

## **Different Social Security Systems and Their Impact on Savings Rates and Economic Growth**

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### **Abstract**

This paper compared with different social security system, Pay-as-you-go (PAYG) and Funded pension system, and they have different degrees of impact on savings rates and economic growth in an overlapping generations model by using discrete optimization method and the Bellman equation, solving the value function makes it possible to obtain its optimal solution.

### **Key words**

Funded pension system; PAYG pension System; OLG model; economic growth

### **1. Introduction**

Social security system can be divided into two modes: one is Pay-as-you-go model (PAYG), the other is the funded system (Funded), these two modes have different effects on savings rates and economic growth. Some scholars believe that pay-as-you-go system will have a negative

impact on savings and economic growth, while the Funded system can offset the negative factors of this influence (Cremer et al., 2002; P. A. Diamond, 1965; M. Feldstein, 1976; Willis, 1989). The Funded system will reduce fertility, making household for human capital investment increased, but the savings rate has no effect (Oshio, 2008; Zhang, 1995).

The savings rate and economic growth in the short and long-term economic performance are an extremely two indicators. High savings rate is meant by an increase in capital stock and production efficiency, also advances the economic growth rate. Economic growth will promote technological progress, and the economic balance growth rate will move to a higher level. (Martin Feldstein, 2011) point out that, social security is a critical factor in the impact of private savings, and social security will also affect the accumulation of material capital, human capital accumulation and technological progress, which will affect economic growth.

Not only the different social security system affects the savings rate and economic growth, the inclusion of different parameters in the model, including the distribution of pensions, the degree of human capital investment, the existence of gifts (Bequests) and whether there is altruism will affect the social security system and the savings rate, and bring different economic growth rate. This article mainly studies the impact of PAYG and Funded pension system and the two different social security systems on the savings rate and economic growth, and analyzes the impact of PAYG and Funded on different equilibrium conditions.

## 2. A Simple Model

Within the framework of an overlapping generations model, there is an obvious relationship between social security system and save rates. We assume that in a closed economy, and the individual life is divided into three phases in the model: First period is the juvenile period  $t - 1$  from birth to work, which is mainly get education from their parents and school; The second period  $t$  is from the middle of the year work to retired, including income, consumption and savings; and the third period is the old age  $t + 1$  retirement until death, and the source of income is mainly dependent on pension and family support. All decisions are made in the  $t$

period, the economy starts for  $t = 0$ , also reflects the historical economic situation. The utility function  $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ , where  $\sigma > 0$  is a monotonously increasing function and CRRA strict concave function.

## 2.1 the consumption function

The utility discount brought by the child is given by  $a(n_t)^{-\epsilon}$ , where  $n_t$  represents the number of children,  $0 < \epsilon < 1$  is a constant parameter,  $a > 0$ ,  $n_t$  for the number of children, the number of children is free choice of parent. In the accumulation of human capital,  $H_t$  is the middle-aged human capital, this period assumes that the work of earning income, the second is the time cost into the raising of children, with  $e_t$  that children need to spend, so that we can get the income it receives in the period  $t$  labo:

$$IL_t = w_t H_t (1 - n_t e_t) (1 - \tau_t) \quad (1)$$

In Eq. (1),  $w_t$  is the wage income,  $\tau_t$  is the government tax, but also the tax levied by the old-age insurance, and its income is mainly used for consumption and savings. As (Cerda, 2005) assume that the personal value function  $V_t$  includes the middle-aged consumption, the consumption of the old age, and the value function of raising children, so that the utility function of the middle age period  $t$  of the representative family can be obtained

$$V_t(b_t, H_t) = \max u_c(c_t^y) + \beta u_c(c_{t+1}^o) + \beta a n_t^{1-\epsilon} V_{t+1}(b_{t+1}, H_{t+1}) \quad (2)$$

s. t.

$$c_t^y = b_t + w_t H_t (1 - n_t e_t) (1 - \tau_t) - s_t \quad (3)$$

$$c_{t+1}^o = (1 + r_{t+1}) s_t + \vartheta w_t H_t (1 - n_t e_t) \tau_t - n_t b_{t+1} \quad (4)$$

$$H_{t+1} = A e_t H_t \quad (5)$$

where  $V_t(b_t, H_t)$  is the middle age value function and influenced by the gift given by the parent of the previous period and affected by the accumulation of human capital. Eq. (3) is the consumption of the young people in the  $t$  period, when the government tax  $0 < \tau_t < 1$ , the cost of raising children  $0 < n_t e_t < 1$ . Eq. (4) is the consumption of the old person in the  $t+1$  period, here  $(1 + r_{t+1})$  should be greater than the pension yield  $\theta$ . Eq. (5) represents the accumulation of next-generation human capital, which is related to capital and labor output, which is related to educational input and the level of parental human resources.

