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The Mathematical Forecast Model of Shandong GDP and the Analysis of its Influencing Factors

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Abstract

Regional GDP is related with many factors. In this paper, using grey forecast model — GM(1,1)forecast model, taking the GDP data of Shandong from 2006 to 2014 for example, GDP from 2015 to 2018 is predicted. At the same time, the GDP influencing factors is carried on the correlation analysis, the correlation degree of various factors to the development of Shandong GDP is calculated, which provides a scientific theory basis for the economic development of Shandong province.

Key words

GDP; GM (1, 1); forecast; correlation degree

1. Introduction

Gross domestic product (GDP) is the core index of the national or regional economic accounting, which is also an important index of the national or regional economic development situation. The growth of GDP indicates that economic development is increasing, and resident consumption is strong. The decrease of GDP indicates that economic development appears recession, and resident consumption is weak. According to GDP situation to adjust strategy, it could stimulate the economic development. Therefore, it is very important to forecast GDP accurately.

Since entering the 21st century, Shandong GDP continues to grow, always keeping ahead. In 2014, it ranked third, following by Guangdong and Jiangsu. Shandong is an economic and population big province in China. The development of its GDP is very important to the economic development of Shandong and China. Predicting Shandong GDP accurately could help to grasp timely the economic development of Shandong, adjust economic policies properly, and guarantee the healthy and stable development of economy.

There are many researches on GDP, and more than 107925 articles about GDP as the key words in how-net inquiry, 291 articles about GDP forecast as the key words, 21 articles about GDP influencing factors. There are relatively a few articles about GDP forecast and GDP influencing factors. Wang Jian has carried on the short-term forecast of GDP using random time series analysis method. Liu Ying and Zhihui Zhang have established the Beukes-Jenkins model by using time series to forecast GDP. Tengger and He Yue combined ARIMA model with ARCH model to forecast GDP using GMDH method. Jiejie Wan thought that China's economic growth has a long-term equilibrium relationship with export, consumption and the total energy consumption. Huijie Song thought gross domestic product mainly depends on five factors such as fiscal expenditure, urban and rural end-of-year savings deposit balance, total export-import volume, prior period GDP and resident consumption. Yuchen Yao analyzed and concluded that the growth of GDP in China was related to total energy consumption and resident consumption level.

Time series analysis method is mainly used in the study of GDP forecast. In this research, GM (1, 1) model is chosen to forecast Shandong GDP, and error analysis is also carried on, which is unique, novel and innovative. At the same time, with the aid of previous researches, the main factors influencing Shandong GDP are determined, using its data to carry on the correlation analysis, which is different from the researches that use cointegration relationship and correlation coefficient. The results are similar. And it could be used for reference.

After checking through Shandong statistic network and the 2014 statistical yearbook of Shandong, collecting the data of Shandong GDP from 2006 to 2013 and its main influencing factors, the forecast GM (1, 1) model of Shandong is set up. GDP in the several future years is predicted and analyzed. And grey correlation analysis method is used to calculate the correlation degree of the development of Shandong province and its main influencing factors, analyzing their relationship.

2. The brief review of GM (1,1) model

2.1 The establishment of GM (1, 1) model

GM (1, 1) model is a common model in mathematics, and is also a differential equation of first order model of a variable. The theory is that processing the discrete data to generate the data of which the random is significantly weakened but has certain regularity, differential equation model is then established, so as to make quantitative predictions for the future.

Accumulate the discrete data sequence $x^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$ composed by n elements, generate new data sequence $x^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\}$ .And $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, $k=1,2,\cdots,n$, the grey derivative of data sequence $x^{(1)}$ can be defined as follows:

$$dx^{(1)}(k)=x^{(0)}(k)=x^{(1)}(k)-x^{(1)}(k-1).$$

The mean value data sequence of data sequence $x^{(1)}$ is: $z^{(1)}=\{z^{(1)}(2),z^{(1)}(3),\cdots,z^{(1)}(n)\}$ .And

$$z^{(1)}(k)=\frac{1}{2}x^{(1)}(k)+\frac{1}{2}x^{(1)}(k-1)\,,\ k=2,\cdots,n.$$

So, the grey differential equation of GM (1, 1) can be written as:

$dx^{(1)}(k)+az^{(1)}(k)=b$, or $x^{(0)}(k)+az^{(1)}(k)=b$,

$x^{(0)}(k)$ is called the grey derivative, a is evolution parameter, $z^{(1)}(k)$ is white background value, b is the grey effect amount.

