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Mathematical Relationship between Dependent and Independent Parameters of Operators Working on Rock Drill Machine by Dimensional Analysis

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Abstract

Hand held power tools are often used in various types of industries. These hand held power tools when used by operators induce high magnitude vibration which is called as segmental vibrations. These segmental vibration plays vital role in productivity of operators. The output, which is called as dependent variables, is depending on various independent variables. The dependent variable is a function of independent variables. Dimensional analysis helps in establishing mathematical relationship in between dependent and independent variables. The dimensional analysis is used in finding the mathematical relationship of various independent and dependent variables when rock drill operators worked in mines. The purpose is to establish mathematical models so that the phenomenon can be understand properly.

Key words

Independent Variables, Dependent Variables, Rock Drill Operators, Productivity, Human Energy Consumption, Dimensional Analysis.

1. Introduction

Power driven hand held tools are used in several industries (e.g. agricultural, construction, logging and manufacturing). These tools are also used in dental and medical work. During the operation with power tools the workers directly come in contact with it, due to this physical contact, the vibration induced by these tools passed in the body of worker through palm and finger this type of vibration influencing part body of worker is called as ‘segmental vibration’ [1]. The role of man as the operator of technical systems is ever increasing; however, the vibrations of machines, acting on man, can reduce the productivity of labor and its quality significantly [2]. The most important sources of hand arm vibration (HAV) are pneumatic tools (air compressed and electrical), for example, grinders, sanders, drills, fettling tools, impact wrenches, jack hammers, and riveting guns. Users of chainsaws, brush saws, hedge cutters, and grass trimmers are also at risk. [3]. Hand-arm vibration syndrome is a condition associated with the use of hand-held vibrating tools. Vibration can cause changes in tendons, muscles, bones and joints as well as affecting nerves, collectively, these effects are known as Hand-Arm Vibration Syndrome (HAVS) [4]. HAVS is also called as Raynaud’s phenomenon of occupational origin, vibration-induced white finger (VWF), dead finger, traumatic vasospastic disease and vibration syndrome. HAVS is a complex syndrome caused by the constriction of blood vessels in the fingers, and involves circulatory, sensory, motor and musculoskeletal disturbances [5]. Man-machine interaction has become an important factor in engineering design. Much emphasis is placed on human safety, and employers are under increasing pressure to create a healthy and safe environment to ensure not only maximum production, but also the minimum injuries, in the mining industry, this becomes more difficult. Working conditions are very hostile, and it is not easy to implement safety procedures and international standards [6]. The human body is a very complex, dynamic and intelligent structure, and thus it is not easy to analyze the effect of vibration on a human being. Vibration can be a source of pleasure or pain; it can heal or impair health. One of the factors that make human vibration so difficult to analyze and quantify, is the fact that the response of the human body varies from person to person. Some people experience symptoms of vibration diseases sooner than other [7].

The purpose of conducting experimentation is to generate output data based on the relevant input variables, which are properly structured in the design of experimentation (DOE). The performance evaluation of RDO (Rock Drill Operator) is difficult for different operating factors and is a complex phenomenon. In this particular case, the valid approach is to collect the experimental field data. This can be purified for proper analysis. The relationship of independent and dependent variables is discussed detail in this chapter, the chapter describes the plan of experimentation and the approach adopted to perform the experiment.

2. Methodology

To understand any complex phenomenon a theoretical approach can be adopted. If a known logic can be applied correlating the various dependent and independent parameters of the system. Though qualitatively, the relationships between the dependent and independent parameters are known, based on the available literature, the generalised quantitative relationships are not known sometimes. Whatever quantitative relationships are known, these are pertaining to a specific anthropometric data and are of statistical nature [8]. Such relevant data is not available for the population in India and for the population working in specific industry. Therefore it is difficult to formulate the quantitative relationship based on the known logic. Hence on account of no possibility of formulation of theoretical model one is left with the only alternative of formulating experimental data based (field-data based) model. It is proposed here to formulate such a model in the present investigation.

