Coordination of the Supply Chain of Seasonal Products with Buy-Back Contract under Weather-Sensitive Demand

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Abstract

Weather is an important determinant of demand for the seasonal products such as outerwear, ice cream products, weatherproof, etc. We consider a manufacturer-retailer supply chain in which a retailer orders from a manufacturer a quantity of a seasonal product, which has to face the stochastic demand that is contingent on the seasonal weather, such as the average seasonal temperature. The retailer has to determine his order quantity, by taking into account the wholesale price, and the possible demand for the seasonal product in the market. In this paper, we examine how a manufacturer can structure a contract to improve the profits of both parties. We develop a model to study this problem, and further design an incentive scheme to facilitate coordination between the two parties. We show that the buy-back contracts can coordinate decentralized supply chain under weather-related demand. In addition, numerical results are reported to show the effectiveness of the buy-back contracts.

Key words

Supply chain of seasonal products, supply chain coordination, buy-back contract, weather-sensitive demand, weather risk management

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1. Introduction

It is well known that there is a relationship between weather and economic activity. The U.S. National Research Council has estimated that 46% of U.S. gross domestic product is affected by weather conditions, such as the average seasonal temperature [1]. In the retail industry, weather affects the business of many retailers, retailers of seasonal products may generate high sales and profit in “favorable” weather conditions but may financially suffer in “unfavorable” weather states. Take the following examples. Demand for propane heating oil is weather-related. If the season is warmer than the normal, then the sales volume will decrease with the temperature state [2]. Demand for soft drinks is also weather-related. Sales of mineral water in a German city were highly correlated with the weekly average temperature [3]. USA Today reported that warmer weather than normal in December 2006 in the northeastern part of the Unites States caused a dramatic fall-off for the demand of coats and sweaters [4].

Above examples are pervasive and anecdotal evidence of the effect of weather on demand. Weather influences sales mainly through this effect on economic activity. When the product of concern is a seasonal product, however, the primary effect from weather may be that of shifting demand earlier or later. Facing strong demand seasonality and uncertainty as a result of weather conditions, sophisticated firms have been using contracts to hedge “weather risk” (the low demand caused by unfavorable weather). With this as a backdrop, in this paper, we analyze the buyback contracts that are adopted by some manufacturer of seasonal products and examine how a manufacturer can structure incentive scheme to facilitate supply chain coordination in a newsvendor context. In this study, our objective is to investigate the effectiveness of the buy-back contracts in increasing the profits of manufacturer and retailer.

The remainder of this paper is organized as follows. The next section presents a brief review of relevant literature. In section 3, we first describe the basic model setting and state our assumptions, and then derive the optimal decisions in the centralized system and decentralized supply chain. In section 4, we propose an incentive scheme that can motivate both parties to adopt coordinated decisions. In section 5, we present numerical examples to offer some managerial insights, and show the effectiveness of the buy-back contracts. Finally, we discuss some implications of our findings and provide future research areas in section 6.
2. Literature Review

Research of weather management mainly addresses the weather forecasting in operations research and management science (OR/MS) [5], using OR/MS tools to integrate weather forecasts in decision-making being the main focus of study. In the interest of space, we list only a few. The papers include Bowers and Mould (1994), who use a combination of Monte Carlo simulation of weather conditions and a project-management model to evaluate managerial alternatives for oil platform installation [6]; Adams et al. (1995), who use a plant-growth model together with historical climate data in a decision-analytic framework to estimate the value hypothetical, improved forecast for El Niño to be $145 for perfect information [7]; Kim and Palmer (1997), who use dynamic programming to optimize hydropower reservoir operation with respect to seasonal flow forecast, improving annual gain by about $1M [8]; Mjelde et al. (1998), who survey studies of the value of climate forecasts in agriculture [9]. A review for recent developments in weather forecasting is given in [5], where has reviewed OR/MS work in integrating weather forecasts into decision processes and has described several open and potentially very rewarding areas for further OR/MS research. Studies in recent years on weather management in decision-making can be found as follows. [10], who analyze the joint optimal ordering and weather hedging decision for seasonal products under weather-related demand; [11], who analyze a weather-conditional rebate program which offers a rebate payment to an early buyer when the weather state in the later selling season turns out to be unfavorable. However, little attention has been devoted to the interaction between weather risk and supply chain management, which we address in this paper.