Substitute the data sequence, and sort out:

$$\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \cdots & \cdots\\ -z^{(1)}(n) & 1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \cdots\\ x^{(0)}(n)\end{bmatrix},$$

And the matrix is as follows:

$$A=\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \cdots & \cdots\\ -z^{(1)}(n) & 1\end{bmatrix},u=\begin{bmatrix}a\\ b\end{bmatrix},B=\begin{bmatrix}x^{(0)}(2)\\ x^{(0)}(3)\\ \cdots\\ x^{(0)}(n)\end{bmatrix},$$

The GM (1, 1) model can be written as: $Au=B$.

The least square method is used to calculate parameter vector.

$$u\therefore u=\begin{bmatrix}a\\ b\end{bmatrix}=(A^{T}A)^{-1}A^{T}B\,,$$

And then substitute back, the time response equation could be obtained.

$$\hat{x}^{(1)}(k+1)=[x^{(1)}(1)-\frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k}+\frac{\hat{b}}{\hat{a}}\,,$$

Restore to the original data sequence, and we could get:

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)=(1-e^{-\hat{a}})[x^{(1)}(1)-\frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k}$$

(1)

According to the formula (1), we could predict the future value of the variable.

2.2 The inspection of GM(1,1)model

The original sequence data is $x^{(0)}=\left\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right\}$,

The forecast data sequence of GM (1, 1) model

$\hat{x}^{(0)}=\left\{\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n)\right\}$, Through calculating, we could get:

①Absolute error data sequence is:

$\left\{\hat{x}^{(0)}(1)-x^{(0)}(1),\hat{x}^{(0)}(2)-x^{(0)}(2),\cdots,\hat{x}^{(0)}(n)-x^{(0)}(n)\right\}$;

②Residual data sequence is:

$\{\varepsilon(1),\varepsilon(2),\cdots,\varepsilon(n)\}=\left\{x^{(0)}(1)-\hat{x}^{(0)}(1),x^{(0)}(2)-\hat{x}^{(0)}(2),\cdots,x^{(0)}(n)-\hat{x}^{(0)}(n)\right\}$;

③Relative error data sequence is:

$\left\{\frac{\hat{x}^{(0)}(1)-x^{(0)}(1)}{x^{(0)}(1)},\frac{\hat{x}^{(0)}(2)-x^{(0)}(2)}{x^{(0)}(2)},\cdots,\frac{\hat{x}^{(0)}(n)-x^{(0)}(n)}{x^{(0)}(n)}\right\}$;

④The standard deviation of original data sequence is:

$S_1=\sqrt{\frac{1}{n}\sum_{k=1}^{n}(x^{(0)}(k)-\bar{x})^2}$, And, $\bar{x}=\frac{1}{n}\sum_{k=1}^{n}x^{(0)}(k)$;

⑤The standard deviation of absolute error data sequence is:

$S_2=\sqrt{\frac{1}{n}\sum_{k=1}^{n}(\varepsilon(k)-\bar{\varepsilon})^2}$, And, $\bar{\varepsilon}=\frac{1}{n}\sum_{k=1}^{n}\varepsilon(k)$;

⑥The standard deviation ratio of original data sequence and absolute error data sequence is:

$c=\frac{S_2}{S_1}$;for the given number c_0 (the specific details are shown in table 1), when $c<c_0$,this model is called mean square deviation qualified model.

⑦Small error probability $P=P(\left|\varepsilon(k)-\bar{\varepsilon}\right|<0.6745S_1)$; for the given number P_0 (the specific details are shown in table 1), when $P>P_0$, this model is called small error probability qualified model.

