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Informationally Efficient Mechanism for the Augmented Inner Product

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Abstract

We study the problem of designing the mechanism for the augmented inner product $F(a,b) = a_1b_1 + a_2b_2 + b_3 + a_1{}^2(b_2 \neq 0)$. For the given goal function $F(a,b) = a_1b_1 + a_2b_2 + b_3 + a_1{}^2(b_2 \neq 0)$, we obtain the reflexive rectangle method correspondence V(•) and the reflexive rectangle method covering C_v which is a partition. After that we construct the message space M, the equilibrium message function g and the outcome function h which constitute the mechanism $\pi = (M, g, h)$. Finally, we demonstrate that the mechanism $\pi = (M, g, h)$ can realize the given goal function F and satisfy informational efficiency and decentralization.

Keywords

Mechanism design, informational efficiency, decentralization, reflexive rectangle method correspondence, reflexive rectangle method covering, condensation.

1. Introduction

A mechanism is a mathematical structure through which economic activity is guided and coordinated [29]. Mechanism design is initially proposed by Leonid Hurwicz (Nobel Laureate in Economics in 2007). Since the 1980s, much attention has been paid to widespread application of mechanism design. Such as auction [1, 2, 3, 9-20], public goods [4, 5], insurance [6, 24], tax [22], sequencing problems [27] and network economics [8]. Economists always tie the good mechanism to incentive compatibility or Pareto optimality or informational efficiency. The mechanism constructed in those research is only satisfied with incentive compatibility or optimality. However, there are a few studies on informationally efficient and decentralized mechanism.

The reasons why we are so interested in designing the mechanism for the augmented inner product $F(a,b) = a_1b_1 + a_2b_2 + b_3 + a_1{}^2(b_2 \neq 0)$ are the following: (1) it plays an important role in economics [29], (2) the mechanism designed for the augmented inner product can satisfy informational efficiency and decentralization, (3) the mechanism for the given augmented one-dimensional inner product $F(a,b) = a_1b_1 + b_2$ has been constructed by Hurwicz and Reiter by making use of condensation method which primarily transforms $G_s(\bar{\theta},\theta) = 0$ to $g_s(m,\theta) = 0$, where $s = 1,2,\cdots,d$ [29]. However the mechanism for the augmented two-dimensional inner product $F(a,b) = a_1b_1 + a_2b_2 + b_3 + a_1{}^2(b_2 \neq 0)$ has not yet been constructed.

Our work is to construct the mechanism for the given goal function $F(a,b) = a_1b_1 + a_2b_2 + b_3 + a_1{}^2(b_2 \neq 0)$, we obtain the reflexive rectangle method correspondence $V(\cdot)$ and the reflexive rectangle method covering C_v which is a partition. What is important is that we construct the informationally efficient and decentralized mechanism based on condensation method [29]. The paper is organized as follows: In section 2, many concepts and lots of important lemmas are given. In section 3, the main results are provided. In section 4, the conclusions are made.

2. Preliminaries

In this section, we introduce some basic concepts and a number of important results which are used through this paper. For more details can be found in the book [29].

2.1 The reflexive rectangle method correspondence

Let Θ be a parameter space, let Θ^1, Θ^2 stand for the type space for the two participants respectively and $\Theta=\Theta^1\times\Theta^2$. Let $F(\bullet):\Theta\to Z$ be a goal function. Let $V(\bullet):\Theta\to C_v$ be a rectangle correspondence, then we have $V(\theta)=A(\theta)\times B(\theta)$ for all $\theta\in\Theta$, where $A(\theta)\subseteq\Theta^1$, $B(\theta)\subseteq\Theta^2$, and $C_v=\{V(\theta)=A(\theta)\times B(\theta):\theta\in\Theta\}$.

Let $\Theta=\Theta^1\times\Theta^2$ and let $L(\bullet):\Theta\to C_l$ be a left rectangle method correspondence. Then for any $\bar\theta=(\overline{\theta^1},\overline{\theta^2})$, we have $L(\bar\theta)=A(\bar\theta)\times B^*(A(\bar\theta);\bar\theta)$, where $\overline{\theta^1}\in A(\bar\theta)\subseteq\Theta^1$, $B^*(A(\bar\theta);\bar\theta)=\cup\{B\subseteq\Theta^2:\overline{\theta^2}\in B,\ A(\bar\theta)\times B\subseteq F^{-1}(F(\bar\theta))\}$ and $C_l=\{L(\bar\theta)=A(\bar\theta)\times B^*(A(\bar\theta);\bar\theta):\bar\theta\in\Theta\}$. Let $R(\bullet):\Theta\to C_r$ be a right rectangle method correspondence. Then for any $\bar\theta=(\overline{\theta^1},\overline{\theta^2})\in\Theta$, we get $R(\bar\theta)=A^*(B(\bar\theta);\bar\theta)\times B(\bar\theta)$, where $\overline{\theta^2}\in B(\bar\theta)\subseteq\Theta^2$, $A^*(B(\bar\theta);\bar\theta)=\cup\{A\subseteq\Theta^1:\overline{\theta^1}\in A,\ A\times B(\bar\theta)\subseteq F^{-1}(F(\bar\theta))\}$ and $C_r=\{R(\bar\theta)=A^*(B(\bar\theta);\bar\theta)\times B(\bar\theta):\bar\theta\in\Theta\}$.

The following three Lemmas are used to construct reflexive rectangle method correspondence.

Lemma 2.1 ([29]). Let $\Theta=\Theta^1\times\Theta^2$. If $A^*(B^*(A(\bar\theta);\bar\theta))=A(\bar\theta)$, then

$$L(\bar\theta)=R(\bar\theta)$$

for all $\bar\theta\in\Theta$.

