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Spurious Regression of Time Series with Shifts in Variance

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Abstract

This paper studies the phenomenon of spurious regression when a pair of independent time series, but with shifts in variance, is found apparently to be related according to standard inference in an OLS regression. It is shown that the asymptotic distribution of t-ratio test is not invariant to shifts in variance. Furthermore, Monte Carlo experiment evidence indicates that, the presence of spurious relationship critically depends on the location and magnitude of change points, regardless of the sample size. Finally, some real data sets from the Shanghai stock database are reported for illustration.

Keywords

Spurious regression; t-ratio test; Variance shifts

1. Introduction

Although financial markets have experienced prominent episodes of instability such as the Great Depression, the financial policy regime shifts, and the start of the European Monetary system, econometric models have typical assumed structural stability. In particular, the study of the conditional variance of financial and economic data has drawn much attention because of its importance in hedging strategies and risk management. However, Lamoreux and Lastrapes (1990) [1] and van Dijk et al. (2002) [2] have given evidence many of the main macro-economic and financial variables across developed countries are characterized by existence of significant variance breaks. Many attempts have followed since then to test and estimate jumps and their sizes with conditional heteroskedasticity; see, inter alia, Inclan and Tiao (1994) [3], Busetti and Taylor (2003) [4], Chen et al. (2005) [5], Jin and Zhang (2011) [6] and Amado and Terasvirta (2014) [7], and the articles cited therein.

On the other hand, the spurious regression phenomenon occurs when there is a statistically significant relationship between two independent random variables. Since the contribution of Granger and Newbold (1974) [8] on the issue of spurious regressions in econometrics, several articles have investigated the phenomenon under a variety of structures for the data generating processes (DGP). Phillips (1986) [9] assumes that the individual series in a spurious regression are driftless random walks, while Haldrup (1994) [10] and Marmol (1998) [11] demonstrate that spurious correlations are evident in OLS regressions involving combinations of time series with integer orders of integration equal to, or greater than, one. Spurious regressions are shown to occur in models with series generated by various combinations of different types of stationary process (with and without linear trends and possibly allowing for time-varying means due to structural breaks or seasonality) by Granger et al. (2001) [12], Hassler (2003) [13], Kim et al. (2004) [14] and Stock and Watson (2007) [15], among many other.

In spite of the distinct advances implied in these literatures, we should acknowledge that some variables do not involve changes in levels or trends, and it is common for them to exhibit the presence of breaks in innovation variance. Therefore, it seems advisable to investigate the effects of such non-constant variance in the framework of t-ratio test on regression models. The primary contribution of this paper lies in, instead of allowing for shifts in levels or trends, investigating spurious regression by taking broken-variance into consideration.

The remainder of the paper is organized as follows. Section 2 introduces our reference data generating process (DGP) and derives the asymptotic distribution in the presence of changes in variance. The finite sample properties of t-ratio test are explored through Monte Carlo simulation in Section 3 and some empirical examples are given in Section 4. Section 5 closes the paper with a review of the main conclusion that can be drawn. Sketches of the proofs and auxiliary results are collected in the appendix.

2. Asymptotic for spurious regression

We consider two independent time series x_t and y_t generated from the following DGP:

$$x_t = \mu_x + u_t, u_t = \varepsilon_t + \xi_t \cdot 1_{\{t>[T\tau]\}}, \tag{1}$$

$$y_t = \mu_y + v_t, v_t = e_t + \eta_t \cdot 1_{\{t>[T\lambda]\}}, \tag{2}$$

Where

$$\varepsilon_t \sim idd\ (0, \sigma_\varepsilon^2), \xi_t \sim idd\ (0, \sigma_\xi^2), \tag{3}$$

$$e_t \sim idd\ (0, \sigma_e^2), \eta_t \sim idd\ (0, \sigma_\eta^2), \tag{4}$$

And, $E(\varepsilon_t \xi_s) = 0$, $E(e_t \eta_s) = 0$ for all t and s. $1_{\{\bullet\}}$ is the indicator function, and $\tau, \lambda \in (0,1)$ are the location of break points of x_t and y_t. Therefore, Esq. (1) and (2) respectively represent changes in variance. In short, at the point in time τ and λ, variance of $\{x_t\}$ increases from σ_ε^2 to $\sigma_\varepsilon^2 + \sigma_\xi^2$, while variance of $\{y_t\}$ increases from σ_e^2 to $\sigma_e^2 + \sigma_\eta^2$. We consider the following OLS regression model given by:

