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# Bayesian Estimation of Reliability of Geometric Distribution under Different Loss Functions

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## Abstract

The aim of this paper is to study the Bayesian estimation of reliability of geometric distribution based on complete sample and record values, respectively. For Bayesian inference, loss function and prior distribution are two important aspects in statistical inference process. The Bayes estimation is discussed under a precautionary loss function, and the Bayesian estimators of reliability of geometric distribution are obtained based on two different prior distributions, namely, the quasi-prior and Beta prior distributions. Finally, numerical simulations are given to illustrate the results.

## Key words

Bayes estimation, geometric distribution, record value, loss function

## 1. Introduction

In life testing experiments, a lot of work has been done under the continuous lifetime models. Sometimes it is neither possible nor convenient to measure the life length of an item continuously until its failure. When failure time data is sometimes discrete either through the grouping of continuous data due to imprecise measurement or because time itself is discrete, the geometric distribution is a natural choice. It possesses most of the nice properties of the exponential distribution, of course in the discrete set up. In such circumstances one measures the life of a device on a discrete scale and considers the number of successful cycles, trials, or operations before failure. Therefore, the number of successful trials before failure is more pertinent than the time of continuous period. The Geometric distribution, owing to its lack of memory property and constant failure rate, and it belongs to the class of long tailed distributions and as such, the occurrence of extreme observations is quite common. Thus it widely used to model discrete reliability. Other applications of the geometric distribution are in Ecology, Information Theory and Cryptography, Labor Economics, Demand Analysis etc. Many authors like Elneweihi and Govindarajulu (1979), Yaqub and Khan (1981), Burke et al. (1992), Xie and Goh (1997), Zhang et al. (2004), Masmoudi et al. (2016) and Hong and Lee (2015) have contributed to the methodology and estimation of the parameter of the geometric distribution. Also, some authors developed many new distribution models based on Geometric distribution, and they studied the applications and statistical inference of these new proposed distributions (Gómez-Déniz and Calderín–Ojeda, 2015; Nadarajah and Bakar, 2015; Chakraborty and Bhati, 2016; Balakrishnan et al., 2015; Shao and Wang, 2015).

The record values can well describe the trend of some random variables sequence, and thus it is widely applied to the fields of weather forecasting, earthquake prediction, sports science, engineering science, etc. (Ahsanullah, 2004). For example, the data comes from the Olympic Games of broking a world record, the maximum rainfall in the meteorology (snow). Research on the change trend of record value and statistical inference theory are of great significance to the development of national economy.

Statistical inference about the value of records has become a hot topic in statistical research. Based on record values, Zhao et al. (2008) studied random order on the basis of single and double sample respectively; Xiong (2008) proposed a Expect Bayes estimation approach to estimate the reliability of geometric distribution under entropy loss function and square loss function when the prior distribution of the reliability is power distribution. Ren and Ren (2010) studied the estimation of exponential distribution parameter based on record values, and obtained the minimum risk equivariant estimation, Bayes estimation and empirical Bayes estimation under entropy loss functions, and further discussed the admissible character of a class of linear form of estimation; Xing (2010) discussed the loss function and risk function of Bayes estimation problem of exponential distribution parameter under Rukhin loss function combining decision-making error with statistical discriminate rule. They also obtained the conditions of the corresponding Bayes estimation for a conservative estimate.

Here we assume that the lifetime of certain items has a Geometric distribution with probability mass function (pmf)

$$P(x \mid R) = (1-R)R^x,\ x = 0,\ 1,\ 2,\ \cdots$$

(1)

The object of the present paper is to obtained Bayes estimators for reliability of Geometric distribution under a precautionary loss function, The rest of this paper is organized as follows. In Section 2, some useful preliminaries are introduced. The Bayes estimators of the reliability are obtained under the different priors and a precautionary loss. In Section 4, discussion analysis is given through numerical examples. Finally, conclusions are given in Section 5.