The inspection level standards of GM (1, 1) model precision are shown in table 1.

Table 1 The inspection level reference table of GM (1, 1) model precision

Precision grade	relative error	The standard deviation ratio c_0	Small error probability P_0	Corresponding correlation degree ρ
Level 1 - very good	0.01	< 0.35	> 0.95	0.90
Level 2 - qualified	0.05	< 0.50	> 0.80	0.80
Level 3 - reluctant	0.10	< 0.65	> 0.70	0.70
Level 4 - unqualified	0.20	≥ 0.65	≤ 0.70	0.60

3. The GM (1, 1) model of Shandong GDP

Through checking Shandong statistic network and 2014 statistical yearbook of Shandong, the total data of Shandong GDP from 2006 to 2013 is sorted out to show in table 2.

Table 2 Shandong GDP from 2006 to 2013 (unit: one hundred million yuan)

year	2006	2007	2008	2009
GDP	21900.19	25776.91	30933.28	33896.65
year	2010	2011	2012	2013
GDP	39169.92	45361.85	50013.24	54684.33

The scatter diagram is shown in diagram 1.

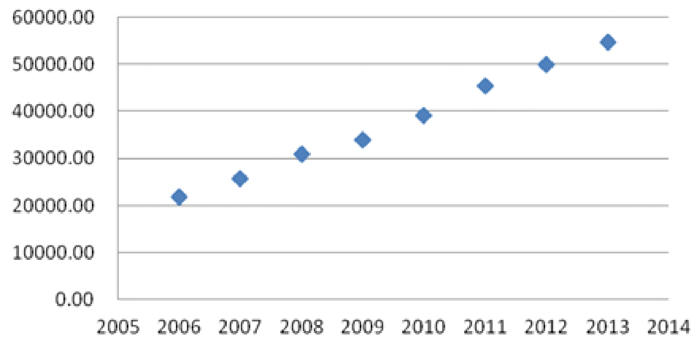

Diagram 1: the total scatter diagram of Shandong GDP from 2006 to 2013

3.1 The establishment of Shandong GDP GM (1, 1) model

3.1.1 Through the total data of Shandong GDP from 2006 to 2013, the original data sequence could be written as follows:

$x^{(0)} = \{21900.19, 25776.91, 30933.28, 33896.65, 39169.92, 45361.85, 50013.24, 54684.33\}$ After accumulation, generation data sequence could be obtained.

$x^{(1)} = \{21900.19, 47677.10, 78610.38, 112507.03, 151676.95, 197038.80, 247052.04, 301736.37\}$

3.1.2 The calculation of time response equation

Input the data into EXCEL and SPSS, use program MATLAB (The program is shown in attachment 1.) to carry on the calculation of the data, and the basic data could be obtained: $u = \begin{bmatrix} -0.121 \\ 22715.943 \end{bmatrix}$,

The time response equation is: $\hat{x}^{(1)}(k+1) = [x^{(1)}(1) - \frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} = 209635.26e^{0.121k} - 187735.07$;

3.1.3 Use program MATLAB (The program is shown in attachment 2.)to calculate simulated value $\hat{x}^{(1)}(k+1)$ and the total predictive value of Shandong GDP $\hat{x}^{(0)}(k+1)$. The results are shown in table 3:

Table 3 Shandong GDP original value from 2004 to 2011, $\hat{x}^{(1)}(k+1)$ and the predictive value of GDP

(Unit: one hundred million yuan)

year	2006	2007	2008	2009
GDP	21900.19	25776.91	30933.28	33896.65
$\hat{x}^{(1)}(k+1)$	21900.19	48864.51	79297.11	113644.1

GDP predictive value	21900.19	26964.32	30432.6	34346.99
year	2010	2011	2012	2013
GDP	39169.92	45361.85	50013.24	54684.33
$\hat{x}^{(1)}(k+1)$	152408.97	196159.96	245538.43	301269.19
GDP predictive value	38764.87	43750.99	49378.47	55730.76

3.2 GM (1, 1) model inspection of Shandong GDP

EXCEL and SPSS are used to calculate the absolute error, the relative error and the residual error. The results are shown in table 4.