Schenck, (1961) suggested the approach for formulating generalized experimental model for any complex physical phenomenon involves following steps.

- Identification of variables or quantities
- Reduction of variables/ dimensional analysis
- Test Planning
- Design of an experimental set up
- Rejection of absurd data
- Formulation of the model

Identification of dependent and independent variables of the phenomenon is based on known qualitative physics of the phenomenon. When system involves a large number of independent variables, the experimentation becomes tedious, time consuming and costly, by deducing dimensional equation for the phenomenon, we can reduce the number of variables.

Dimensional analysis is a method which describes a natural phenomenon by a dimensionally correct equation among certain variables which affect the phenomenon. It reduces the number of variables and arranges them into dimensionless groups. Dimensional analysis finds its applications in nearly all fields of engineering and is most extensively used in fluid mechanics and heat transfer. It is based on the principle of dimensional homogeneity and uses the dimensions of relevant variables affecting the phenomenon [10].

The exact mathematical form of this equation will be the targeted model. Test Planning comprises of deciding test envelope, test points, test sequence and experimental plan for the deduced set of dimensional equations. It is necessary to evolve physical design of an experimental set up having provision of setting test points, adjusting test sequence, executing proposed experimental plan, noting down the responses and to deduce the dependent pi-terms of the dimensional equation. Upon getting the experimental results, adoption of the appropriate method for test data checking, rejection of the erroneous data identified, from the gathered data involved. Based on the purified data as mentioned above one has to formulate quantitative relationship between the dependent and independent pi terms of the dimensional equation [9].

2.1 Identification of variables

The term variable means any physical quantity which undergoes changes during the experimentation. When physical quantities change independent of the other quantities, then it is called as independent variable. If a physical quantity changes in response to the variation of one or more number of variables, then it is termed as dependent or response variable. If a physical quantity that affects our test is changing in random and uncontrolled manner, then it is called an extraneous variable. The variables affecting the effectiveness of the phenomenon under consideration are anthropometric data of an operator, features of hand tool, and environmental (conditions) variables. The dependent or the response variables in this case are

- Productivity
- Human energy consumed

There are many independent variables involved in this system. For better presentation and manipulation of these variables in this study they have been grouped in the following categories.

- Anthropometric features and associated variables of operator
- Operating conditions (with gloves and without gloves)
- Variable associated with tool
- Environmental variables

Productivity 'P'

It is measured in terms of number of holes drilled for particular specified amount of time. The productivity can be measured manually by counting the number of holes drilled made over a specified amount of time.

Human Energy 'HE'

The human energy 'HE' consumed by the person performing the task is measured in Kj. The human energy can be estimated by many methods. But the heart rate measurement is the simplest and most suitable in the context of this research method for estimation of the human energy [11]

Anthropometric Features and Associated Variables

The related human dimensions decide the anthropometric variables of operator. The various variables associated with the operator selected are Grip strength (Gs), Weight of individual (Wi), Height of individual (Stature) (H). Weight and height of individual will be used to calculate Body mass index (BMI).

Variable Associated With Tool

The independent variables of tool taken are weight of tool (Wt), magnitude of vibration measured in terms of acceleration (a_t) and Noise level (N_L).

Environmental Factors / Variables

Apart from the above independent variables the various environmental variables are also listed to find out their effect if any on the dependent variables viz. Relative humidity denoted by 'Φ', Air speed in m/sec, denoted by 'Va', Dry bulb temperature denoted by 'Td' and Light intensity denoted by 'L_i'.

2.2 Reduction of Independent Variables

There are several quite simple ways in which a given test can be made compact in operating plan without loss in generality or control. The best known and the most powerful tool of these is dimensional analysis. Deducing the dimensional equation for a phenomenon reduces the number of variables in the experiments. The exact mathematical form of this dimensional equation is the targeted model. This is achieved by applying Buckingham's pi theorem [9].