## 2. Preliminary Knowledge

It is well-known that, for Bayes estimators, the performance depends on the form of the prior distribution and the loss function assumed.

### 2.1 Prior Distribution

In the Bayesian approach, we further assume some prior knowledge about the reliability parameter $R$ is available to the investigator from past experiences with the underlying queuing system. The prior knowledge can often be summarized in terms of the so-called prior densities on the parameter space of $R$. In the following discussion, we assume the following priors:

(i) The quasi-prior distribution:

For the situation where the experimenter has no prior information about the parameter $R$, one may use the quasi density as given by

$$p_1(R;c) \propto \frac{1}{R^c},\ R > 0, c > 0$$

(2)

Hence, $c = 0$ leads to a diffuse prior and $c = 1$ to a non-informative prior.

(ii) The Beta prior distribution:

The most widely used prior distribution of $R$ is the Beta prior distribution with parameters $a$ and $b(>0)$, denoted by $Beta(a, b)$, the corresponding probability density function (pdf) is given by the following formula:

$$p_2(R;a,b) = \frac{1}{B(a,b)} R^{a-1}(1-R)^{b-1},\ a,b > 0$$

(3)

## 2.2 Loss Function

In Bayesian analysis, a commonly used loss function is the squared error loss function $L(\hat{q},q)=(\hat{q}-q)^2$, which is symmetrical, and associates equal importance to the losses due to overestimation and under estimation of equal magnitude. However, such a restriction may be impractical. For example, in the estimation of reliability and failure rate function, an overestimate is usually much more serious than an underestimate; In this case, the use of a symmetrical loss function might be inappropriate, which has been recognized by Basu and Ebrahimi (1991) and Soliman (2005). A useful asymmetric loss known linear exponential (LINEX) loss function was introduced in Zellner (1986), which has been found to be appropriate in the situation where overestimation is more serious Under-estimation or Vice-versa. The LINEX loss function has been widely used in many fields. Also, in recent time there are a lot of authors studied LINEX loss functions for different distribution. Such as, Jaheen (2004) studied the exponential model based on record statistics, where the Bayes estimators obtained on the basis of the square error loss and LINEX loss functions. Also, Li et al. (2007) derived Bayes estimators for Burr XII distribution based on progressively Type-II censored samples under LINEX error loss function. You and Zhou (2015) firstly derived the Bayes estimator of the location parameter in double-exponential family under the LINEX loss function, and then constructed the corresponding empirical Bayes estimator. They also shown that the empirical Bayes estimator is asymptotically optimal with some convergence rate.

Another useful asymmetric loss function is the General Entropy (GE) loss. This loss function was used in several papers, as an example see Dey et al. (1987), Dey and Lin (1992) and Soliman (2005). And Norstorm (1996) proposed an asymmetric loss function known as precautionary loss function with the following form:

$$L(\hat{q},q)=\frac{(\hat{q}-q)^2}{\hat{q}} \tag{4}$$

Where $\hat{q}$ is an estimator of $q$. The loss function (4) infinitely near to the origin to prevent underestimation, thus giving conservative estimators, especially when low failure rates are being estimator. It is very useful when underestimation may lead to serious consequences. This loss function was used by several authors; among of them Yarmohammadi and Pazira (2010), Pandey and Rao (2009), Mohsin et al. (2012).

**Lemma 1.** Under the precautionary loss function (4), where $\hat{q}$ is a estimator of $q$ , then for every prior distribution $p(q)$ of $q$ . The Bayes estimator of $q$ is given by

$$\hat{q}_B = [E(q^2 \mid X)]^{1/2} \tag{5}$$

Provided that $E(q^2 \mid X)$ exist, and is finite.

**Proof**. Under the precautionary loss function (4), the Bayes risk of $\hat{q}$ is $r(\hat{q}) = E[E(L(\hat{q}, q) \mid X)]$.

To miminize $r(\hat{q})$ , we only need $E(L(\hat{q}, q) \mid X)$ almost obtained minimum.