$$y_t = \alpha + \beta x_t + \varsigma_t. \tag{5}$$

The general null hypothesis on β is routinely formulated as $H_0 : \beta = 0$, and tested against the alternative hypothesis $H_1 : \beta \neq 0$. For convenience, we denote the ratio of the post-break to pre-break standard deviation as $k_x = \sqrt{\frac{\sigma_\varepsilon^2 + \sigma_\xi^2}{\sigma_\varepsilon^2}}$ and $k_y = \sqrt{\frac{\sigma_e^2 + \sigma_\eta^2}{\sigma_e^2}}$. Before establishing our main theorems, three fundamental quantities will be given by the following functions.

$$\sigma_x^2 = \sigma_\varepsilon^2 \cdot (\tau + (1-\tau)k_x^2)\ , \tag{6}$$

$$\sigma_y^2 = \sigma_e^2 \cdot (\lambda + (1-\lambda)k_y^2)\ , \tag{7}$$

And

$$\sigma_{xy}^2 = \begin{cases} \sigma_\varepsilon^2\sigma_e^2 \cdot (\tau + (\lambda-\tau)k_x^2 + (1-\lambda)(k_x^2+k_y^2)), & \tau \le \lambda, \\ \sigma_\varepsilon^2\sigma_e^2 \cdot (\tau + (\tau-\lambda)k_y^2 + (1-\tau)(k_x^2+k_y^2)), & \tau > \lambda. \end{cases}, \tag{8}$$

Hereafter, the functions σ_x^2, σ_y^2 and σ_{xy}^2 could be referred as the variance profile of the processes, since they depend solely on the time series behavior of the volatility. Notice that the variance profile satisfies $\sigma_x^2 = \sigma_\varepsilon^2$, $\sigma_y^2 = \sigma_e^2$ and $\sigma_{xy}^2 = \sigma_\varepsilon^2\sigma_e^2$ under the homoscedasticity while they deviate in the presence of heteroscedasticity. Moreover, the quantities σ_x^2 and σ_y^2 in (6) and (7) are respectively the limit of $T^{-1}\sum_{t=1}^T x_t^2$ and $T^{-1}\sum_{t=1}^T y_t^2$, may therefore be interpreted as the asymptotic average variance. Similarly, as showed in (8), σ_{xy}^2 would also be understood as the limiting of $T^{-1}\sum_{t=1}^T x_t^2 y_t^2$.

Now, we start with following lemmas which collect useful results for subsequent analysis (all proofs are provided in the Appendix). The notation $\xrightarrow{w}$ stands for convergence in distribution.

Lemma 2.1 If u_t and v_t are generated by (1)-(2), then we have that:

(i) $T^{-1/2}\sum_{t=1}^T u_t \xrightarrow{w} \sigma_x B(1)$, $T^{-1/2}\sum_{t=1}^T v_t \xrightarrow{w} \sigma_y^2 B(1)$,

(ii) $T^{-1/2}\sum_{t=1}^T u_t v_t \xrightarrow{w} \sigma_{xy}^2 B(1)$,

where $B(\cdot)$ is a standard Brownian motion.

On routine applications of $t-ratio$ test in regression models, we demonstrate that spurious regression can happen due to neglected of structural changes in variance. The following theorem collects the asymptotic behavior of the estimated parameters and test in model (5).

Theorem 2.1 Suppose x_t and y_t are respectively generated by (1)-(4) with break fraction. If the conditions of Lemma 2.1 are satisfied, then we have that

(a) $T^{1/2}\hat{\beta} \xrightarrow{W} \frac{\sigma_{xy}}{\sigma_x^2} \cdot B(1)$,

(b) $t_{\hat{\beta}} \xrightarrow{w} \frac{\sigma_{xy}}{\sigma_x \sigma_y} \cdot B(1)$ ·

This theorem provides some interesting results. The remarkable finding is that, dose not diverge as the sample size approach to infinity. The asymptotic distribution of is not standard normal and more complicated, which is intensely sensitive to the location of shifts and the ratio the order of $\hat{\beta}$ is $T^{-1/2}$, implying that is consistent and converges to its true value zero.