2.3. Dimensional Analysis

Dimensional analysis is a method which describes a natural phenomenon by a dimensionally correct equation among certain variables which affect the phenomenon. It reduces the number of variables and arranges them into dimensionless groups. Each physical phenomenon can be expressed by an equation giving relationship between different quantities; such quantities are dimensional and non dimensional [10, 12]. In a study dimensional analysis was carried out to established mathematical relationship between dependent and independent parameters of women workers working on spinning wheel [13]. In another studies mathematical model was developed for sewing machine operators, considering women factors. In these studies also dimensional analysis was carried out to established mathematical relation between dependent and independent variables [14, 15].

The various physical quantities used can be expressed as fundamental quantities or primary quantities. The fundamental quantities are mass, length, time and temperature, designated by the letter M, L, T respectively. The quantities which are expressed in terms of the fundamental or primary quantities are called derived or secondary quantities such as velocity, area, acceleration etc. The expression for a derived quantity in terms of the primary quantities is called the dimension of the physical quantity.

A physical equation is the relationship between two or more physical quantities, any correct equation expressing a physical relationship between quantities must be dimensionally homogeneous (according to Fourier's principle of dimensional homogeneity) and numerically equivalent. Dimensional homogeneity states that every term in an equation when reduced to fundamental dimensions must contain identical power of each dimension.

Two method of dimensional analysis are

i) Rayleigh's method

ii) Buckingham's Π method/ theorem

i) Rayleigh's method

This method gives a special form of relationship among the dimensionless group, and has the inherent drawback that it does not provide any information regarding the number of dimensionless groups to be obtained as a result of dimensional analysis. Due to this reason this method has become obsolete and is not favoured for use.

ii) Buckingham's Π method/Theorem:

When a large number of physical variables are involved Rayleigh's method of dimensional analysis becomes increasingly laborious and cumbersome. Buckingham's method is an improvement over the Rayleigh's method. Buckingham designated dimensionless group by the Greek capital letter Π (Pi). It is often called Buckingham Π- method. The advantage of this method over Rayleigh's method is that it let us know, in advance of the analysis, as how many dimensionless groups are to be expected. Buckingham's Π- theorem states as follows

If there are n variables (dependent and independent variables) in a dimensionally homogeneous equation and if these contain m fundamental dimensions (such as M, L, T etc) then the variables are arranged into (n-m) dimensionless terms. Those dimensionless terms are called Π-terms.

Mathematically, if any variable X_1, depends on independent variables $X_2, X_3, X_4, \ldots X_n$ the functional equation may be written as

$X_1 = f_1 (X_2, X_3, X_4, \ldots X_n)$

Above equation can also be written as

$f_1 (X_1, X_2, X_3, X_4, \ldots X_n) = 0$

It is a dimensionally homogeneous equation and contains n variables. If there are 'm' fundamental dimensions then according to Buckingham's Π-terms is equal to (n-m).

$f_1 (\Pi_1, \Pi 2, \Pi 3 \ldots \Pi_{n-m}) = 0$

Each dimensionless Π-terms is formed by combining m variables out of the total n variables with one of the remaining (n-m) variables i.e. each Π-term contains (m+1) variables. These m variables which appear repeatedly in each of Π-terms are consequently called repeating variables and are chosen from among the variables such that they together involve all the fundamental dimensions and they themselves do not form dimensionless parameters.