Let

$$f(\hat{q}) = E(L(\hat{q}, q) \mid X) = E(\hat{q} - 2q + \frac{q^2}{\hat{q}} \mid X)$$

$$= \hat{q} - 2E[q \mid X] + \frac{1}{\hat{q}} E[q^2 \mid X]$$

Then, we have

$$f'(\hat{q}) = 1 - \frac{1}{\hat{q}^2} E[q^2 \mid X]$$

And $f''(\hat{q}) = \frac{2}{\hat{q}^3} E[q^2 \mid X] \ge 0$ is always true.

Thus the solution of $f'(\hat{q}) = 0$ , i.e. $\hat{q}_B = [E(q^2 \mid X)]^{1/2}$ is the minimum value of $f'(\hat{q})$ . Therefore $\hat{q}_B = [E(q^2 \mid X)]^{1/2}$ is the Bayes estimator of $q$ under the precautionary loss function (4).

## 3. Bayes Estimation

### 3.1 Bayes Estimation Based on Complete Samples

In this section, we are interested in estimating the estimation of the reliability $R$ of the Geometric distribution (1) under the precautionary loss function (4), and the unique Bayes estimator of $R$, say $\hat{R}_B$, is given by $\hat{R}_B = [E(R^2 \mid X)]^{1/2}$ .

**Theorem 1**. Let $X_1, X_2, \cdots, X_n$ be a sample drawn from the Geometric distribution (1), $x_1, x_2, \cdots, x_n$ is the corresponding observation value. $t = \sum_{i=1}^{n} x_i$ is the observation of $T = \sum_{i=1}^{n} X_i$ . Then under the precautionary loss (4), we have

(i) On the basis of quasi-prior (2), the Bayes estimator of $R$ is $\hat{R}_1 = \left[ \frac{(T-c+2)(T-c+1)}{(T-c+n+3)(T-c+n+2)} \right]^{1/2}$

(ii) On the basis of Beta prior (3), the Bayes estimator of $R$ is $\hat{R}_2 = \left[ \frac{(T+a+1)(T+a)}{(T+a+b+n+1)(T+a+b+n)} \right]^{1/2}$ .

**Proof**. The likelihood function of $R$ is given as

$$L(R) = (1-R)^n R^t$$

(6)

Then It is easily shown that the maximum likelihood estimator of $R$ is given as

$$\hat{R}_{ML} = \frac{T}{T+n} .$$

For the case (i), we consider the prior density of $R$ is quasi-prior (2). The likelihood function is combined with the prior (2) by using the Bayes theorem to obtain the posterior density:

$$h_1(R/x) \propto l(R/x) \cdot p_1(R) \propto (1-R)^n R^{t-c},$$

Then

$$h_1(R/x) = \frac{(1-R)^n R^{t-c}}{\int_0^1 (1-R)^n R^{t-c} dR}$$

(7)

It is obvious that $R|X$ is distributed with Beta distribution $Beta(T-c+1, n+1)$. Then, under the precautionary loss (4), the Bayes estimator of $R$ is

$$\hat{R}_1 = [E(R^2 \mid X)]^{1/2} = [\int_0^1 R^2 h_1(R \mid x) dR]^{1/2}$$

$$= \frac{\int_0^1 R^{T-c+2}(1-R)^n \, dR}{\int_0^1 R^{T-c}(1-R)^n \, dR}$$

The, we can get

$$\hat{R}_1 = \left[ \frac{(T-c+2)(T-c+1)}{(T-c+n+3)(T-c+n+2)} \right]^{1/2}$$

(8)

For the case (ii), we consider the prior density of $R$ is Beta distribution with parameters $a$ and $b$ ( > 0 ). The posterior density of $R$ can be obtained by using Bayes theorem combining with Eq. (8) as:

$$h_2(R|x) \propto l(R|x) \cdot p_2(R) \propto (1-R)^{n+b-1} R^{t+a-1}$$

Then

$$h_2(R \mid x) = \frac{(1-R)^{n+b-1} R^{t+a-1}}{\int_0^1 (1-R)^{n+b-1} R^{t+a-1} dR}$$