From above Lemma 2.1, the reflexive rectangle method correspondence $V(\bullet)$ can be defined by $V(\bullet)=L(\bullet)=R(\bullet)$.

Lemma 2.2 ([29]). Let $\Theta=\Theta^1\times\Theta^2$, let $A_1\subseteq\Theta^1$ and let $B_1=B^*(A_1;\bar\theta)$. If $A_2=A^*(B_1;\bar\theta)$, then

$$B^*(A_2;\bar\theta)=B_1$$

for all $\bar{\theta} \in \Theta$.

Lemma 2.3 ([29]). Let V(•):Θ→ C_v be the the reflexive rectangle method correspondence. Then the set $C_v = \{V(\bar{\theta}): \bar{\theta} \in \Theta\}$ be a reflexive rectangle method covering.

2.2 The reflexive rectangle method covering

Let V(•):Θ→ C_v be a correspondence and let G(•;•) be a function of the correspondence V(•) such that V($\bar{\theta}$)={θ∈ $\Theta: G(\bar{\theta},\theta) = 0\}$ for all $\bar{\theta} \in \Theta$. The next three facts are used to verify the reflexive rectangle method covering C_v is a partition.

Lemma 2.4 ([29]). Let G(•):Θ $\times \Theta \to R^d$, let $G^1(\bullet): \Theta \times \Theta^1 \to R^{d1}$, and let $G^2(\bullet): \Theta \times \Theta^2 \to R^{d2}$, where $d_1 + d_2 = d$. Then $G(\bar{\theta},\theta) = 0$ if and only if

$$\begin{cases} G^1(\bar{\theta}, a) = 0 \\ G^2(\bar{\theta}, b) = 0 \end{cases}$$

for all $\bar{\theta}, \theta \in \Theta$.

Lemma 2.5 ([29]). The correspondence V(•) is symmetric if and only if the function G(•;•) is symmetric.

Note that the function G(•;•) is symmetric can be defined by $G(\bar{\theta},\theta) = 0 \leftrightarrow G(\theta,\bar{\theta}) = 0$.

Lemma 2.6 ([29]). The reflexive rectangle method covering C_v is a partition if and only if:

(1) The reflexive rectangle method covering C_v is generated by a symmetric and self-belonging correspondence V(•):Θ→ C_v.

(2) The reflexive rectangle method covering C_v is irreducible.

Recall that the correspondence V(•) can be said to satisfy self-belonging if $\theta \in V(\theta)$.

2.3 The condensation setting

Let x=($x_1, x_{2,\ldots,} x_m$)∈ X, and let x_i be a primary variable. Let y = ($y_1, y_2, \cdots, y_n$)∈ Y, and let y_j be a secondary variable, where i∈ $\{1,2,\cdots,m\}$, j∈ $\{1,2,\cdots,n\}$. Let w = ($w_1, w_2, \cdots, w_r$)∈ W.

The following results on condensation theory are used to construct the mechanism.

Definition 2.7 ([29]). Let $\Phi^{\sigma}(x,y)$ be a function on $X \times Y$. Then the Hessian matrix H^{σ} is defined by

$$
H^{\sigma} = \begin{array}{c|cccc}
\Phi^{\sigma} & \partial y_1 & \partial y_2 & \cdots & \partial y_n \\
\hline
\partial x_1 & \partial^2\Phi^{\sigma}/\partial x_1 \partial y_1 & \partial^2\Phi^{\sigma}/\partial x_1 \partial y_2 & \cdots & \partial^2\Phi^{\sigma}/\partial x_1 \partial y_n \\
\partial x_2 & \partial^2\Phi^{\sigma}/\partial x_2 \partial y_1 & \partial^2\Phi^{\sigma}/\partial x_2 \partial y_2 & \cdots & \partial^2\Phi^{\sigma}/\partial x_2 \partial y_n \\
\vdots & \cdots & \cdots & \cdots & \cdots \\
\partial x_m & \partial^2\Phi^{\sigma}/\partial x_m \partial y_1 & \partial^2\Phi^{\sigma}/\partial x_m \partial y_2 & \cdots & \partial^2\Phi^{\sigma}/\partial x_m \partial y_n
\end{array}
$$

The block Hessian matrix H(x, y) is given by

$$H(x,y)=(H^1(x,y) \vdots \; H^2(x,y) \vdots \; \cdots \vdots \; H^N(x,y)),$$

and the block bordered Hessian matrix BH(x, y) is denoted by

$$BH(x,y)=(\Phi_x^1 \vdots \; \Phi_x^2 \vdots \; \cdots \vdots \; \Phi_x^N \vdots \; H^1 \vdots \; H^1 \vdots \; \cdots \vdots \; H^N).$$

Where

$$\Phi_x^{\sigma}(x,y)=\begin{pmatrix} \frac{\partial \Phi^{\sigma}}{\partial x_1} \\ \vdots \\ \frac{\partial \Phi^{\sigma}}{\partial x_m} \end{pmatrix}$$

is called a gradient vector and $\sigma \in \{1,2,\cdots,N\}$.

Definition 2.8 ([29]). Let $\Phi^{\sigma}(x, y)$ be a function on $X \times Y$, let $\Gamma^{\sigma}(w,y)$ be a function on $W \times Y$, let $A_i(\bullet): X \to W_i$. Assume the function $\Phi^1(x,y), \cdots, \Phi^N(x,y)$ and the function $\Gamma^1(w,y), \cdots, \Gamma^N(w,y)$ is continuous and differentiable, for arbitrary $(x, y) \in X \times Y$, if

$$
\begin{cases}
\Phi^1(x,y) = \Gamma^1((A_1(x), A_2(x), \cdots, A_r(x)), y) \\
\Phi^2(x,y) = \Gamma^2((A_1(x), A_2(x), \cdots, A_r(x)), y) \\
\vdots \\
\Phi^N(x,y) = \Gamma^N((A_1(x), A_2(x), \cdots, A_r(x)), y)
\end{cases},
$$

then the function $\Phi^{\sigma}(x, y)$ is called a condensed function, the function $\Gamma^{\sigma}(w,y)$ is called a condensation form, the correspondence $A(x) = (A_1(x), A_2(x), \cdots, A_r(x))$ is called a condensation correspondence, where $\sigma \in \{1,2,\cdots,N\}$ and $i \in \{1,2,\cdots,r\}$.