Let in the above case X_2, X_3 and X_4 are the repeating variables if the fundamental dimensions m (M LT) =3, then each term is written as

$\Pi_1 = X_2 a_1, X_3\, b_1, X_4 c_1, X_1$

$\Pi_2 = X_2 a_2, X_3 b_2, X_4 c_2, X_5$

$\Pi\ n\text{-}m = (X_2 a_{n-m}, X_3 b_{n-m}, X4 c_{n-m} \ldots X_n)$

Where a_1, b_1, c_1; a_2, b_2, c_2 etc are the constants, which are determined, by considering dimensional homogeneity. These values are substituted in equation and values of $\Pi_1, \Pi_2, \Pi_3, \ldots \Pi n\text{-}m$ is obtained. These values of Π's are substituted above equations. The final general equation for the phenomenon may then be obtained by expressing anyone of the Π-terms as a function of the other as

$\Pi_1 = \Phi(\Pi_2, \Pi_3, \Pi_4 \ldots \Pi n\text{-}m)$

$\Pi_2 = \Phi(\Pi_1, \Pi_3, \Pi_4 \ldots \Pi n\text{-}m)$

Selection of repeating variables:-

i) M repeating variables must contain jointly all the fundamental dimensions involved in the phenomenon. Usually the fundamental dimensions are M, L and T. However if only two dimensions are involved, there will be '2' repeating variables and they must contain together the two dimensions involved.

ii) The repeating variables must not form the non-dimensional parameters among themselves.

iii) As per as possible dependent variables should not be selected repeating variables.

iv) No two repeating variables should have the same dimension.

v) The repeating variables should be chosen in such a way that one variable contains geometric property (e.g. Length l, diameter d, height H etc.). Other variables contain flow property (velocity v, acceleration a etc) and third variables contains fluid property (e.g. mass m, density ρ, weight density etc.).

2.4 Dimensional Analysis of Operators Working On Rock Drill Machine

Hand held power tool is a man-machine system it includes system (hand held power tool), causes/input/independent parameters, effect/output/dependent parameters and extraneous variables.

The parameters considered are classified as productivity per second (P) and human energy consumed per second (HE) of operators as dependent parameters and Body Mass Index (BMI), Weight of tool (Wt), Magnitude of vibration of tool measures in terms of acceleration (a_t), Noise level (N_L), Velocity of air (Va), Dry bulb temp (Td), Humidity of air (Φ), Intensity of light (Li), Effective time (Te) as independent variable.

Table 1 Variable With Symbol Unit And Dimensions.

Sr. No.	Variable	Symbol	Unit	Dimensions
1.	Productivity	P	Holes/Sec.	T^{-1}
2.	Human Energy consumed per second	HE	NM	ML^2T^{-3}
3.	Body Mass Index	BMI	N/M^2	$ML^{-1}T^{-2}$
4.	Grip Strength	Gs	N	MLT^{-2}
5.	Weight of Tool	Wt	N	MLT^{-2}
6.	Acceleration of tool	a_t	m/s^2	LT^{-2}
7.	Velocity of air	Va	m/s	LT^{-1}
8.	Dry bulb temp.	Td	^0C	ϴ
9.	Humidity of air	Φ	Ratio	$M^0L^0T^0\,\Theta^0I^0$
10.	Noise Level	N_L	Ratio	$M^0L^0T^0\,\Theta^0I^0$
11.	Light Intensity	Li	I/L^2	IL^{-2}
12.	Effective Time	Te	Sec.	T

Dependent parameters are Productivity (measure in terms of no. of holes drilled per second) and human energy consumed per second. Productivity 'P' is a function of independent variables as given below.

Functional Relation of Dependent Parameter, Productivity (P)

$P = f$ (BMI, GS, Wt, a_t, Va, Td, Φ, N_L, Li, Te)…………………………….. (1)

Or f_1 (BMI, GS, Wt, a_t, Va, Td, Φ, N_L, Li, Te, P) =0……………………… (2)

Here the number of variables are, m=11

The numbers of fundamental dimensions in above variables are n=5, these are M, L, T, Θ and I, therefore as per Buckingham's Π method/ theorem number of Π terms are

m -n =11 - 5 = 6

Therefore f_1 ($\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5, \Pi_6$) = 0 …………………………………… (3)

Let Te, Va, Wt, Td and Li are repeating variables which contain all five fundamental dimensions i.e. M, L, T, Θ and I. These repeating variables will make dimensionless to the other variables.