(9)

It is obvious that $R|X$ is distributed with Beta distribution $Beta(T+a, n+b)$ . Then, under the precautionary loss function (4), the Bayes estimator of $R$ is

$$\hat{R}_2 = [E(R^2 \mid X)]^{1/2} = [\int_0^1 R^2 h_2(R \mid x) dR]^{1/2}$$

$$= \frac{\int_0^1 R^{T+a+1}(1-R)^{n+b+1} \, dR}{\int_0^1 R^{T+a-1} R^{n+b-1} dR}$$

$$= \left[ \frac{(T+a+1)(T+a)}{(T+a+b+n+1)(T+a+b+n)} \right]^{1/2}$$

(10)

## 3.2 Bayes Estimation Based on Record Value

Let $X_1, X_2, \cdots, X_n$ be a independent and identically distributed random variable sequence from the population $X$, which comes form the distribution $F(x;q)$ and probability density function $f(x;q)$. For any postive number $n \geq 1$, set $U(1) = 1$, $U(n+1) = \min\{j: j > U(n), X_j > X_{U(n)}\}$. Here $X_{U(n)}$ is called the $n$-th upper record valuem and $U(n)$ is called the $n$-th record time.

Suppose that $X_1, X_2, \cdots, X_n$ be a i.i.d. random sample comes from geometric distribution (1). $X_{U(1)}, X_{U(2)}, \ldots, X_{U(n)}$ is the observed $n$ first record value sample, and $x_{U(1)}, x_{U(2)}, \ldots, x_{U(n)}$ is the corresponding observation. Then for given $x_{U(1)}=X_{U(1)}$, $x_{U(2)}=X_{U(2)}$, …, $x_{U(n)}=X_{U(n)}$, the likelihood function of reliability $R$ can be obtained as follows[1]:

$$L(R;\underline{x}) = L(R; x_{U(1)}, x_{U(2)}, \cdots, x_{U(n)}) = \prod_{i=1}^{n-1} \frac{P(X = x_i)}{P(X > x_i)} \cdot P(X = x_n)$$

By equation (1), we have

$$P(X > x) = \sum_{k=x+1}^{\infty} P(X = k)$$

$$= \sum_{k=x+1}^{\infty} P(X = k) - \sum_{k=1}^{x} P(X = k) = R^{x+1}$$

Then we can easily obtain the following result:

$$L(R;\underline{x}) = \prod_{i=1}^{n-1} \frac{R^{x_{U(i)}}(1-R)}{R^{x_{U(i)}+1}} \cdot R^{x_{U(i)}}(1-R)$$

$$= (1-R)^n R^{x_{U(i)}-n+1}$$

Let $L'(R;x) = 0$, then we have

$$L'(R;x) = (1-R)^{n-1} R^{x_{U(n)}-n}[-nR + (1-R)(x_{U(n)} - n + 1)] = 0$$

The we solve the maximum likelihood estimator(MLE) of reliability $R$ as follows:

$$\hat{R}_{ML} = \frac{X_{U(n)} - n + 1}{X_{U(n)} + 1} = 1 - \frac{n}{T} \tag{11}$$

where $T= X_{U(n)}+1$. It is easily to prove that the random variable $T= X_{U(n)}+1$ is the completely statistics of $R$, and $T$ distributed with negative binomial distribution $NB(n,R)$ with the following formula:

$$P(T = t) = C_{t-1}^{n-1} \cdot R^{t-n}(1-R)^n,\ t = n,\ n+1,\ \cdots$$

Let $G(T) = 1 - \dfrac{n-1}{T-1}$, then we can easily prove that

$$E[G(T)] = 1 - \sum_{t=n}^{\infty} \frac{n-1}{t} C_{t-1}^{n-1} R^{t-n} (1-R)^n = R$$