3. Discussion

In this section, we will numerically discuss the effects of variance changes on the size of t-ratio test in model (5) by means of Monte Carlo experiments. For each simulated sample, the percentage of rejection are obtained by 3000 replications at 5% nominal level, i.e., the percentage of t-ratio teat such that $|t_{\hat{\beta}}| > 1.96$. In order to weaken the initialization influence, we would generate these time series of length $T + 200$ and trim the first 200 observation to achieve the simulated sequences.

Data are generated from the DGP (1)-(4) with mean zero normal sequences $\{\varepsilon_t\}$ and $\{\eta_t\}$ using the rndKMn function of Gauss 5.0. Without loss of generality, we assume $\mu_x = \mu_y = 0$, $\sigma_\varepsilon^2 = \sigma_e^2 = 1$ in all cases. In order to examine the effect of structural breaks, we consider two case $\sigma_\xi^2 = \sigma_\eta^2 = 3$ and $\sigma_\varepsilon^2 = \sigma_\eta^2 = 8$ .Thus, the ratio of standard deviation k_x and k_y take this value among $\{2,3\}$. According to Theorem 2.1, these tests do not depend on the sample size, so we just report results for $T = 1000$ .Moreover, the location structural changes varies τ , λ among

$\{0.1, 0.3, 0.5, 0,7, 0.9\}$, so that both early and late shifts are taken into account.

Table 1-2 provide the percentage of rejection of $t_{\hat{\beta}}$ convergence for each break fraction and ratio of standard deviation profile. As expected, the phenomenon of spurious regression appears either the ratio of standard deviation k_x or k_y increases. More special, the spurious correlation is more severe if $k_x = k_y = 3$. For example, if $\tau = 0.5$ and $\lambda = 0.7$, the discrepancy between $k_x = k_y = 2$ and $k_x = k_y = 3$ is 36.5%,that confirms spurious regression is sensitive to the ratio of standard deviation.

Table 1. Rejection percentage of $| t_{\hat{\beta}} | > 1.96$, when $k_y = 2$

		λ				
k_x	τ	0.1	0.3	0.5	0.7	0.9
2	0.1	0.121	0.126	0.124	0.134	0.142
	0.3	0.152	0.168	0.153	0.158	0.150
	0.5	0.188	0.192	0.187	0.179	0.190
	0.7	0.214	0.212	0.218	0.204	0.208
	0.9	0.235	0.229	0.230	0.236	0.233
3	0.1	0.199	0.217	0.221	0.208	0.212
	0.3	0.263	0.268	0.259	0.266	0.263
	0.5	0.329	0.313	0.309	0.311	0.325
	0.7	0.397	0.382	0.369	0.381	0.378
	0.9	0.421	0.426	0.417	0.422	0.430

Table 2. Rejection percentage of $| t_{\hat{\beta}} | > 1.96$, when $k_y = 3$

		λ				
k_x	τ	0.1	0.3	0.5	0.7	0.9
2	0.1	0.368	0.389	0.387	0.375	0.385
	0.3	0.451	0.438	0.428	0.439	0.436
	0.5	0.492	0.476	0.489	0.478	0.503
	0.7	0.525	0.527	0.532	0.526	0.531
	0.9	0.541	0.552	0.548	0.543	0.551
3	0.1	0.397	0.419	0.435	0.439	0.442
	0.3	0.477	0.482	0.464	0.496	0.503
	0.5	0.514	0.520	0.531	0.544	0.552

	0.7	0.587	0.598	0.583	0.588	0.590
	0.9	0.614	0.618	0.606	0.609	0.613

What is surprising is that, the rate of divergence of $t_{\hat{\beta}}$ varies according to the location of break τ. When $k_x = 2, k_y = 2$ and $\lambda = 0.7$, the rejection rate are 37.5% and 54.3% for $\tau = 0.1, 0.9$. The late break fraction τ does induce the larger rejection rate. While the rejection percentage seems to do not depend on the other break fraction λ, which is an interesting result for our proposed test procedures. If $k_x = k_y = 3$ and $\tau = 0.7$, the rejection power are 59.8%, 58.3% and 58.8% for $\lambda = 0.3, 0.5, 0.7$.Thus, the location of breaks τ plays a dominated role in occurrence of spurious regression.