Definition 2.9 ([29]). Let $a = (a^*, a^{**}) \in \Theta^1$ and let $b = (b^*, b^{**}) \in \Theta^2$. If the sub-vector a^{**} and b^{**} satisfy the following conditions:

(1) The Jacobin sub-matrix $J^1 = \partial G^1/\partial a^*$ and $J^2 = \partial G^2/\partial b^*$ are nonsingular.

(2) There is at least one nonempty residual sub-vector between a^{**} and b^{**}.

Then the sub-vector a^{**} and b^{**} are called the residual sub-vector, where the Jacobin matrix can be defined by

$$J=\begin{pmatrix} \frac{\partial G^1}{\partial a^*} & \frac{\partial G^1}{\partial a^{**}} & \frac{\partial G^1}{\partial b^*} & \frac{\partial G^1}{\partial b^{**}} \\ \frac{\partial G^2}{\partial a^*} & \frac{\partial G^2}{\partial a^{**}} & \frac{\partial G^2}{\partial b^*} & \frac{\partial G^2}{\partial b^{**}} \end{pmatrix}.$$

Lemma 2.10 ([29]). Let H be the block Hessian matrix and let BH be the block bordered Hessian matrix, then the following statements hold:

(1) There exists a condensation correspondence $A(\bullet) = (A_1(\bullet), A_2(x\bullet), \cdots, A_r(\bullet))$ if and only if $R(BH) \leq r$;

(2) There exists a condensation correspondence $A(\bullet) = (A_1(\bullet), A_2(x\bullet), \cdots, A_r(\bullet))$ if $R(BH) \leq r$, $R(H) = r$.

2.4 The property of mechanism

Let $\pi = (M, g, h)$ be a mechanism. Write M for the message space, denote the equilibrium message function by g, and h stands for the outcome function.

Lemma 2.11 ([29]). Let $F(\bullet): \Theta \to Z$ be a goal function, and let $\mu(\bullet): \Theta \to M$ be a equilibrium message correspondence. Then the mechanism $\pi = (M, g, h)$ can be said to realize the goal function F if the following conditions hold:

(1) There exists a message $m \in M$ such that $g(m,\theta) = 0$ for all $\theta \in \Theta$;

(2) There exists a equilibrium message $m \in \mu(\theta)$ such that $h(m) = F(\theta)$ for all $\theta \in \Theta$.

Lemma 2.12 ([29]). Let $F(\bullet):\Theta\to Z$ be a goal function, and let C be a covering. Then the covering C is called a F-maximal covering if and only if the covering C is a reflexive rectangle method covering.

In addition, informational efficiency can be influenced by the coarsening of the covering C, the dimension of the message space M, the equation efficiency and the redundancy.

3. Main results

Let $\Theta=\Theta^1\times\Theta^2$ be a parameter space, and let $\theta=(a,b)\in\Theta$, $a=(a_1, a_2)\in\Theta^1$, $b=(b_1,b_2,b_3)\in\Theta^2$.

Lemma 3.1 Let $F(a,b)=a_1b_1+a_2b_2+b_3+a_1{}^2(b_2\neq 0)$ be a goal function on Θ and let $V(\bullet):\Theta\to C_v$ be a rectangle correspondence. Then there exists the reflexive rectangle method correspondence $V(\bullet):\Theta\to C_v$ such that

$$V(\bar{\theta})=\{a\in\Theta^1: G^1(\bar{\theta},a)=0\}\times\{(b_1,b_2,b_3): b_j=\bar{b_j}, j=1,2,3\}$$

for all $\theta\in\Theta$, where $G^1(\bar{\theta},a)=a_1\overline{b_1}+a_2\overline{b_2}+\overline{b_3}+a_1{}^2-\bar{F}$ and $\bar{F}=F(\bar{\theta})$.

Proof. For each $\bar{\theta}\in\Theta$, suppose that $V(\bar{\theta})=A(\bar{\theta})\times B(\bar{\theta})$.

Firstly, let $A(\bar{\theta})$ be a maximal set which satisfies $A(\bar{\theta})\times\{\bar{b}\}\subseteq F^{-1}(F(\bar{\theta}))$.

Then for arbitrary $a\in A(\bar{\theta})$, we have

$$F(a,\bar{b})=F(\bar{a},\bar{b})=\bar{F},$$

specifically,

$$a_1\overline{b_1}+a_2\overline{b_2}+\overline{b_3}+a_1{}^2=\bar{F}. \qquad (1)$$

Let

$$G^1(\bar{\theta},a)=a_1\overline{b_1}+a_2\overline{b_2}+\overline{b_3}+a_1{}^2-\bar{F}=0 \qquad (2)$$

by (2), we obtain

$$A(\bar{\theta})=\{a\in \Theta^1: G^1(\bar{\theta},a)=0\}. \tag{3}$$

Secondly, from (1), we have

$$a_2=\frac{1}{\bar{b}_2}(\bar{F}-\ a_1\overline{b_1}-\overline{b_3}-a_1{}^2), \tag{4}$$

fixed $A(\bar{\theta})=\{a\in \Theta^1: G^1(\bar{\theta},a)=0\}$, let $B(\bar{\theta})$ be a maximal set that satisfies $A(\bar{\theta})\times B(\bar{\theta})$ $\subseteq F^{-1}(F(\bar{\theta}))$. Then for any $a\in A(\bar{\theta})$, and any $b\in B(\bar{\theta}))$, we get

$$F(a,b)=\bar{F},$$

namely,

$$a_1b_1+a_2b_2+b_3+a_1{}^2=\bar{F}. \tag{5}$$

Substituting (4) into (5), then we have

$$(b_1-\frac{b_2}{\bar{b}_2})a_1+(1-\frac{b_2}{\bar{b}_2})a_1{}^2+[\frac{b_2}{\bar{b}_2}(\bar{F}-\overline{b_3})+b_3-\ \bar{F}]=0. \tag{6}$$