Pi Term Relating To Grip Strength

Let $\Pi_1 = (Td)^{a_1}, (Te)^{b_1}, (Va)^{c_1}, (Wt)^{d_1}, (Li)^{e_1}, Gs$ ……………………………… (4)

$\Theta^0 M^0 L^0 T^0 I^0 = [\Theta]^{a_1} [T]^{b_1}, [LT^{-1}]^{c_1}, [MLT^{-2}]^{d_1}, [IL^{-2}]^{e_1}, [MLT^{-2}]$

Equating power of Θ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_1$

$a_1 = 0$

$I = 0 = e_1$

$e_1 = 0$

$M = 0 = d_1 + 1$

$d_1 = -1$

$L = 0 = c_1 + d_1 - 2e_1 + 1$

$0 = c_1 - 1 - 0 + 1$

$c_1 = 0$

$T = 0 = b_1 - c_1 - 2d_1 - 2$

$0 = b_1 + 2 - 2$

$b_1 = 0$

Substituting the values of a_1, b_1, c_1, d_1 and e_1 in equation 4

$\Pi_1 = (Td)^0, (Te)^0, (Va)^0, (Wt)^{-1}, (Li)^0, Gs$ …………………………………… (5)

$\Pi_1 = Gs/Wt$ ………………………......……………………………… (6)

Dimensionless since $[MLT^{-2}]/[MLT^{-2}]$

Pi Term Relating To Body Mass Index

Let $\Pi_2 = (Td)^{a_2}, (Te)^{b_2}, (Va)^{c_2}, (W_t)^{d_2}, (Li)^{e_2}$ BMI ……………………... (7)

$\Theta^0 M^0L^0T^0 I^0 = [\Theta]^{a_2}, [T]^{b_2}, [LT^{-1}]^{c_2}, [MLT^{-2}]^{d_2}, [IL^{-2}]^{e_2}, [ML^{-1}T^{-2}]$

Equating power of Θ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_2$

$a_2 = 0$

$M = 0 = d_2 + 1$

$d_2 = -1$

$I = 0 = e_2$

$e_2 = 0$

$L = 0 = c_2 + d_2 - 2e_2 - 1$

$L = 0 = c_2 - 1 - 1$

$c_2 = 2$

$T = 0 = b_2 - c_2 - 2d_2 - 2$

$T = 0 = b_2 - 2 + 2 - 2$

$b_2 = 2$

Substituting the values of a_2, b_2, c_2, d_2 and e_2 in equation 7

$\Pi_2 = (Td)^0, (Te)^2, (Va)^2, (W_t)^{-1}, (Li)^0$ BMI …………………………... (8)

$\Pi_2 = (BMI*T_e^2*Va^2)/W_t$ {Dimensionless as $[ML^{-1}T^{-2}* T^2* (LT^{-1})^2]/[MLT^{-2}]$}… (9)

Pi Term Relating To acceleration of tool

Let $\Pi_3 = (Td)^{a_3}, (Te)^{b_3}, (Va)^{c_3}, (W_t)^{d_3}, (Li)^{e_3} a_t$ …………………………….. (10)

$\Theta^0 M^0L^0T^0 I^0 = [\Theta]^{a_3} [T]^{b_3}, [LT^{-1}]^{c_3}, [MLT^{-2}]^{d_3} [IL^{-2}]^{e_3}, [LT^{-2}]$

Equating power of Θ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_3$

$a_3 = 0$

$M = 0 = d_3$

$d_3 = 0$

$I = 0 = e_3$

$e_3 = 0$

$L = 0 = c_3 + d_3 - 2e_3 + 1$

$0 = c_3 + 0 + 0 + 1$

$c_3 = -1$

$T = 0 = b_3 - c_3 - 2d_3 - 2$

$0 = b_3 + 1 – 0 - 2$

$b_3 = 1$

Substituting the values of a_3, b_3, c_3, d_3 and e_3 in equation 10

$\Pi_3 = (Td)^0, (Te)^1, (Va)^{-1}, (W_t)^0, (Li)^0\, a_t$ …………………………………… (11)