Table 4 The model inspection table

year	Original GDP	Predictive GDP	Absolute error	Residual error	Relative error
2006	21900.19	21900.19	0	0	0
2007	25776.91	26964.32	1187.41	-1187.41	0.046065
2008	30933.28	30432.60	-500.68	500.68	-0.01619
2009	33896.65	34346.99	450.34	-450.34	0.013286
2010	39169.92	38764.87	-405.05	405.05	-0.01034
2011	45361.85	43750.99	-1610.86	1610.86	-0.03551
2012	50013.24	49378.47	-634.77	634.77	-0.01269
2013	54684.33	55730.76	1046.43	-1046.43	0.019136

3.2.1 SPSS is used to calculate the standard deviation of original data sequence.(The output results are shown in attachment 3.)

$S_1 = 11667.99$;

3.2.2 SPSS is used to calculate the standard deviation of absolute error data sequence.(The output results are shown in attachment 4.)

$S_2 = 932.97$;

3.2.3 Calculate the standard deviation ratio of original data sequence and absolute error data sequence.

$c = \frac{S_2}{S_1} = 0.07996 < 0.35$, Belongs to level 1;

3.2.4 Calculate small error probability.

$P = P(\left|\varepsilon(k) - \bar{\varepsilon}\right| < 0.6745 S_1) = P(\left|\max \varepsilon(k) - \bar{\varepsilon}\right| < 0.6745 S_1) = P(1552.46 < 7870.06) = 1 > 0.95$,Belongs to level 1;

In conclusion: this GM (1, 1) model passes the inspection. The inspection level belongs to level 1, which could be used to predict the future GDP of Shandong.

3.3 The prediction results of Shandong GDP in future years

Using the GM (1, 1) forecast model above to predict Shandong GDP, the results are shown in table 5. The contrast diagram is as shown in diagram 2.

Table 5 The original value and predictive value of Shandong GDP from 2006 to 2018 (Unit: one hundred million yuan)

year	2006	2007	2008	2009	2010	2011	2012
GDP	21900.19	25776.91	30933.28	33896.65	39169.92	45361.85	50013.24
Predictive value	21900.19	26964.32	30432.6	34346.99	38764.87	43750.99	49378.47
year	2013	2014	2015	2016	2017	2018	
GDP	54684.33						
Predictive value	55730.76	62897.01	70988.25	80119.11	90424.42	102055.26	

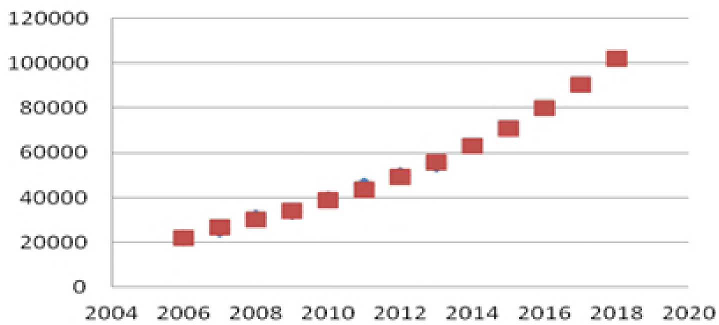

Diagram 2: The contrast diagram of original value and predictive value of Shandong GDP

4. The analysis of the main factors affecting the development of Shandong GDP

4.1 Establish the factor data sequence affecting the development of Shandong GDP

Analyze the influencing factors affecting the development of Shandong GDP, and establish reference data sequence and comparative data sequence. Here we choose the original data sequence of Shandong GDP x_0 as the reference data sequence, comparative data sequences are total amount of the first industry x_1,the industry total amount x_2,the construction total amount x_3, total amount of the third industry x_4, total amount of fixed assets investment x_5,foreign exchange earnings from tourism x_6. The specific details are shown in table 6.