It is obvious (6) is an identity with respect to a_1, we therefore obtain

$$b_1-\frac{b_2}{\bar{b}_2}\overline{b_1}=0, \tag{7}$$

$$1-\frac{b_2}{\bar{b}_2}=0, \tag{8}$$

$$\frac{b_2}{\bar{b}_2}(\bar{F}-\overline{b_3})+b_3-\ \bar{F}. \tag{9}$$

By (7), (8), (9). Let

$$G^{21}(\bar{\theta},b_1)=b_1-\overline{b_1}=0, \tag{10}$$

$$G^{22}(\bar{\theta},b_2)=b_2-\overline{b_2}=0, \tag{11}$$

$$G^{23}(\bar{\theta},b_3)=b_3-\overline{b_3}=0. \tag{12}$$

Then we can obtain

$$B(\bar{\theta})=\{(b_1,b_2,b_3):b_j=\bar{b}_j, j=1,2,3\}. \tag{13}$$

It means $A(\bar{\theta})\times B(\bar{\theta})$ is a left rectangle, namely $A(\bar{\theta})\times B(\bar{\theta})=L(\bar{\theta})$.

Thirdly fixed $B(\bar{\theta}) = \{(b_1, b_2, b_3): b_j = \bar{b_j}, j = 1,2,3\}$, let $A'(\bar{\theta})$ be a maximal set which satisfies $A'(\bar{\theta}) \times B(\bar{\theta}) \subseteq F^{-1}(F(\bar{\theta}))$. Then for any $a \in A'(\bar{\theta})$ and any $b \in B(\bar{\theta})$, we obtain

$$F(a,b)=F(\bar{a}, \bar{b}),$$

namely,

$$a_1 b_1 + a_2 b_2 + b_3 + {a_1}^2 = \bar{F}.$$

As $b \in B(\bar{\theta})$, namely $b_1=\overline{b_1}$, $b_2=\overline{b_2}$, $b_3=\overline{b_3}$, we have

$$a_1\overline{b_1} + a_2\overline{b_2} + \overline{b_3} + {a_1}^2=\bar{F}.$$

We can infer that $A'(\bar{\theta}) = A(\bar{\theta})$. It reveals that $A(\bar{\theta})\times B(\bar{\theta})$ is a right rectangle, namely $A(\bar{\theta})\times B(\bar{\theta})=R(\bar{\theta})$.

From Lemma 2.1 and Lemma 2.2, we can conclude that the rectangle

$A(\bar{\theta})\times B(\bar{\theta}) = \{a \in \Theta^1: G^1(\bar{\theta}, a) = 0\} \times \{(b_1, b_2, b_3): b_j = \bar{b_j}, j = 1,2,3\}$ is a reflexive rectangle, and the rectangle correspondence $V(\bullet):\Theta \to C_v$ is the reflexive rectangle method correspondence.

The proof has been completed.

Lemma 3.2 Let $V(\bullet):\Theta \to C_v$ be the reflexive rectangle method correspondence as described in Lemma 3.1. Then the reflexive rectangle method covering $C_v = \{V(\bar{\theta}): \bar{\theta} \in \Theta\}$ is a partition.

Proof. From Lemma 2.3 and Lemma 3.1, we know that the set $C_v = \{V(\bar{\theta}): \bar{\theta} \in \Theta\}$ is a reflexive rectangle method covering. From Lemma 2.5 and Lemma 2.6, we only need to prove $G(\bar{\theta},\theta) = 0 \leftrightarrow G(\theta,\bar{\theta}) = 0$.

Firstly we prove $G(\bar{\theta},\theta) = 0 \to G(\theta,\bar{\theta}) = 0$.

By (2), (10), (11), (12), we have

$$G^1(\bar{\theta},a) = F(a,\bar{b}) - F(\bar{a},\bar{b}) = 0,$$

$$G^{21}(\bar{\theta}, b_1) = b_1 - \overline{b_1} = 0,$$

$$G^{22}(\bar{\theta}, b_2) = b_2 - \overline{b_2} = 0,$$

$$G^{23}(\bar{\theta}, b_3) = b_3 - \overline{b_3} = 0.$$

Since b = $\bar{b}$, we arrive at

$$G^1(a,\bar{\theta}) = F(\bar{a},b) - F(a,b) = F(\bar{a},\bar{b}) - F(a,\bar{b}) = G^1(\bar{\theta},a) = 0, \tag{14}$$

$$G^{21}(\bar{\theta}, b_1) = b_1 - \overline{b_1} = \overline{b_1} - b_1 = G^{21}(b_1, \bar{\theta}) = 0, \tag{15}$$

$$G^{22}(\bar{\theta}, b_2) = b_2 - \overline{b_2} = \overline{b_2} - b_2 = G^{22}(b_2, \bar{\theta}) = 0, \tag{16}$$

$$G^{23}(\bar{\theta}, b_3) = b_3 - \overline{b_3} = \overline{b_3} - b_3 = G^{23}(b_3, \bar{\theta}) = 0. \tag{17}$$

By Lemma 2.4, we therefore obtain

$$G(\theta,\bar{\theta}) = 0.$$

Furthermore, by (14), (15), (16), (17), we can immediately deduce that

$$G(\theta,\bar{\theta}) = 0 \rightarrow G(\bar{\theta},\theta) = 0.$$

The assertion is established.