The envelope theorem and derivation Eq. (2) shows that the first order condition (FOC) gives

$$n_t \beta u_c(c_{t+1}^o) = \beta a n_t^{1-\epsilon} \frac{\partial V_{t+1}}{\partial b_{t+1}} \quad (6)$$

$$w_t H_t n_t \left[ u_c(c_t^y) \left( 1 - \tau_t \left( 1 - \frac{\theta}{1+r_{t+1}} \right) \right) \right] = A \beta a n_t^{1-\epsilon} \frac{\partial V_{t+1}}{\partial H_{t+1}} H_t \quad (7)$$

$$w_t H_t e_t \left[ u_c(c_t^y) \left( 1 - \tau_t \left( 1 - \frac{\theta}{1+r_{t+1}} \right) \right) \right] = A a (1 - \epsilon) n_t^{-\epsilon} V_{t+1}$$

(8)

$$\frac{\partial V}{\partial H_t} = w_t H_t (1 - n_t e_t) \left[ u_c(c_t^y) \left( 1 - \tau_t \left( 1 - \frac{\theta}{1+r_{t+1}} \right) \right) \right] \quad (9)$$

$$\frac{\partial V}{\partial b_t} = u_c(c_t^y)$$

(10)

Eq. (6) means the marginal cost of the old gift, also understood as the marginal utility loss of the elderly to reduce consumption, the right is to reduce the marginal benefits of child gifts. The right of Eq. (7) and Eq. (8) represent the marginal benefit of raising a child, putting into human capital accumulation and the number of children's choice. For the value function  $V_t(b_t, H_t)$ ,  $H_{t+1}$  and  $n_t$  First-order derivative, the left is the cost of raising children and education investment, in Sinn (2000) article has a more detailed explanation and derivation process. Eq. (9)

represents the marginal cost of investing in its own human capital. Eq. (10) shows the marginal cost of the gift in the t period, as the envelope condition.

## 2.2 Production function

In this paper, we choose the more commonly used C-D function as production function (see, e.g., Weil (1989) and Blanchard (1985), the model is gives

$$Y_t = AK_t^a \tilde{L}_t^{1-a} \cdot \left(\frac{F}{L_t}\right)^\mu \quad (11)$$

where A represents the technical level factor, K is the capital element, a is the efficiency of the use of capital or labor,  $\tilde{L} = L_t e^{xt}$  is the effective labor input, manifested as the change of labor input factor at time t, X for the impact of this total change. If delayed retirement is a new element into the production function, this element is denoted by R, and  $(F/L_t)^\mu$  means that the input of the new factor due to the input of the new element Increase,  $\mu$  means the rate at which this factor increases.  $m(t)$  that the policy changes brought about by the rate of labor supply.

Then the total population of the labor supply at time t gives

$$L(t) = L(0) \cdot e^{nt} \cdot \exp \int_0^t m(v) dv \quad (12)$$

where n is the natural growth rate of labor.  $\exp \int_0^t m(v) dv$  is the rate of change in the number of labor caused by the change of family planning policy from 0 to t period. L (0) for the number of labor without delay before the supply of labor. This is an increase in the supply of labor in the labor market, but in the case of equilibrium labor supply is L (0).

The price of the element is equal to its marginal output, so that the wage rate under its optimal condition can be obtained

$$\omega = (1 - a) \cdot A \hat{k}^a \cdot \left[\left(\frac{F}{L_t}\right)^\mu\right] e^{xt} \quad (13)$$

where  $\hat{k}$  represents the ratio of capital to labor, it can be said that the changes in the per capita capital stock, we can also find new capital density gives

$$\hat{k} = \left[ \frac{\alpha A \cdot (F/L_t)^\mu}{r + \delta} \right]^{1/(1-\alpha)} \quad (14)$$

where  $r + \delta$  is the cost of capital.

## 2.3 Government and Market

The existence of the government has an intermediary role that will levy taxes on labor, as a social security payments and transfers of payments, in the current payment system (PAYG) of the social security system, the government received the current funds for the current Pension payment, the income and expenditure formula can be expressed as

$$P_t n_{t-1} = \tau_t w_t n_t \quad (15)$$

it is t-1 period person who we pay for salary tax, the t period pays for the current period. (see, e.g., P. Diamond & Geanakoplos, 2003), the PAYG system can also be expressed as:

$$T_t + n\tau_t = r_{t-1}G \quad (16)$$

where t-1 period need to pay  $r_{t-1}$  interest, and the purpose of the government is to ensure the balance of the budget. In the market, assume that the period t savings in the future performance of t + 1 period of capital stock

$$s_t = (1 + n)k_{t+1} \quad (17)$$

## 3. Equilibrium solution of model

We solve the PAYG social security system under the equilibrium solution, consider factors such as intertemporal utility function, budget constraint and government tax, followed by a Funded of social security system, and the result of two systems were compared.