$\Pi_3 = (a_t*Te)/Va$ {Dimensionless as $[LT^{-2}][T]/[LT^{-1}]$ }………………………. (12)

Pi Term Relating To Humidity of Air

Let $\Pi_4 = (Td)^{a_4}, (Te)^{b_4}, (Va)^{c_4}, (W_t)^{d_4}, (Li)^{e_4}\, \Phi$ ………………………………. (13)

$\Theta^0 M^0 L^0 T^0 I^0 = [\Theta]^{a_4} [T]^{b_4}, [LT^{-1}]^{c_4}, [MLT^{-2}]^{d_4}, [IL^{-2}]^{e_4}, [\Theta^0 M^0 L^0 T^0]$

Equating power of Θ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_4$

$a_4 = 0$

$M = 0 = d_4$

$d_4 = 0$

$I = 0 = e_4$

$e_4 = 0$

$L = 0 = c_4 + d_4 - 2e_4$

$c_4 = 0$

$T = 0 = b_4 - c_4 - 2d_4 - 0$

$b_4 = 0$

Substituting the values of a_4, b_4, c_4, d_4 in equation 13

$\Pi_4 = (Td)^0, (Te)^0, (Va)^0, (W_t)^0, (Li)^0, \Phi$ …………………………………….. (14)

$\Pi_4 = \Phi$ {Dimensionless as $[\Theta^0 M^0 L^0 T^0 I^0]$} …………………………….. (15)

Pi Term Relating To Noise Level

Let $\Pi_5 = (Td)^{a_5}, (Te)^{b_5}, (Va)^{c_5}, (W_t)^{d_5}, (Li)^{e_5} N_L$ …………………………….. (16)

$\Theta^0 M^0 L^0 T^0 I^0 = [\Theta]^{a_5}, [T]^{b_5}, [LT^{-1}]^{c_5}, [MLT^{-2}]^{d_5}, [IL^{-2}]^{e_5}, [\Theta^0 M^0 L^0 T^0 I^0]$

Equating power of Θ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_5$

$M = 0 = d_5 + 0$

$d_5 = 0$

$I = e_5 = 0$

$e_5 = 0$

$L = 0 = c_5 + d_5 - 2e_5 + 0$

$0 = c_5 + 0 + 0 + 0$

$c_5 = 0$

$T = 0 = b_5 - c_5 - 2d_5 + 0$

$0 = b_5 + 0$

$b_5 = 0$

Substituting the values of a_5, b_5, c_5, d_5 and e_5 in equation 16

Let $\Pi_5 = (Td)^0, (T)^0, (Va)^0, (W_t)^0, (Li)^0 N_L$ …………………………………… (17)

$\Pi_5 = N_L$ {Dimensionless as $[\Theta^0 M^0 L^0 T^0 I^0]$} ……………………………… (18)

Pi Term Relating To Productivity

Let $\Pi_6 = (Td)^{a_6}, (T_e)^{b_6}, (Va)^{c_6}, (W_t)^{d_6}, (Li)^{e_6} P$ ……………………………….. (19)

$\Theta^0 M^0 L^0 T^0 I^0 = [\Theta]^{a_6} [T]^{b_6}, [LT^{-1}]^{c_6}, [MLT^{-2}]^{d_6}, [IL^{-2}]^{e_6} [T^{-1}]$