Theorem 3.3 Let $F(a,b) = a_1 b_1 + a_2 b_2 + b_3 + {a_1}^2 (b_2 \neq 0)$ be a goal function on Θ. Then there exists the mechanism π = (M, g, h) such that the message space M is denoted by

$$M = \{(m_1, m_2, m_3, m_4): m_1 = w_1, m_2 = w_2, m_3 = w_3, m_4 = w_4, \bar{\theta} \in \Theta\};$$

the equilibrium message function g is denoted by

$$g^1(m,a) = {a_1}^2 - a_1 m_1 - a_2 m_2 - m_3 - m_4,$$

$$g^{21}(m,b) = b_1 + m_1,$$

$$g^{22}(m,b) = b_2 + m_2,$$

$$g^{23}(m,b) = b_3 + m_3;$$

the outcome function h is denoted by

$$h(m) = m_4.$$

Where $w_1 = -\overline{b_1}, w_2 = -\overline{b_2}$, $w_3 = -\overline{b_3}, w_4 = \overline{F}$ and $\overline{F} = F(\overline{\theta})$.

Proof. The proof of Theorem 3.3 can be divided into the following five steps.

Step one. Determine the number of the condensation correspondence.

By Lemma 3.1, let

$$G^1(\overline{\theta}, a) = a_1\overline{b_1} + a_2\overline{b_2} + \overline{b_3} + a_1{}^2 - \overline{F} = \Phi^1,$$

$$G^{21}(\overline{\theta}, b_1) = b_1 - \overline{b_1} = \Phi^2,$$

$$G^{22}(\overline{\theta}, b_2) = b_2 - \overline{b_2} = \Phi^3,$$

$$G^{23}(\overline{\theta}, b_3) = b_3 - \overline{b_3} = \Phi^4,$$

$$F(\overline{\theta}) = \overline{a_1}\overline{b_1} + \overline{a_2}\overline{b_2} + \overline{b_3} + \overline{a_1}{}^2 = \Phi^5,$$

and let $\overline{\theta} = (\overline{a}, \overline{b}) = (\overline{a_1}, \overline{a_2}, \overline{b_1}, \overline{b_2}, \overline{b_3})$ be the primary variable, let $\theta = (a_1, a_2, b_1, b_2, b_3)$ be the secondary variable.

From Definition 2.7, we have

$$H^1 = \begin{array}{c|ccccc} \Phi^1 & \partial a_1 & \partial a_2 & \partial b_1 & \partial b_2 & \partial b_3 \\ \hline \partial\overline{a_1} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{a_2} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{b_1} & 1 & 0 & 0 & 0 & 0 \\ \partial\overline{b_2} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{b_3} & 0 & 0 & 0 & 0 & 0 \end{array}, \qquad (18)$$

$$H^{\sigma} = \begin{array}{c|ccccc} \Phi^{\sigma} & \partial a_1 & \partial a_2 & \partial b_1 & \partial b_2 & \partial b_3 \\ \hline \partial\overline{a_1} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{a_2} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{b_1} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{b_2} & 0 & 0 & 0 & 0 & 0 \\ \partial\overline{b_3} & 0 & 0 & 0 & 0 & 0 \end{array}, \tag{19}$$

where σ = 2,3,4,5.

Then we get

$$H = (H^1 \vdots \quad H^2 \vdots \quad H^3 \vdots \quad H^4 \vdots \quad H^5 \vdots). \tag{20}$$

It is clear that R(H) = 2.

Moreover, we have

$$\Phi_{\theta} = \begin{array}{c|ccccc} & \partial\Phi^1 & \partial\Phi^2 & \partial\Phi^3 & \partial\Phi^4 & \partial\Phi^5 \\ \hline \partial\overline{a_1} & -\overline{b_1} - 2\overline{a_1} & 0 & 0 & 0 & \overline{b_1} + 2\overline{a_1} \\ \partial\overline{a_2} & -\overline{b_2} & 0 & 0 & 0 & \overline{b_2} \\ \partial\overline{b_1} & a_1 - \overline{a_1} & -1 & 0 & 0 & \overline{a_1} \\ \partial\overline{b_2} & a_2 - \overline{a_2} & 0 & -1 & 0 & \overline{a_2} \\ \partial\overline{b_3} & 0 & 0 & 0 & -1 & 1 \end{array}, \tag{21}$$

Then we can obtain

$$BH = (\Phi_{\theta}^1 \quad \Phi_{\theta}^2 \quad \Phi_{\theta}^3 \quad \Phi_{\theta}^4 \quad \Phi_{\theta}^5 \quad H^1 \quad H^2 \quad H^3 \quad H^4 \quad H^5), \tag{22}$$

we therefore get

$$\mathrm{BH} \rightarrow \begin{pmatrix} -\overline{b_1} - 2\overline{a_1} & 0 & 0 & 0 \\ -\overline{b_2} & 0 & 0 & 0 \\ a_1 - \overline{a_1} & -1 & 0 & 0 \\ a_2 - \overline{a_2} & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Hence R(BH) = 4, it is obvious that R(BH)≠R(H).

Now we need amplify the secondary variable $\theta=(a_1, a_2, b_1, b_2, b_3)$ into the Variable $\theta=(a_1, a_2, b_1, b_2, b_3, \widehat{y_1}, \widehat{y_2}, \widehat{y_3}, \widehat{y_4}, \widehat{y_5})$.