In order to give intuitive idea for the influence of the location of changes τ and ratio of standard deviation k_x, k_y, we provide the rejection frequency with sample size $T = 100$ in Figure 1-2. As expected, all figures clearly show spurious relationship occurs when these time series involves breaks in variance. These figures confirm the conclusion that the larger ratio of standard deviation provides higher rejection percentage. A surprising finding which could not provide by Theorem 2.1 is that there is a peak. More special, the highest rejection percentage may occur near the region of $\tau = 0.9$.

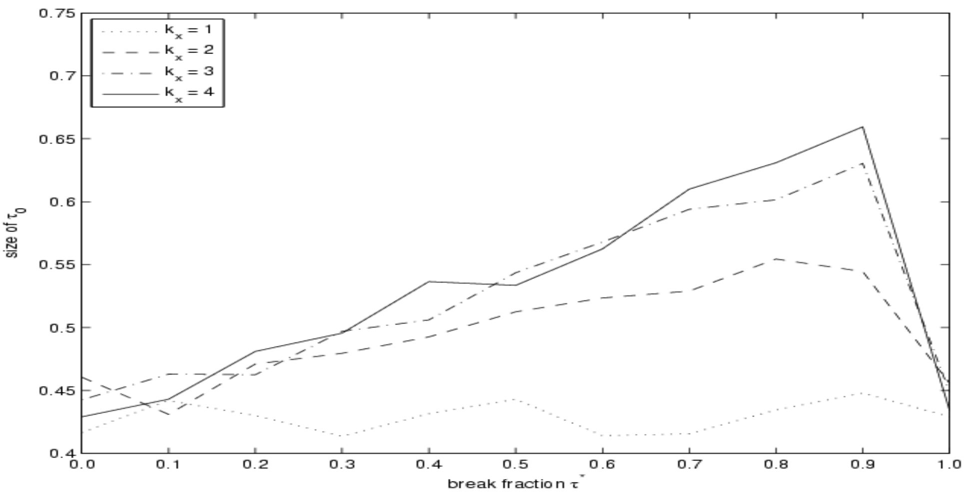


Fig.1. The rejection rate for $k_y = 2$, when $T = 1000$ and $k_y = 1,2,3,4$

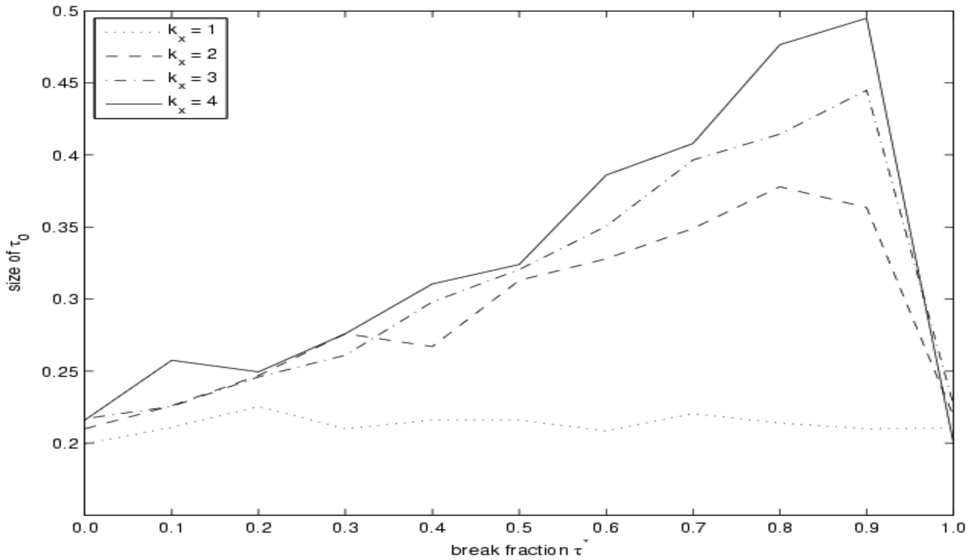
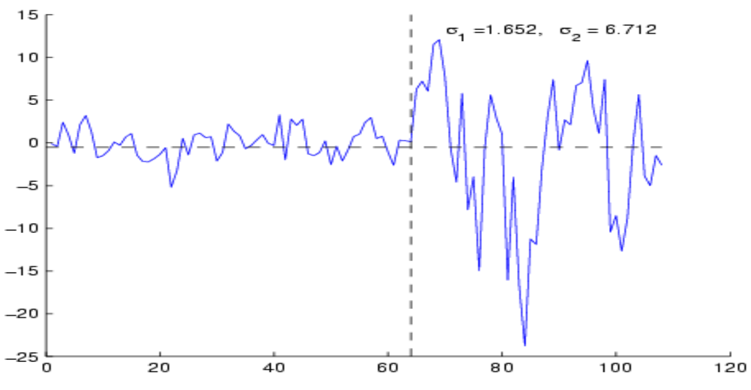