Equating power of Θ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_6$

$a_6 = 0$

$M = 0 = d_6$

$d_6 = 0$

$I = 0 = e_6$

$e_6 = 0$

$L = 0 = c_6 + d_6 - 2e_6$

$0 = c_6 + 0 - 0$

$c_6 = 0$

$T = 0 = b_6 - c_6 - 2d_6 - 1$

$0 = b_6 - 1$

$b_6 = 1$

Substituting the values of a_6, b_6, c_6, d_6 and e_6 in equation 4.19

$\Pi_6 = (Td)^0, (T_e)^1, (a_t)^0, (W_t)^0, (Li)^0, P$ ………………………….. ……….. (20)

$\Pi_6 = P*T_e$ {Dimensionless as $[T]\,[T^{-1}]$} ……………………………………… (21)

2.5 Model for Dependent Parameter Productivity (P)

Substituting the values of all Pi terms in equation 4.3

$f_1\,((Gs/Wt)\,((BMI*T_e^2*Va^2)/W_t)\,((a_t*Te)/Va)\,(\Phi)\,(N_L)\,(P*T_e))$

$f_1\,((Gs/W_t)(Wt/W_i)(Va*H^{1/2}/a_t)(\Phi)(Te*(a_t)^{1/2}/(H)^{1/2})(P*(H)^{1/2}/(a_t)^{1/2}(N_L) = 0.$ (22)

$(P*T_e) = f_1\,((Gs/Wt)\,((BMI*T_e^2*Va^2)/W_t)\,((a_t*Te)/Va)\,(\Phi)\,(N_L))$………… (23)

$P = Te^{-1} f_1\,((Gs/Wt)\,((BMI*T_e^2*Va^2)/W_t)\,((a_t*Te)/Va)\,(\Phi)\,(N_L))$ ………… (24)

Equation number 23 can be written as

$\Pi_6 = k\,(\Pi_1^{a_1}, \Pi_2^{b_1}, \Pi_3^{c_1}, \Pi_4^{d_1}, \Pi_5^{e_1})$ ………………………………… (25)

Neglecting the Pi term related to noise level i.e. Π_5 [Considering it extraneous variables]

$\Pi_6 = k\,(\Pi_1^{a_1}, \Pi_2^{b_1}, \Pi_3^{c_1}, \Pi_4^{d_1})$ ……………………………………… (26)

2.6 Functional Relation of Dependent Parameter, Human Energy Consumed Per Second (HE)

The human energy consumed per second (HE) can be expressed as a functional relation

$HE = f\,(BMI, GS, Wt, a_t, Va, Td, \Phi, N_L, Li, Te,)$ ……………………............... (27)

Or $f_1\,(BMI, GS, Wt, a_t, Va, Td, \Phi, N_L, Li, Te, HE) = 0$…………………… (28)

Here the number of variables, m = 11

Let the number of fundamental dimensions be n=5, these are M, L, T Θ, I therefore as per Buckingham's Π method, number of Π terms are,

m - n =11 - 5 = 6

Therefore $f_1\,(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5)$………………………………………. (29)

Each Π term is composed of (n+1) i.e. six quantities, out of these six, five are repeating variables.

Let Te, Va, Wt, Td and Li are repeating variables which contain all five fundamental dimensions i.e. M, L, T, ϴ and I. These repeating variables will make dimensionless to the other variables. Only the Pi term related to human energy need to be calculated, the values of Π_1, Π_2, Π_3, Π_4 and Π_5 are previously calculated which are as below.

$\Pi_1 = G_s /Wt$

$\Pi_2 = (BMI*T_e^{2}*Va^{2)}/W_t$

$\Pi_3 = (a_t*Te)/Va$

$\Pi_4 = \Phi$

$\Pi_5 = N_L$

Pi Term Relating To Human Energy consumed per second

Let $\Pi_7 = (Td)^{a_7}, (Te)^{b_7}, (Va)^{c_7}, (W_t)^{d_7}, (Li)^{e_7}, HE$ ……………………………… (30)

$\Theta^0 M^0L^0T^0 I^0 = [\Theta]^{a_7}, [T]^{b_7}, [LT^{-1}]^{c_7}, [MLT^{-2}]^{d_7}, [IL^{-2}]^{e_7} [ML^2T^{-3}]$