### 3.1 Pay-as-you-go social security (PAYG)

The government's income budget condition gives

$$T_t + n\tau_t = r_{t-1}G \quad (18)$$

as shown above Eq. (18) , The left is the government's current social security income, which is related to the labor force and social security rates, On the right is the current government expenditure, is paid to the previous period  $t - 1$  to pay the pension of the residents.

Young people and the elderly will be subject to budget constraints, its Agents budget constraints as follows

$$c_t^y = b_t + w_t H_t (1 - n_t e_t) (1 - \tau_t) - s_t - \tau_t w_t n_t \quad (19)$$

$$c_{t+1}^o = (1 + r_{t+1})s_t + \vartheta w_t H_t (1 - n_t e_t) \tau_t - n_t b_{t+1} \quad (20)$$

the budget for young people is that their spending is equal to the previous generation of gifts  $b_t$ , plus part of their wages  $w_t$ , except for the portion of human capital investment  $H_t$ , minus the income and taxes of their dependent offenders, minus the portion of their savings, and finally Out of the pension section  $\tau_t w_t n_t$ . The consumption of the elderly  $c_{t+1}^o$ , including its savings  $(1 + r_{t+1})s_t$  at a young age, plus the pension it receives  $\vartheta w_t H_t (1 - n_t e_t) \tau_t$ , less its gift to the next generation  $n_t b_{t+1}$ .

Then we can get the value function, the problem

$$V_t(b_t, H_t) = \max_{b_{t+1}, n_t, H_t, s_t} \left\{ \begin{array}{l} \ln[b_t + w_t H_t (1 - n_t e_t) (1 - \tau_t) - s_t - \tau_t w_t n_t] \\ + \beta \{ \ln[(1 + r_{t+1})s_t + \vartheta w_t H_t (1 - n_t e_t) \tau_t - n_t b_{t+1} + P_t n_{t-1}] \} \\ + \beta a n_t^{1-\epsilon} V_{t+1}(b_{t+1}, H_{t+1}) \end{array} \right\} \quad (21)$$

where  $b_t \geq 0, s_t \geq 0$  and  $n_t \geq 0$ , Eq. (21) for the derivative, we can get the First-Order conditions(FOC)

$$b_{t+1}: \beta n_t + \beta a n_t^{1-\epsilon} \frac{\partial V_{t+1}(b_{t+1}, H_{t+1})}{\partial b_{t+1}} \geq 0,$$

$$\text{then } \beta(1 + n_t) / c_{t+1}^y \geq a / c_{t+1}^o \quad (22)$$

get a simplified Eq. (23) formula, use  $V_t(b_t, H_t)$  derivative savings rate  $s_t$

$$s_t: 1 / c_t^y \geq \beta(1 + r_{t+1}) / c_{t+1}^y \quad (23)$$

using the envelop theorem, we can get:

$$\begin{aligned}
 H_t & : \frac{w_t(1-n_t e_t)(1-\tau_t)}{b_t + w_t H_t(1-n_t e_t)(1-\tau_t) - s_t - \tau_t w_t n_t} + \\
 & \ln[(1+r_{t+1})s_t + \vartheta w_t H_t(1-n_t e_t)\tau_t - n_t b_{t+1} + P_t n_{t-1}] + \\
 & \frac{\beta \vartheta w_t(1-n_t e_t)\tau_t}{(1+r_{t+1})s_t + \vartheta w_t H_t(1-n_t e_t)\tau_t - n_t b_{t+1} + P_t n_{t-1}} \quad (24)
 \end{aligned}$$

differentiating the Eq. (21) with  $n_t$

$$\begin{aligned}
 n_t : & \left[ \frac{w_t v_t(1-\tau_t - s_t)}{c_t^y} + \beta(1+r_{t+1})s_t w_t v_t / c_{t+1}^y \right. \\
 & \left. + \beta b_{t+1} / c_{t+1}^y \right] \geq \beta(1+n_t) \quad (25)
 \end{aligned}$$

Eq. (22) means that in their own consumption and between the gifts maximize the total utility. Eq. (23) shows between consumption and savings seeking to maximize. Eq. (24) and (25) represent when you choose to have children, you need to consume some of the current consumer spending, is always paid, will affect its budget constraints and leisure expenses, but raising children will bring it to the positive utility, in the two between a trade-off.