Fig. 2. The rejection rate for $k_y = 3$, when $T = 1000$ and $k_y = 1, 2, 3, 4$

4. Empirical studies

Next, we test for the presence of spurious regression phenomenon considered in Section 2 using two independent sets of weekly stock prices in Shanghai Stock database. These real data respectively collected by Western Mining ($P_{1,t}$) and Guangji Pharmaceutical ($P_{2,t}$) are from 13 November 2006 to 22 December 2008. The sample size of each group in this analysis is 107, which can be found in http://business.sohu.com/. The logarithmic difference transformation of these data are written as $\tilde{P}_{i,t} = \ln P_{i,t} - \ln P_{i,t-1}$, where $1 \le t \le 107$ and $i = 1, 2$. Figure 3shows the scatter plot of the transformed data set together with a mean regression function.



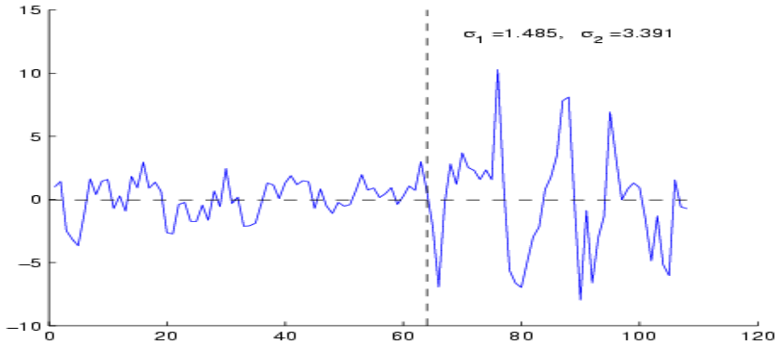


Fig.3. The left and right panel represent the logarithmic difference transformation of $\tilde{P}_{1,t}$ and $\tilde{P}_{2,t}$ respectively

A visual inspection of these time series plot in Figure 3 seems to suggest that there might be some change points in variance. Applying the test statistic proposed by Inclan and Tiao (1994) [3], which can be regarded as a variant of the CUSUM test, the suspicion is confirmed. According to previous conclusion, we need not know the exact position of these breaks. As it shows that, the dashed lines depict the sample standard deviation of the pre-break series ($\hat{\sigma}_{1,1} = 1.652$, $\hat{\sigma}_{2,1} = 1.485$) and that of the post-break series ($\hat{\sigma}_{1,2} = 6.712$, $\hat{\sigma}_{2,2} = 3.391$). Therefore, the Western Mining and Guangji Pharmaceutical sequences have variance breaks relatively late in the sample.

Applying t-ratio test to examine for possible relationship between sequences $\tilde{P}_{1,t}$ and $\tilde{P}_{2,t}$, this delivers $t_{\hat{\beta}} = 2.542$ and the p-value is less than 1%. Thus, we should accept the alternative hypothesis at 5% that there is strong evidence for the presence of spurious regression. Hence, all indicate that if time series undergo the structural breaks in variance, the spurious relationship may occur in these regressions.