When demand for seasonal product is sensitive to weather conditions (i.e. demand is uncertainty), double marginalization leads to a loss of efficiency in seasonal product supply chain. There is an extensive literature on contract mechanisms to mitigate this efficiency loss in a newsvendor context, in which relevant literatures focus on contract structures that coordinate the supply chain or improve performance vis-à-vis scenario with decentralized decisions. In the interest of brevity, we refer the reader to surveys by [12-15].

We investigate the supply chain of seasonal product coordination between the manufacturer and the retailer by buy-back contracts under weather-related demand. Under buy-back contract, the retailer returns all or some of the excess inventory to the manufacturer for a full or partial refund. [16] provide a brief history of buy-back contract and discuss their advantages that are mitigating the retailer’s risk, safeguarding the brand.
from deterioration of its image due to stale and discounted product, and facilitating collection of more accurate demand data. The literature in recent years on buy-back contracts mainly includes [17-20].

The literature on weather index-linked contracts or weather derivatives used in contracts is quite limited. Chen and Yano analyze weather rebate contracts for news vendor setting that can achieve supply chain coordination and allow an arbitrary allocation of profits between the two parties. They show that the weather-linked rebate can take many different forms, and this flexibility allows the manufacturer to design contracts that are Pareto improving [1]. Elias et al. developed four models to modeling the stochastic behavior of temperature with an aim to the valuation of temperature-based weather options [21]. Fu et al. address the joint determination of pricing and ordering decisions in a news vendor setting, where a retailer (news vendor) sells the seasonal products and faces demand risk due to weather uncertainty [22]. However, in which don’t consider a two-party supply chain [21, 22]. Our results complement other findings in the incentive mechanism by the buy-back contracts, which achieve also supply chain of seasonal product coordination under weather-related demand.

We next discuss the manufacturer’s and retailer’s optimal decisions and profits when prices are exogenous.

3. The Basic Model

We consider a simple supply chain with two firms, a manufacturer and a retailer who sells a seasonal product with single period under weather-related demand uncertainty. The manufacturer, as Stackelberg leader, decides the contract terms and provides a wholesale price, $\omega$. The retailer chooses the order quantity $q$ and sells the seasonal product at a unit price, $p$. In this basic model, both the wholesale price and the retail price are exogenous.

This paper is concerned with weather risk caused by non-catastrophic weather volatility. The demand for seasonal product is affected by the average seasonal weather which is uncertain prior to the selling season. Favorable weather generates strong sales and hence high profits, whereas unfavorable weather significantly shrinks demand, and even causes losses. Although the demand of seasonal product can be influenced by temperature, rain, snow, fog, and wind, without loss of generality, we take temperature as an example throughout this paper, and for other weather indices can be similarly formulated.
In this remainder of this section, we first give the key notations and assumptions, and then analyze the optimal order quantity in centralized decision-making supply chain. Finally, we derive the specific order quantity in decentralized decision-making supply chain, and compare and analyze the effects of adverse weather on the optimal order quantity under centralized and decentralized settings, respectively.