Table 6: Shandong GDP and the original data of its influencing factors from 2006 to 2013

(Unit: one hundred million yuan)

	2006	2007	2008	2009	2010	2011	2012	2013
x_0	21900.19	25776.91	30933.28	33896.65	39169.92	45361.85	50013.24	54684.33
x_1	2138.90	2509.14	3002.65	3226.64	3588.28	3973.85	4281.70	4742.63
x_2	11378.82	13283.72	15894.95	16896.14	18861.45	21275.89	22798.33	24222.16
x_3	1195.21	1363.81	1677.03	2005.69	2377.04	2741.22	2937.40	3200.31
x_4	7187.26	8620.24	10358.64	11768.18	14343.14	17370.89	19995.81	22519.23
x_5	11136.06	12537.02	15435.93	19030.97	23276.69	26769.73	31255.96	36789.07
x_6	80.84	102.79	96.64	120.59	145.89	164.75	184.56	169.15

4.2 The nondimensional processing of original data sequence

Carry on the no dimensional processing of the original data of Shandong GDP and its influencing factors from 2006 to 2013, that is, for each data sequence, use the first number to divide the rest numbers. The results are shown in table 7.

Table 7 The no dimensional processing results of original data

	2006	2007	2008	2009	2010	2011	2012	2013
x_0	1.00	1.18	1.41	1.55	1.79	2.07	2.28	2.50
x_1	1.00	1.17	1.40	1.51	1.68	1.86	2.00	2.22

x_2	1.00	1.17	1.40	1.48	1.66	1.87	2.00	2.13
x_3	1.00	1.14	1.40	1.68	1.99	2.29	2.46	2.68
x_4	1.00	1.20	1.44	1.64	2.00	2.42	2.78	3.13
x_5	1.00	1.13	1.39	1.71	2.09	2.40	2.81	3.30
x_6	1.00	1.27	1.20	1.49	1.80	2.04	2.28	2.09

4.3 Calculate the correlation between Shandong GDP and its main influencing factors

According to table 7 the reference data sequence x_0 ,comparative data sequence x_1 , x_2 , x_3 , x_4 , x_5 , x_6 we could get that the relative difference of comparative data sequence x_i and reference data sequence x_0 at the moment of k is called correlation coefficient of x_i to x_0 at the moment of k, that is, $\omega_i(k)$. The calculation formula of $\omega_i(k)$ is as follows:

$$\omega_i(k) = \frac{\min_i \min_k |x_0(k) - x_i(k)| + \alpha \max_i \max_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \alpha \max_i \max_k |x_0(k) - x_i(k)|},$$

α is the identification coefficient, which is between 0 and 1, here we choose $\alpha = 0.5$.

Calculate the correlation coefficient, and the results are shown in table 8.

Table 8 The correlation coefficient results

$\omega_1(k)$	1.00	0.93	0.93	0.78	0.56	0.33
$\omega_2(k)$	1.00	0.94	0.94	0.70	0.56	0.31
$\omega_3(k)$	1.00	0.73	0.92	0.46	0.35	0.38
$\omega_4(k)$	1.00	0.94	0.91	0.78	0.60	0.33
$\omega_5(k)$	1.00	0.89	0.95	0.71	0.57	0.33
$\omega_6(k)$	1.00	0.88	0.93	0.86	0.64	0.33
$\omega_7(k)$	1.00	0.94	0.85	0.71	0.53	0.33
$\omega_8(k)$	1.00	0.69	0.49	0.77	0.95	0.33
$\omega_9(k)$	1.00	0.93	0.93	0.78	0.56	0.33
$\omega_{10}(k)$	1.00	0.94	0.94	0.70	0.56	0.31

$k = 1, 2, \cdots, 8$, let grey correlation degree of comparison data sequence x_i to reference data sequence x_0 be $\rho_i = \frac{1}{8}\sum_{i=1}^{8} \omega_i(k)$,the calculated results are shown in table 9.

Table 9 The correlation degree of the main factors affecting the development of Shandong GDP

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6
0.66	0.66	0.57	0.68	0.68	0.77

Therefore, the size relation of corresponding correlation is as follows: $\rho_6 > \rho_5 = \rho_4 > \rho_1 = \rho_2 > \rho_3$.

