination method, we obtain the idle probability, the busy probability, the repair probability and some other performance measures for the stationary distribution of the system. It is found that customers' impatience can drastically decrease the waiting customers in the second queue.

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