### 3.1. Notations and Assumptions

In this paper, the superscript * denotes the optimal value, the subscript $I$ denotes the integrated / centralized problem, the subscript $d$ denotes the decentralized problem, the subscript $m$ denotes the manufacturer, the subscript $r$ denotes the retailer. The other key notations and variables used in the paper are summarized in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\omega$</td>
<td>wholesale price per unit</td>
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<tr>
<td>$q$</td>
<td>order quantity (decided by the retailer)</td>
</tr>
<tr>
<td>$p$</td>
<td>retail price per unit</td>
</tr>
<tr>
<td>$D(w, \varepsilon)$</td>
<td>demand (random variable)</td>
</tr>
<tr>
<td>$w$</td>
<td>weather index</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>random variable influencing demand (unrelated to temperature)</td>
</tr>
<tr>
<td>$f(\cdot)$</td>
<td>probability density function of random variable $\varepsilon$</td>
</tr>
<tr>
<td>$F(\cdot)$</td>
<td>cumulative distribution function of random variable $\varepsilon$</td>
</tr>
<tr>
<td>$\bar{w}$</td>
<td>the strike temperature</td>
</tr>
<tr>
<td>$s$</td>
<td>salvage value per unit leftover</td>
</tr>
<tr>
<td>$c$</td>
<td>unit production cost of manufacturer</td>
</tr>
<tr>
<td>$b$</td>
<td>buy-back price per unit</td>
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We now give the following assumptions that are made in this paper:

**Assumption 1** The manufacturer and the retailer are both risk neutral, and as long as $\pi_i > 0, i = m, r$, they all prefer to participate in the market transaction.

**Assumption 2** We suppose that there is no shortage cost incurred by the manufacturer or retailer, apart from the lost gross margin.

**Assumption 3** Let $D(w, \varepsilon)$ denote the random demand parameterized by temperature index $w$ and random source $\varepsilon$, which is independent of $w$. We assume that both the demand function and the random variables are continuous. We suppose that higher temperatures have an adverse impact on demand of seasonal product, i.e. $D(w, \varepsilon)$ is stochastically...
decreasing in \( w \). Further, there is a strike or threshold temperature \( w \) at which the demand begins to reduce. We initially assume it is set exogenously.

**Assumption 4** We also assume all the information to be common knowledge to both the manufacturer and the retailer.

**Assumption 5** To avoid trivial solutions, we assume that \( s < c < \omega < p \).

### 3.2. The Centralized Decision-making Supply Chain Model

The centralized decision-maker decides the quantity \( q_i \) that maximizes the entire supply chain profit. The stochastic profit for the entire supply chain profit is

\[
\pi_i(q) = p \min\left(q, D(w, \varepsilon)\right) + s\left(q - D(w, \varepsilon)\right) - cq
\]

(1)

i.e., the unit price multiplied by the quantity sold, plus the salvaging, and then less gross production cost. From (1), we can then rewrite the entire supply chain’s profit as

\[
\pi_i(q) = (p - c)q - (p - s)\left(q - d(w)\varepsilon\right)
\]

\[
= (p - c)q - (p - s)\int_0^{q/d(w)}\left(q - d(w)x\right)dx
\]

\[
= (p - c)q - \frac{1}{2}\left(p - s\right)\frac{q^2}{d(w)}
\]

(2)

Taking the second order derivative on Equation (2) with respect to \( q \), we can obtain

\[
\frac{d^2\pi_i(q)}{dq^2} = -\frac{(p - s)}{d(w)} < 0
\]

(3)

As a result, \( \pi_i(q) \) is concave regarding \( q \) and the optimal \( q_i^* \) satisfies the first order condition (F.O.C). According to the F.O.C, the first-best ordering quantity of the entire supply chain system is

\[
q_i^* = \frac{(p - c)d(w)}{p - s}
\]

(4)

### 3.3. The Decentralized Decision-making Supply Chain Model

In the absence of a coordination contract, we characterize the optimal decisions of the manufacturer and the in the decentralized system. The manufacturer’s profit is
\[ \pi_{m_1}(q) = (\omega - c)q \]

(5)

i.e., the gross margin per unit multiplied by the quantity sold. Without a coordination contract, the retailer’s objective is to maximize his profit as follows by setting an optimal an optimal order quantity \( q_d \)

\[ \pi_{r_1}(q) = p \min(q, D(w, \varepsilon)) + s(q - D(w, \varepsilon))^+ - \omega q \]