5. Conclusion

In this paper, spurious regression is considered between sequences driven by variance shifts in regressions model. We prove that the asymptotic distribution of $t_{\hat{\beta}}$ is not standard normal and more complicated, which do not diverge as the sample size approaches to infinity. The Monte Carlo simulation studies and some empirical examples have been conducted to investigate the performance of our test procedures and show the existence of spurious relationship may be unambiguous. Especially, these series involve variance breaks relatively late in the sample. Given that many important economic and financial time series show some strong evidence for

Volatilities with breaks, the potential to encounter such spurious relationship in practical applications seems to be very high.

6. Appendix

Proof of Lemma 2.1: Part (i) Taking into account changes in variance, we first derive limiting distribution of $T^{-1/2}\sum_{t=1}^{T} u_t$.

Note that

$$T^{-1/2}\sum_{t=1}^{[Tr]} \varepsilon_t \xrightarrow{w} \sigma_\varepsilon B(r) , \quad T^{-1/2}\sum_{t=1}^{[Tr]} \xi_t \xrightarrow{w} \sigma_\xi B(r) ,$$

hold true. To more explicitly express the effects of these shifts in variance, we have to perform some transformations,

$$T^{-1/2}\sum_{t=1}^{[T]} u_t = T^{-1/2}\sum_{t=1}^{[T]} \varepsilon_t + T^{-1/2}\sum_{t=[T\tau]+1}^{[T]} \xi_t \xrightarrow{w} \sigma_\varepsilon B(1) + \sigma_\eta [B(1) - B(\tau)] \cdot$$

Since ε_t and ξ_t are independent, the following equation is derived that:

$$\sigma_\varepsilon B(1) + \sigma_\eta [B(1) - B(\tau)] \sim N(0, \sigma_\varepsilon^2 + \sigma_\eta^2 (1-\tau)) ,$$

and

$$\sigma_\varepsilon^2 + \sigma_\xi^2 (1-\tau) = \sigma_\varepsilon^2 (\tau + (1-\tau) k_x^2) \equiv \sigma_x^2 ,$$

where $k_x^2 = \frac{\sigma_\varepsilon^2 + \sigma_\xi^2}{\sigma_\varepsilon^2}$ .Thus, we can get

$$T^{-1/2}\sum_{t=1}^{[T]} u_t \xrightarrow{w} \sigma_x B(1) \cdot$$

and

$$T^{-1}\sum_{t=1}^{[T]} u_t^2 \xrightarrow{w} \sigma_x^2 \cdot$$

One can similarly verify that $T^{-1/2}\sum_{t=1}^{T} v_t \xrightarrow{w} \sigma_y B(1)$. Therefore, the proof of item (i) in Lemma 2.1 is established.

Since ξ_t and ε_t are independent, we have $E(\varepsilon_t+\xi_t)^2 e_t^2=(\sigma_\varepsilon^2+\sigma_\xi^2)\sigma_e^2$.

Thus $T^{-1/2}\sum_{t=[Ts]+1}^{[Tr]}(\varepsilon_t+\xi_t)e_t \xrightarrow{w} \sigma_e\sqrt{(\sigma_\varepsilon^2+\sigma_\xi^2)}(B(r)-B(s))$.

Similarly, we have

$$T^{-1/2}\sum_{t=[Ts]+1}^{[Tr]}(\varepsilon_t+\xi_t)(e_t+\eta_t)\xrightarrow{w}\sqrt{(\sigma_\varepsilon^2+\sigma_\xi^2)(\sigma_e^2+\sigma_\xi^2)}(B(r)-B(s)).$$

To prove item (ii) of Lemma 2.1, without loss of generality, we assume $\tau\le\lambda$, then

$$\sum_{t=1}^{[T]}u_tv_t=\sum_{t=1}^{T}((\varepsilon_t+\xi_t 1_{\{t>[T\tau]\}})(e_t+\eta_t 1_{\{t>[T\lambda]\}}))=\sum_{t=1}^{[T\tau]}\varepsilon_t e_t+\sum_{t=[T\tau]+1}^{[T\lambda]}(\varepsilon_t+\xi_t)e_t+\sum_{t=[T\lambda]+1}^{[T]}(\varepsilon_t+\xi_t)(e_t+\eta_t)$$