Then let

$$\widehat{\Phi^1}=a_1\overline{b_1} + a_2\overline{b_2} + \overline{b_3} + a_1{}^2 - \widehat{y_1}\overline{F},$$

$$\widehat{\Phi^2}=b_1 - \widehat{y_2}\overline{b_1},$$

$$\widehat{\Phi^3}=b_2 - \widehat{y_3}\overline{b_2},$$

$$\widehat{\Phi^4}=b_3 - \widehat{y_4}\overline{b_3},$$

$$\widehat{\Phi^5}=\widehat{y_5}(\overline{a_1}\overline{b_1} + \overline{a_2}\overline{b_2} + \overline{b_3} + \overline{a_1}{}^2).$$

By Definition 2.7, we can obtain

$$\widehat{H^1} = \begin{array}{c|cccccccccc} \widehat{\Phi^1} & \partial a_1 & \partial a_2 & \partial b_1 & \partial b_2 & \partial b_3 & \partial\widehat{y_1} & \partial\widehat{y_2} & \partial\widehat{y_3} & \partial\widehat{y_4} & \partial\widehat{y_5} \\ \hline \partial\overline{a_1} & 0 & 0 & 0 & 0 & 0 & -\overline{b_1} - 2\overline{a_1} & 0 & 0 & 0 & 0 \\ \partial\overline{a_2} & 0 & 0 & 0 & 0 & 0 & -\overline{b_2} & 0 & 0 & 0 & 0 \\ \partial\overline{b_1} & 1 & 0 & 0 & 0 & 0 & -\overline{a_1} & 0 & 0 & 0 & 0 \\ \partial\overline{b_2} & 0 & 1 & 0 & 0 & 0 & -\overline{a_2} & 0 & 0 & 0 & 0 \\ \partial\overline{b_3} & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{array}, \quad (23)$$

$$\widehat{H^2} = \begin{array}{c|cccccccccc}
\widehat{\Phi^2} & \partial a_1 & \partial a_2 & \partial b_1 & \partial b_2 & \partial b_3 & \partial \widehat{y_1} & \partial \widehat{y_2} & \partial \widehat{y_3} & \partial \widehat{y_4} & \partial \widehat{y_5} \\
\hline
\partial \overline{a_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{b_1} & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\partial \overline{b_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{b_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} ,(24)$$

$$\widehat{H^3} = \begin{array}{c|cccccccccc}
\widehat{\Phi^3} & \partial a_1 & \partial a_2 & \partial b_1 & \partial b_2 & \partial b_3 & \partial \widehat{y_1} & \partial \widehat{y_2} & \partial \widehat{y_3} & \partial \widehat{y_4} & \partial \widehat{y_5} \\
\hline
\partial \overline{a_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{b_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{b_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
\partial \overline{b_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} ,(25)$$

$$\widehat{H^4} = \begin{array}{c|cccccccccc}
\widehat{\Phi^4} & \partial a_1 & \partial a_2 & \partial b_1 & \partial b_2 & \partial b_3 & \partial \widehat{y_1} & \partial \widehat{y_2} & \partial \widehat{y_3} & \partial \widehat{y_4} & \partial \widehat{y_5} \\
\hline
\partial \overline{a_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{a_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\partial \overline{b_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} ,(26)$$

$\partial\overline{b_2}$	0	0	0	0	0	0	0	0	0	0
$\partial\overline{b_3}$	0	0	0	0	0	0	0	0	-1	0

$\widehat{H^5} =$

$\widehat{\Phi^5}$	∂a_1	∂a_2	∂b_1	∂b_2	∂b_3	$\partial\widehat{y_1}$	$\partial\widehat{y_2}$	$\partial\widehat{y_3}$	$\partial\widehat{y_4}$	$\partial\widehat{y_5}$
$\partial\overline{a_1}$	0	0	0	0	0	0	0	0	0	$\overline{b_1} + 2\overline{a_1}$
$\partial\overline{a_2}$	0	0	0	0	0	0	0	0	0	$\overline{b_2}$
$\partial\overline{b_1}$	0	0	0	0	0	0	0	0	0	$\overline{a_1}$
$\partial\overline{b_2}$	0	0	0	0	0	0	0	0	0	$\overline{a_2}$
$\partial\overline{b_3}$	0	0	0	0	0	0	0	0	0	1

.(27)

Then we get

$$\widehat{H} = (\widehat{H^1} : \quad \widehat{H^2} : \quad \widehat{H^3} : \quad \widehat{H^4} : \quad \widehat{H^5} :). \tag{28}$$

Thus we have

$$\widehat{H} \rightarrow \begin{pmatrix} 0 & 0 & 0 & -\overline{b_1} - 2\overline{a_1} \\ 0 & 0 & 0 & -\overline{b_2} \\ 1 & 0 & 0 & -\overline{a_1} \\ 0 & 1 & 0 & -\overline{a_2} \\ 0 & 0 & -1 & -1 \end{pmatrix}. \tag{29}$$

It is clear that $R(\widehat{H}) = 4$.

Furthermore we arrive at

	$\partial\widehat{\Phi^1}$	$\partial\widehat{\Phi^2}$	$\partial\widehat{\Phi^3}$	$\partial\widehat{\Phi^4}$	$\partial\widehat{\Phi^5}$
$\partial\overline{a_1}$	$-\widehat{y_1}\overline{b_1} - 2\widehat{y_1}\overline{a_1}$	0	0	0	$\widehat{y_5}\overline{b_1} + 2\widehat{y_5}\overline{a_1}$
$\partial\overline{a_2}$	$-\widehat{y_1}\overline{b_2}$	0	0	0	$\widehat{y_5}\overline{b_2}$

$$\widehat{\Phi_\theta} = \begin{array}{c|ccccc} \partial\overline{b_1} & a_1 - \widehat{y_1}\overline{a_1} & -\widehat{y_2} & 0 & 0 & \widehat{y_5}\overline{a_1} \\ \partial\overline{b_2} & a_2 - \widehat{y_1}\overline{a_2} & 0 & -\widehat{y_3} & 0 & \widehat{y_5}\overline{a_2} \\ \partial\overline{b_3} & 1-\widehat{y_1} & 0 & 0 & -\widehat{y_4} & \widehat{y_5} \end{array} \quad . \tag{30}$$