(6)

i.e., the unit price multiplied by the quantity sold, plus the salvaging, and then less gross procurement cost. From (6), we can then rewrite the retailer’s profit as

\[
\pi_{r_1}(q) = (p - \omega)q - (p - s)(q - D(w, \varepsilon))^+ = (p - \omega)q - (p - s) \int_0^{q/d(w)} q - d(w)x \, dx = (p - \omega)q - \frac{1}{2}(p - s)q^2/d(w)
\]

(7)

Taking the second order derivative on Equation (7), we can easily obtain

\[ d^2\pi_{r_1}(q)/dq^2 = -(p - s)/d(w) < 0. \]

As a result, \( \pi_{r_1}(q) \) is concave regarding \( q \) and the optimal \( q_d^* \) satisfies the first order condition (F.O.C). According to the F.O.C, the first-best ordering quantity of the retailer is

\[ q_d^* = \left( \frac{p - \omega}{p - s} \right) d(w) \]

(8)

Note that the optimal order quantity (4) for the centralized supply chain are exactly the same as (8) for the decentralized system, as long as the wholesale price \( \omega \) is replaced by the unit production cost \( c \). We can establish the following theorem.

**Theorem1** Compared with the centralized supply chain system, the retailer in the decentralized system orders less, i.e. \( q_d^* < q_c^* \).

**Proof:** Because \( \omega > c \), the result \( q_d^* < q_c^* \) follows immediately from (4) and (8). \( \square \)

Theorem1 implies that the retailer’s order quantity in decentralized system is always less than that in the centralized system, since under the influence of adverse weather the
The retailer will always choose a conservative order quantity in order to maximize his own profit. The corollary below follows from theorem 1.

Corollary 1 As the adverse weather index $w$ increases, the optimal order quantity $q^*_d, q^*_c$ decreases.

It is easy to see the intuition of Corollary 1. A higher weather index means that the demand is more sensitive, which therefore induces the retailer to order a less quantity. Thus, we need to design a reasonable contract mechanism to motivate the retailer to choose a higher order quantity, and to achieve the supply chain coordination.

4. Supply Chain Coordination Based on Buy-back Contract

It follows from Theorem 1 that, if the order quantity adopted in the decentralized system are the same as those in the centralized system, then the supply chain coordination will be achieved. Coordination between the manufacturer and the retailer should result in the optimal decision in the centralized system to be adopted, so that both parties benefit and the profit loss caused by incoordination is eliminated. In order to motivate the retailer to order more, in this section, we investigate the possibility of achieving supply chain coordination with buy-back contracts.

Buy back contracts are also called returns polices, but, unfortunately, both names are somewhat misleading since they both imply the units remaining at the end of season are physically returned to the manufacturer. However, buy back is sometime referred to as “markdown money” when the retailer’s salvage value is higher. In fact, an important implicit assumption is that the manufacturer can verify the number of remaining unites and the cost of such monitoring does not negate the profits created by the contract. With a buy back contract the manufacturer charges the retailer $\omega_h$ per unit purchased, but pays the retailer $b$ per unit remaining at the end of the season. With a buy back contract the manufacturer’s profit function is

$$\pi_{m1} (q) = (\omega_h - c)q - b(q - D(w, \epsilon))$$

$$\pi_{m2} (q) = (\omega_h - c)q - b(q - D(w, \epsilon))$$

(9)

With a buy back contract the retailer’s profit function is
Now, we propose a specific incentive scheme (buy-back contracts), which consists of two parts: (1) a wholesale price contract, and (2) a buy back arrangement. Specifically, our wholesale price contract suggests that the manufacturer should offer his wholesale price as

\[ \omega_h = b + \lambda c + (1 - \lambda)s \]

(11)

Where \( b \) is the compensation price of any unsold products from the retailer at the end of the season. The parameter \( \lambda \) is a constant taking value in \((0,1)\), which represents the share of net profit the retailer wishes to take. Consequently, \( 1 - \lambda \) is the share of the net profit for the manufacturer.