Equating power of ϴ, M, L, T and I from both sides,

Therefore

$\Theta = 0 = a_7$

$a_7 = 0$

$M = 0 = d_7 + 1$

$d_7 = -1$

$I = 0 = e_7$

$e_7 = 0$

$L = 0 = c_7 + d_7 - 2e_7 + 2$

$0 = c_7 - 1 + 2$

$c_7 = -1$

$T = 0 = b_7 - c_7 - 2d_7 - 3$

$0 = b_7 + 1 + 2 - 3$

$b_7 = 0$

Substituting the values of a_7, b_7, c_7, d_7 and e_7 in equation (30)

$\Pi_7 = (Td)^0, (Te)^0, (Va)^{-1}, (W_t)^{-1}, (Li)^{0,} HE$ ……………………………… (31)

$= HE/ (Wt*Va)$ {Dimensionless as $[ML^2T^{-3}]/ [MLT^{-2}][LT^{-1}]$}……….. (32)

2.7 Model for Dependent Parameter Human Energy (HE)

Substituting the values of all Pi terms in equation number 4.31 we have

f_2 ((G$_s$ /Wt) ((BMI*T_e^2*Va$^{2)}$/W$_t$) ((a$_t$*Te)/Va) (Φ) (N$_L$) (HE/ (Wt*Va)))….. (33)

HE/ (Wt*Va) = f_2 ((G$_s$ /Wt) ((BMI*T_e^2*Va$^{2)}$/W$_t$) ((a$_t$*Te)/Va) (Φ) (N$_L$))… (34)

HE = 1/(Wt*Va) f_2 ((G$_s$ /Wt) ((BMI*T_e^2*Va$^{2)}$/W$_t$) ((a$_t$*Te)/Va) (Φ) (N$_L$))… (35)

Equation 34 can be written as

$\Pi_7 = k_2\ (\Pi_1{}^{a}{}_2, \Pi_2{}^{b}{}_2, \Pi_3{}^{c}{}_2, \Pi_4{}^{d}{}_2, \Pi_5{}^{e}{}_2)$ ……………………………………… (36)

Neglecting the Pi term related to noise level i.e. Π_5 [Considering it extraneous variables]

$\Pi_7 = k_2\ (\Pi_1{}^{a}{}_2, \Pi_2{}^{b}{}_2, \Pi_3{}^{c}{}_2, \Pi_4{}^{d}{}_2)$……………………………………………… (37)

The deduced equations for the independent and dependent Pi terms are presented in Table 2

Table 2. Non-Dimensional Pi Terms for Dependent and Independent Parameters.

Type of variables	Variable with Symbol	Description of non dimensional Pi terms	Equation of Pi terms
Dependent	Productivity-P	Π_6 - Pi term relating to productivity	P*T_e
	Human Energy-HE	Π_7 - Pi term relating to human energy consumed per second	HE/ (Wt*Va)
Independent	Grip Strength-Gs	Π_1-Pi term relating to grip strength	Π_1= Gs /Wt
	Body Mass Index - BMI	Π_2-Pi term relating to body mass index	Π_2= (BMI*T_e^2*Va^2)/W$_t$
	Acceleration of tool - a_t	Π_3-Pi term relating to acceleration of tool	Π_3 = (a_t*Te)/Va
	Humidity of air- Φ	Π_4-Pi term relating to humidity of air	Π_4 = Φ
	Noise Level- N_L	Π_5 – Pi term relating to noise level	Π_5 = N_L

4. Conclusions

Dimensional analysis can be used as a tool to establish the relationship in between independent and dependent variables. It can be used in reducing the number of variables controlling the system. These reduce number of variables can be used to derive the formula for calculating output i.e. dependent quantity of the system under study. Dimensional analysis can be used in formulating the data based model for rock drill operators working in rock mines.

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