Then we obtain

$$\widehat{BH}=\left(\widehat{\Phi_\theta^1} \quad \widehat{\Phi_\theta^2} \quad \widehat{\Phi_\theta^3} \quad \widehat{\Phi_\theta^4} \quad \widehat{\Phi_\theta^5} \quad \widehat{H^1} \quad \widehat{H^2} \quad \widehat{H^3} \quad \widehat{H^4} \quad \widehat{H^5}\right). \tag{31}$$

Take

$\widehat{y_1} = 1,\ \widehat{y_2} = 1, \widehat{y_3} = 1, \widehat{y_4} = 1, \widehat{y_5} = 1.$

Then we have

$$\widehat{BH} \to \begin{pmatrix} -\overline{b_1} - 2\overline{a_1} & 0 & 0 & 0 \\ -\overline{b_2} & 0 & 0 & 0 \\ a_1 - \overline{a_1} & -1 & 0 & 0 \\ a_2 - \overline{a_2} & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

It is easy that $R(\widehat{BH}) = 4$.

By Lemma 2.10, we can obtain that the number of the condensation correspondence is four.

Step two. Construct the condensation correspondence.

We choose a special matrix $\widetilde{H}$ which satisfies $R(\widetilde{H}) = 4$ in the block Hessian Matrix $\widehat{H}$. Let

$$\widetilde{H} = \begin{array}{c|cccc} & (\widehat{\Phi^2},\widehat{y_2}) & (\widehat{\Phi^3},\widehat{y_3}) & (\widehat{\Phi^4},\widehat{y_4}) & (\widehat{\Phi^5},\widehat{y_5}) \\ \hline \partial\overline{a_2} & 0 & 0 & 0 & \overline{b_2} \\ \partial\overline{b_1} & -1 & 0 & 0 & \overline{a_1} \\ \partial\overline{b_2} & 0 & -1 & 0 & \overline{a_2} \\ \partial\overline{b_3} & 0 & 0 & -1 & 1 \end{array} \quad . \tag{32}$$

Then the condensation correspondence can be given by

$$A_1(\bar{\theta}) = A_1(\bar{a}, \bar{b}) = \frac{\partial \widehat{\Phi^2}}{\partial \widehat{y_2}} = \frac{\partial (b_1 - \widehat{y_2}\overline{b_1})}{\partial \widehat{y_2}} = -\overline{b_1},$$

$$A_2(\bar{\theta}) = A_2(\bar{a}, \bar{b}) = \frac{\partial \widehat{\Phi^3}}{\partial \widehat{y_3}} = \frac{\partial (b_2 - \widehat{y_3}\overline{b_2})}{\partial \widehat{y_3}} = -\overline{b_2},$$

$$A_3(\bar{\theta}) = A_3(\bar{a}, \bar{b}) = \frac{\partial \widehat{\Phi^4}}{\partial \widehat{y_4}} = \frac{\partial (b_3 - \widehat{y_4}\overline{b_3})}{\partial \widehat{y_4}} = -\overline{b_3},$$

$$A_4(\bar{\theta}) = A_4(\bar{a}, \bar{b}) = \frac{\partial \widehat{\Phi^5}}{\partial \widehat{y_5}} = \frac{\partial (\widehat{y_5}\bar{F})}{\partial \widehat{y_5}} = \bar{F}$$

Step three. Construct the equations and introduce the intermediate variable. Let $w_i \in W$ be the intermediate variables, and $i \in \{1, 2, 3, 4\}$. From Definition 2.9, we take $\bar{\theta} = (\bar{\theta}^*, \bar{\theta}^{**}) \in \Theta$, where $\bar{\theta}^* = (\overline{a_2}, \overline{b_1}, \overline{b_2}, \overline{b_3})$ and $\bar{\theta}^{**} = (\overline{a_1})$.

Then the equations can be constructed as follows.

$$\begin{cases} w_1 = A_1(\bar{\theta}) \\ w_2 = A_2(\bar{\theta}) \\ w_3 = A_3(\bar{\theta}) \\ w_4 = A_4(\bar{\theta}) \end{cases} \rightarrow \begin{cases} w_1 = -\overline{b_1} \\ w_2 = -\overline{b_2} \\ w_3 = -\overline{b_3} \\ w_4 = \bar{F} \end{cases}. \tag{33}$$

Solving the equations (33), we can obtain

$$\begin{cases} \overline{a_2} = \frac{w_4 + w_1\overline{a_1} + w_3 - \overline{a_1}^2}{-w_2} \\ \overline{b_1} = -w_1 \\ \overline{b_2} = -w_2 \\ \overline{b_3} = -w_3 \end{cases}. \tag{34}$$

Step four. Construct the condensation form $\Gamma^i(w, \theta)$, $i \in \{1, 2, 3, 4, 5\}$.

Let $\Gamma^i(w, \theta) = \widetilde{\Phi^i}\left(\bar{\theta}, (\theta, \hat{y})\right)$, where $i \in \{1, 2, 3, 4, 5\}$ and $\hat{y} = (\widehat{y_1}, \widehat{y_2}, \widehat{y_3}, \widehat{y_4}, \widehat{y_5})$. Then we have

$$\Gamma^1(w, \theta) = {a_1}^2 - a_1 w_1 - a_2 w_2 - w_3 - w_4,$$

$$\Gamma^2(w, \theta) = b_1 + w_1,$$

$$\Gamma^3(w,\theta) = b_2 + w_2,$$

$$\Gamma^4(w,\theta) = b_3 + w_3,$$

$$\Gamma^5(w,\theta) = w_4.$$

Step five. Construct the message space M, the equilibrium message function g and the outcome function h.