In addition to the wholesale price contract, our incentive contracts shows that, for each unsold unit of seasonal products at the end of the season, the manufacturer compensates the retailer an amount \( b \) as follows:

\[ b = (1 - \lambda)(p - s) \]

(12)

The wholesale contract above has an alternative form as follows, which is equivalent to (11), but is different in implementation.

\[ \omega_h = \lambda c + (1 - \lambda)p \]

(13)

Theorem 2 shows that the buy-back contracts above can coordinate the supply chain of seasonal product under weather-related demand.

Theorem 2 For any \( 0 \leq \lambda \leq 1 \), the buy-back contracts, i.e., the wholesale price contract (11) (or equivalently (13)) together with the compensation contract (12) can induce the retailer to order up to \( q^* \), so contracts with those structure coordinate the supply chain.

Proof: Substituting (11) (or (13)) and (12) into (9), under the proposed buy-back contracts, the manufacturer’s profit can be calculated as follows:
\[ \pi_{m2}(q) = \left( \lambda c + (1-\lambda) p - c \right)q - (1-\lambda)(p-s)(q-D(w, \varepsilon)) \]
\[ = \left( (1-\lambda) p - (1-\lambda)c \right)q - (1-\lambda)(p-s)(q-D(w, \varepsilon)) \]
\[ = (1-\lambda) \left[ (p-c)q - (p-s)(q-D(w, \varepsilon)) \right] \]
\[ = (1-\lambda) \pi_i(q) \]

(14)

Equation (14) implies that under proposed contract mechanism, the manufacturer and the retailer’s optimal will be \( \pi_{m2}(q) = (1-\lambda) \pi_i^*(q) \), \( \pi_{r2}(q) = \lambda \pi_i^*(q) \), respectively. Because \( \pi_{m2}(q) \) ( \( \pi_{m2}(q) \) ) is an affine function of \( \pi_i^*(q) \), it is easy to see that the decentralized and centralized solutions coincide.

\[ \square \]

Theorem 2 implies that the buy-back contracts of the form (12) and (13) eliminate the effect of double marginalization that would prevent supply chain coordination from being achieved. Moreover, the contract has sufficient flexibility to allow for any division of the supply chain’s profit between the manufacturer and the retailer through parameter \( \lambda \). For a fixed \( \lambda \), we now discuss how the proposed contracts (12) and (13) can be implemented in practice.

Contract (13) can be implemented as follows: The retailer can observe the manufacturer’s unit cost \( c \) and the manufacturer can observe the retailer’s unit selling price \( p \). The manufacturer can adjust his wholesale price according to (13). The buy-back arrangement (12) is a compensation contract. This is a common practice in industry, which can be easily implemented based on the retailer’s unit selling price \( p \) and salvage value per unit leftover \( s \).

Recall that the manufacturer’s motive for offering the buy-back contracts is to induce the retailer to order more seasonal products. Although the Theorem 2 shows that the buy-back contracts coordinate the supply chain, the retailer sometime may be unwilling to participate in the contract if it is accompanied by a higher wholesale price. Thus, we now wish to determine whether it is possible to construct a coordinating buy-back contracts that are Pareto improving for both the manufacturer and the retailer.

If \( (1-\lambda) \pi_i^*(q) = \pi_{m2}^*(q) \geq \pi_{m1}^*(q) \), the manufacturer is better off under the buy-back contracts. When \( \lambda = 1 - \pi_{m1}(q)/\pi_i^*(q) \), \( \pi_{m1}^*(q) = \pi_{m2}^*(q) \) and \( \pi_{r2}^*(q) = \pi_i^*(q) - \pi_{m1}^*(q) \), meaning that the retailer takes all the incremental profit while the manufacturer’s profit remains the
same as that without the incentive scheme. In this case, the parameter $\lambda_{\text{max}}$ denotes the upper bound as follows:

$$\lambda_{\text{max}} = 1 - \frac{\pi_{m1}^*(q)}{\pi_f^*(q)}$$

(15)