We can find in the equilibrium conditions to solve the balance growth

$$\gamma_b = \frac{a[1-a(1-\epsilon)]\{a\vartheta(1+\beta)+(1+\vartheta)[a\tau_t-\beta(1-\tau_t)]\}}{(1-\vartheta)[1-a(1-\epsilon)(a+\beta)]} \quad (26)$$

and

$$\gamma_q = \frac{a\beta\vartheta(1-\tau_t)}{1-a(1-\epsilon)(1+n)} \quad (27)$$

Eq.(26)and (27) is the steady-state balanced growth path under equilibrium conditions, this requirement needs the following two conditions: bequests are positive; fertility is positive; In the steady-state growth equilibrium, the  $r$ ,  $s$ ,  $n$ ,  $H$ ,  $Y$  are constant.

When bequests are positive, the saving rete equal to



$$s_t = a\vartheta / (1 - \vartheta)$$

(28)

and endogenous population growth is equal to

$$1 + n = \frac{x}{v_t(1 - \tau_p)(1 - \epsilon) + x}$$

(29)

labor productivity  $e$

$$e = \frac{s[D(1 - \epsilon)]^{1 - \vartheta}}{(1 + n)A\{\gamma_q[1 - \epsilon(1 + n)]\}^{\vartheta}} \quad (30)$$

similarly, we can also find, in the steady-state balanced growth rate under the condition of PAYG pension system

$$1 + g = \left[ \left( A\{\gamma_q[1 - \epsilon(1 + n)]\}^{\vartheta} \right)^{1 - \epsilon} [(1 - \epsilon)D]^{\epsilon} \left( \frac{s}{1 + n} \right)^{\epsilon\vartheta} \right]^{\frac{1}{1 - \vartheta(1 - \epsilon)}}$$

(31)

From the above, we can draw the conclusion that due to the increase in the rate of occurrence and per capita capital investment, household income will increase, growth rate also will be a certain degree of increase. But under unfunded social security with operative bequests, the growth rate of per capita income may or may not increase with high level.

### 3.2 Funded Social security

In the case of a fund system, residents need to save part of their own pension, which can be invested in the capital market, but it will also have other investment in the capital market caused by a certain squeeze effect, will also make the residents of the income budget curve of a certain change, affecting the purchase of people's purchasing power.

When bequests are positive

$$s_t = a\vartheta / (1 - \vartheta) - \tau_t \quad (32)$$

we can get the save rate, compared with the PAYG social security system, we can see that there will be a higher savings rate. Also in the absence of a bequests, the resulting savings rate. When bequests are zero

$$s = \frac{\beta\{1 - a[1 - \vartheta(1 - \epsilon)]\}}{[1 - a(1 - \epsilon)](1 + \beta) - \tau_t} \quad (33)$$

we can see that the Funded social security system has a higher savings rate, the reasons include that some people want to keep a better life, so they need to save more, and this social security system is not a universal social security system, for some people who do not have old-age insurance, may choose to save more, after retirement save is their only income sources.

Solved the following indicators to compare:

$$\gamma_q = \frac{a\epsilon\vartheta((1-\tau_t)(1+\beta))}{[1-a(1-\epsilon)+a\epsilon\vartheta(1+n)]} \quad (34)$$

And

$$\begin{aligned} \gamma_b = & \frac{\{a\vartheta(1 + \beta) + (1 + \vartheta)[a\tau_t - \beta(1 - \tau_t)]\}}{(1 - \vartheta)[1 - a(1 - \epsilon)(a + \beta)]} \\ & + \frac{a\beta\vartheta((1 - \tau_t)(1 - \epsilon))}{\vartheta[1 - a(1 - \epsilon)(a + \beta)]} \end{aligned} \quad (35)$$

Also

$$1 + n = \frac{x}{v_t[1 - a(1 - \tau_t)](1 - \epsilon) + x} \quad (36)$$

in the Funded pension system, the steady-state balanced growth rate higher than the PAYG pension system: The main reason for this is that not only the savings rate is higher in the funded pension system, but also the capital investment per capita is higher, and the population growth rate is relatively lower, but this can be compensated by other fertility policies, but also can enhance the unit productivity, making economic growth faster.

#### 4. Conclusion

Through the model comparison we can get, the pay-as-you-go system is often associated with lower rates of economic growth, because relative to the funded pension system, its fertility

rate is often higher, and the unit of human capital investment is often less, relatively speaking, people work less hard than PAYG and work less time. But PAYG's social security system will bring people higher utility and satisfaction with life will be improved, and has some of the functions of income redistribution.

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