Based on the analysis above, the message space M is denoted by

$$M = \{(m_1,m_2,m_3,m_4):m_1=w_1,m_2=w_2,m_3=w_3,m_4=w_4,\bar{\theta} \in\Theta\};$$

the equilibrium message function g is denoted by

$$g^1(m,a) = \Gamma^1(w,\theta)|_{w=m} = {a_1}^2 - a_1m_1 - a_2m_2 - m_3 - m_4,$$

$$g^{21}(m,b) = \Gamma^2(w,\theta)|_{w=m} = b_1 + m_1,$$

$$g^{22}(m,b) = \Gamma^3(w,\theta)|_{w=m} = b_2 + m_2,$$

$$g^{23}(m,b) = \Gamma^4(w,\theta)|_{w=m} = b_3 + m_3;$$

the outcome function h is denoted by

$h(m) = \Gamma^5(w,\theta)|_{w=m} = \overline{a_1}\overline{b_1} + \overline{a_2}\overline{b_2} + \overline{b_3} + \overline{a_1}^2 = m_4.$

This completes the proof of the Theorem 3.3.

Theorem 3.4 Let $F(a,b) = a_1b_1 + a_2b_2 + b_3 + {a_1}^2 (b_2 \neq 0)$ be a goal function on Θ, and let π = (M, g, h) be the mechanism described in Theorem 3.3. Then the following statements hold.

(a) The mechanism π = (M, g, h) can realize the goal function F.

(b)The mechanism π = (M, g, h) satisfies informational efficiency and decentralization.

Proof. (a) From Theorem 3.3, for each θ = (a, b)∈Θ, assume that there exists a message m satisfies the following equations

$$\begin{cases} {a_1}^2 - a_1m_1 - a_2m_2 - m_3 - m_4 = 0 \\ b_1 + m_1 = 0 \\ b_2 + m_2 = 0 \\ b_3 + m_3 = 0 \end{cases}, \qquad (35)$$

solving the equations (35), then we have

$$\begin{cases} m_1 = -b_1 \\ m_2 = -b_2 \\ m_3 = -b_3 \\ m_4 = a_1b_1 + a_2b_2 + b_3 + {a_1}^2 \end{cases} . \qquad (36)$$

It means there exists an equilibrium message m which is corresponding to θ for all$\theta \in \Theta$.

In addition, suppose that there exists an equilibrium message m that is corresponding toθfor allθ $\in \Theta$, then we get

$$h(m)=m_4=a_1b_1 + a_2b_2 + b_3 + {a_1}^2=F(a,b)=F(\theta).$$

By Lemma 2.11, we can infer that the mechanism π = (M, g, h) constructed by Theorem 3.3 can realize the given goal function $F(a,b) = a_1b_1 + a_2b_2 + b_3 + {a_1}^2 (b_2 \neq 0)$.

This completes the proof of (a).

(b) From Theorem 3.3, we have

$$g^1(m,a) = {a_1}^2 - a_1m_1 - a_2m_2 - m_3 - m_4,$$

$$g^{21}(m,b) = b_1 + m_1,$$

$$g^{22}(m,b) = b_2 + m_2,$$

$$g^{23}(m,b) = b_3 + m_3.$$

Note that the equilibrium message function $g^1(m,a) = {a_1}^2 - a_1m_1 - a_2m_2 - m_3 - m_4$ depends only on the type variable a = (a_1, a_2), and the equilibrium message function $g^{21}(m,b) = b_1 + m_1$, $g^{22}(m,b) = b_2 + m_2$ and $g^{23}(m,b) = b_3 + m_3$ depend only on the type variable b = (b_1, b_2, b_3). Hence the mechanism $\pi =$ (M, g, h) can be satisfied with decentralization.

Furthermore, on one hand, by Lemma 2.12, we know the reflexive rectangle method covering C_v is a F-maximal covering. On the other hand, from Theorem 3.3, we obtain dim(M) = 4. Finally from Lemma 3.2, it means that there are no the redundant rectangles. Therefore minimal informational size can be achievable by the mechanism π = (M, g, h).

This completes the proof of (b).

Discussion

The augmented inner product is defined and proposed by Leonid Hurwicz (Nobel Laureate in Economics in 2007) in the economic design theory. It plays an important role in economics and it is a discrete function. But there are many goal functions which are continuous in many studies.

Next, our mechanism is constructed based on the economic design theory and condensation method which have logical and rigorous mathematical deduction and characterized by the agents without strategic behaviors. However, the mechanism design theory in some research can be illustrated by the agents with behaviors.

Finally, a good mechanism can be measured by incentive compatibility or Pareto optimality or informational efficiency. Our mechanism which is designed for the augmented inner product can be written π = (M, g, h), it is mainly expressed by the message space M, the equilibrium message function g and the outcome function h, and the mechanism π = (M, g, h) is satisfied with informational efficiency and decentralization. But in many research, the mechanism is mainly constructed by the different expressions and meets incentive compatibility or Pareto optimality or optimality.

In conclusion, the analysis in this paper shows that informationally efficient mechanism plays a crucial role for a good mechanism and informational efficiency can be used to reduce the cost of the economic and information.

Conclusions

In this paper, we focus on designing the mechanism π = (M, g, h) for the augmented inner product $F(a,b) = a_1b_1 + a_2b_2 + b_3 + a_1^2(b_2 \neq 0)$. We construct the reflexive rectangle method correspondence V (•) and the reflexive rectangle method covering C_v which is a partition. Furthermore, we construct the message space M, the equilibrium message function g and the outcome function h which constitute the mechanism π = (M, g, h). Finally we prove the mechanism π = (M, g, h) can realize the goal function F and satisfy informational efficiency and decentralization.

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