Now consider $\lambda\pi_f^*(q) = \pi_{r2}^*(q)$, the retailer is better off under the buy-back contracts. When $\lambda = \frac{\pi_{r1}^*(q)}{\pi_f^*(q)}$, $\pi_{m2}^*(q) = \pi_f^*(q) - \pi_{r1}^*(q)$ and $\pi_{r1}^*(q) = \pi_{r2}^*(q)$, meaning that the manufacturer takes all the incremental profit while the retailer’s profit remains the same as that without the incentive scheme. In this case, the parameter $\lambda_{\text{min}}$ denotes the lower bound as follows:

$$\lambda_{\text{min}} = \frac{\pi_{r1}^*(q)}{\pi_f^*(q)}$$

(16)

We now have the following theorem on whether the buy-back contracts are guaranteed to make both parties better off.

**Theorem 3** The buy-back contracts are Pareto improving for both the manufacturer and the retailer if condition (12) and (13) hold and $\lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]$.

Note that the final choice of $\lambda \in [\lambda_{\text{min}}, \lambda_{\text{min}}]$ depends on the bargaining powers of the manufacturer and the retailer, but any such $\lambda$ ensures that both of them are better off by coordinating with each other.

5. **Numerical Experiments**

More management insights are gained based on the following numerical experiments. In this section, we analyze the results of our numerical example, which we have conducted to investigate the impacts of adverse weather for the optimal order quantities and the effectiveness of the buy-back contracts for the manufacturer and the retailer. The stochastic demand function is taken as an additive form $D(w) = a - kw + \varepsilon$, where $a > 0, k > 0$, and $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ is a uniformly distribute random variable. The other related parameters are valued as follows.
TABLE 2. The parameters

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<th></th>
<th>4.</th>
<th>0</th>
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<td>.5</td>
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We draw the Figure 1-2 by MATLAB 7.0, and interpret their management insights by combing Theorem 1-3 and Corollary 1.

Fig. 1. Effect of adverse weather on order quantity and the benefits of manufacturer, retailer, and supply chain system

The results for the examples are reported in Figure 1-2. From the results given in Figure 1(a), we reveal the effects of adverse weather on the optimal order quantity under centralized and decentralized. The optimal order quantity increases as adverse weather $w$ increases in two different decision-making modes, which is consistent with the Corollary 1. Compared with centralized system, the retailer always devotes a lower order quantity in the decentralized system. This observation is consistent with theoretical finding (cf. Theorem 1). Figure 1(b) depicts the optimal profits for both the decentralized and centralized system with respect to different $w$. For both decision-making modes, the optimal profits decrease as the adverse weather $w$ increases. Compared with centralized system, the profits under the decentralized system are lower. This is because the profits loss is incurred by the double marginalization phenomenon.
Figure 2. Effect of buy-back contracts on supply chain of seasonal product coordination

Figure 2 depicts the effect of buy-back contracts on supply chain coordination. Figure 2 shows an interesting pattern with respect to the parameter $\lambda$. During the interval $0.444 \leq \lambda \leq 0.556$, the manufacturer and the retailer always obtain the profits with the buy-back contract. This means both of them are better off, i.e. achieving Pareto improving. This observation is consistent with theoretical finding (cf. Theorem 3).

6. Conclusions

Retailers of seasonal products face a challenge in matching their order quantities with uncertainty market demand that is influenced not only by inherent randomness but also by the weather during the selling season. In this scenario, we propose the buy-back contracts for newsvendor setting that can achieve supply chain of seasonal product coordination and allow an arbitrary allocation of profits between the manufacturer and the retailer. Our study has generated some useful insights based on reasonable assumptions. However, our study has a limitation that all information is common knowledge to both parties. In fact, information could be incomplete in reality. Thus, the study considers the situation under incomplete information settings, which will be very interesting fields.
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