That is, foreign exchange earnings from tourism x_6 ,total amount of fixed assets investment x_5 ,total amount of the third industry x_4 ,total amount of the first industry x_1 ,the industry total amount x_2 ,the construction total amount x_3 .

5. Results and discussion

5.1 The prediction results of Shandong GDP

Using the established GM (1, 1) forecast model of Shandong province above to predict GDP from 2014 to 2018, the results are 6.289701 trillion yuan in 2014, 7.098825 trillion yuan in 2015, 8.011911 trillion yuan in 2016, 9.042442 trillion yuan in 2017, and 10.205526 trillion yuan in 2018, and the average rate of growth is 12.86%, which grows faster.

5.2 The relationship between the development of Shandong GDP and its main influencing factors

According to the principle of grey correlation analysis, the bigger the correlation is, the closer the relationship between the variables and Shandong GDP is, the more important to the development of Shandong GDP is. To summarize the correlation of the main influencing factors, the results are shown as follows:

(1) $\rho_6 = 0.77$ is the biggest, which indicates that foreign exchange earnings from tourism has the largest correlation with Shandong GDP, and has the biggest influence on the development of Shandong GDP.

(2) $\rho_5 = \rho_4 = 0.68$, $\rho_1 = \rho_2 = 0.66$ belongs to the middle level, the correlation is pretty much the same thing, which indicates that total budget amount of public finance, total amount of fixed assets investment, total amount of the third industry, total amount of the first industry, the industry total amount, total production value of farming, forestry, herd and fishery have big correlation with Shandong GDP, and have large influence on the development of Shandong GDP.

(3) $\rho_3 = 0.57$ is the smallest, which indicates that the construction total amount has the smallest correlation with Shandong GDP, and has little influence on the development of Shandong GDP.

Conclusion

In a word, during a short term, the correlation relationship between the development of Shandong GDP and its main influencing factors is consistent, mainly influenced by foreign exchange earnings from tourism, total amount of fixed assets investment, total amount of the third industry. Other factors have little influence on the development of Shandong GDP. But during a long term, with the changes of social form and the main influencing factors, the forecast model and correlation analysis may need to be modified, and the main influencing factors might be consumption, investment, government spending and net exports.

Attachment

1. program 1 MATLAB

```
>>A= [-34788.645,-63143.74,-95558.705,-132091.99,-174357.875,-222045.42,-274394.205; 1, 1, 1, 1, 1, 1, 1]';
>> B= [25776.91, 30933.28, 33896.65, 39169.92, 45361.85, 50013.24, 54684.33]';
>> u= (A'*A) ^ (-1)*A'*B
u =
  1.0e+004 *
   -0.0000
    2.2716
>> format long, u
u =
  1.0e+004 *
  -0.00001212634818
```

2.27159430889993

2. program 2 MATLAB

```
>> 209635.26*exp (0.121)-187735.07
ans =
   4.886450694660537e+004
>> 209635.26*exp (0.121*2)-187735.07
ans =
   7.929710679751315e+004
>> 209635.26*exp (0.121*3)-187735.07
ans =
   1.136440971373544e+005
>> 209635.26*exp (0.121*4)-187735.07
ans =
   1.524089660997397e+005
>> 209635.26*exp (0.121*5)-187735.07
ans =
   1.961599629353434e+005
>> 209635.26*exp (0.121*6)-187735.07
ans =
   2.455384279049105e+005
>> 209635.26*exp (0.121*7)-187735.07
ans =
   3.012681936040223e+005
>> 209635.26*exp (0.121*8)-187735.07
ans =
   3.641661955324348e+005
>> 209635.26*exp (0.121*9)-187735.07
ans =
   4.351544474469704e+005
>> 209635.26*exp (0.121*10)-187735.07
ans =
   5.152735570431243e+005
>> 209635.26*exp (0.121*11)-187735.07
```

ans =

6.056979800900328e+005

>> 209635.26*exp (0.121*12)-187735.07

ans =

7.077532366272179e+005

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