Hence, we obtain

$$T^{-1/2}\sum_{t=1}^{[T]}u_tv_t\xrightarrow{w}\sigma_\varepsilon\sigma_e B(\tau)+\sigma_e\sqrt{(\sigma_\varepsilon^2+\sigma_\xi^2)}(B(\lambda)-B(\tau))+\sqrt{(\sigma_\varepsilon^2+\sigma_\xi^2)(\sigma_e^2+\sigma_\eta^2)}(B(1)-B(\lambda))$$

$$\sim N(0,\sigma_\varepsilon\sigma_e+(\sigma_\varepsilon^2+\sigma_\xi^2)\sigma_e^2(\lambda-\tau)+(\sigma_\varepsilon^2+\sigma_\xi^2)(\sigma_e^2+\sigma_\eta^2)(1-\lambda)),$$

which follows because these (iid) sequences ε_t,ξ_t,e_t and η_t are independent each other.

Some algebra yields

$$(\sigma_\varepsilon^2\sigma_\xi^2)+(\sigma_\varepsilon^2+\sigma_\xi^2)\sigma_e^2(\lambda-\tau)+(\sigma_\varepsilon^2+\sigma_\xi^2)(\sigma_e^2+\sigma_\eta^2)(1-\lambda)=\sigma_\varepsilon^2\sigma_e^2(\tau+(\lambda-\tau)k_x^2+(1-\lambda)k_x^2k_y^2)\equiv\sigma_{xy}^2$$

Then, the proof is completed.

Proof of Theorem 2.1 By some algebra, the OLS statistics are given by

$$\hat\beta=\frac{\sum_{t=1}^{T}x_tx_t-T^{-1}\sum_{t=1}^{T}x_t\sum_{t=1}^{T}y_t}{\sum_{t=1}^{T}(x_t-\bar x)^2},\qquad \hat\alpha=T^{-1}\sum_{t=1}^{T}y_t-\hat\beta T^{-1}\sum_{t=1}^{T}x_t,$$

$$s^2=T^{-1}\sum_{t=1}^{T}\zeta_t^2=T^{-1}\sum_{t=1}^{T}(y_t-\bar y)^2-\hat\beta T^{-1}\sum_{t=1}^{T}(x_t-\bar x)^2,\quad s_{\hat\beta}^2=\frac{s^2}{\sum_{t=1}^{T}(x_t-\bar x)^2},\quad t_{\hat\beta}=\frac{\hat\beta}{s_{\hat\beta}},$$

Then, we have

$$T^{-1/2}\hat\beta=\frac{T^{-1/2}\sum_{t=1}^{T}u_tv_t-(T^{-1/2}\sum_{t=1}^{T}u_t)(T^{-1/2}\sum_{t=1}^{T}v_t)}{T^{-1}\sum_{t=1}^{T}u_t^2+Op(1)}\xrightarrow{w}\frac{\sigma_{xy}}{\sigma_x^2}\cdot B(1)$$

and

$$\hat\alpha=T^{-1}\sum_{t=1}^{T}y_t-T^{-1/2}\cdot T^{1/2}\hat\beta\cdot T^{-1}\sum_{t=1}^{T}x_t\xrightarrow{w}\mu_y$$

where the weak convergence is based on $T^{-1}\sum_{t=1}^{T}x_t\xrightarrow{w}\mu_x$ and $T^{-1}\sum_{t=1}^{T}y_t\xrightarrow{w}\mu_y$.

Applying the asymptotic behavior of $\hat{\beta}$, then

$$s^2 = T^{-1}\sum_{t=1}^{T} v_t^2 - T^{-1}\cdot(T^{1/2}\hat{\beta})^2(T^{-1}\sum_{t=1}^{T} u_t^2) + Op(1) \xrightarrow{w} \sigma_y^2 \cdot$$

Hence, we obtain

$$Ts_{\hat{\beta}}^2 = \frac{s^2}{T^{-1}\sum_{t=1}^{T} u_t^2 + Op(1)} \xrightarrow{w} \frac{\sigma_y^2}{\sigma_x^2},$$

Combining these results with the asymptotic behavior of $\hat{\beta}$, we obtain

$$t_{\hat{\beta}} = \frac{T^{-1/2}\hat{\beta}}{(Ts_{\hat{\beta}}^2)^{1/2}} \xrightarrow{w} \frac{\sigma_{xy}}{\sigma_x\sigma_y} B(1)$$
.

Then, the proof is completed.

Acknowledgements

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