Then $G(T)$ is the minimum variance unbiased estimator for reliability $R$, we denote $G(T)$ by $\hat{R}_U$, i.e. $\hat{R}_U = 1 - \frac{n-1}{T-1}$

**Theorem 2.** Let $\underline{X}$=($X_{U(1)}$, $X_{U(1)}$, …, $X_{U(n)}$) be the first n upper record values comes from geometric distribution (1), and $\underline{x}$=($x_{U(1)}$, $x_{U(1)}$, …, $x_{U(n)}$) be the corresponding observation values, $T = X_{U(n)} + 1$. Then

(i) Under the squared error loss function, Bayes estimator of reliability $R$ is

$$\hat{R}_{BS} = 1 - \frac{n+b}{T+a+b} \tag{12}$$

(ii) Under the precautionary loss function (4), the Bayes estimator of $R$ is

$$\hat{R}_{BP} = 1 - \frac{n+b-1}{T+a+b-1} \tag{13}$$

**Proof.** Using Bayes theorm, the posterior probability density function is derived as follows:

$$p(R \mid \underline{x}) \propto L(R;\underline{x}) \cdot p(R) \propto R^{(a+x_{U(n)}+1-n)+1} (1-R)^{n+b-1},$$

Then $R|\underline{X}$ distributed with Beta distribution $Bea$t($a$+$T$+$n$, $n$+$b$).

(i) Under the squared error loss function (8), Bayes estimator of reliability $R$ is

$$\hat{R}_{BS} = E[R \mid \underline{X}] = \frac{a+T-n}{T+a+b-1}.$$

(ii) Under the precautionary loss function, the Bayes estimator of $R$ is

$$\hat{R}_{BP} = [E(R^2 \mid X)]^{1/2} = [\int_0^1 R^2 p(R \mid X) dR]^{1/2}$$

$$= \left[ \frac{\int_0^1 R^2 R^{(a+T+1-n)-1} (1-R)^{n+b-1} dR}{\int_0^1 R^{(a+T+1-n)-1} (1-R)^{n+b-1} dR} \right]^{1/2}$$

$$= \left[ \frac{B(a+T+3-n, n+b)}{B(a+T+1-n, n+b)} \right]^{1/2}$$

$$= \left[ \frac{(a+T-n+2)(a+T-n+1)}{(T+a+b+2)(T+a+b+1)} \right]^{1/2}.$$

## 4. Discussion

**Example 1** (Estimation based on complete sample)

We generated 2000 samples of size $n$=50 from the geometric distribution as mentioned in section 1. Table 1and 2 show Bayesian estimates of the parameter for different values of the parameter $R$ under the prior distributions (i) :quasi-prior density, and (ii): Beta prior density.

**Table1**. The Bayesian estimates with $r$ = 0.5, $n$ =50

| $\hat{R}_{ML}$ | $c$ | $\hat{R}_1$ | $(a, b)$ | $\hat{R}_2$ |
|---|---|---|---|---|
| 0.4970 | 0.0 | 0.4996 | (1.0,1.0) | 0.4996 |
| 0.4936 | 1.0 | 0.4912 | (1.0,1.0) | 0.4963 |
| 0.4953 | 2.0 | 0.4878 | (1.0,1.5) | 0.4956 |
| 0.4935 | 2.5 | 0.4834 | (1.0,2.0) | 0.4914 |
| 0.4745 | 3.0 | 0.4818 | (2.0,1.0) | 0.5022 |

**Table2**. The Bayesian estimates with $r$ = 0.8, $n$ =50

| $\hat{R}_{ML}$ | $c$ | $\hat{R}_1$ | $(a, b)$ | $\hat{R}_2$ |
|---|---|---|---|---|
| 0.7964 | 0.0 | 0.7945 | (1.0,1.0) | 0.7945 |
| 0.7969 | 1.0 | 0.7941 | (1.0,1.0) | 0.7949 |
| 0.7974 | 2.0 | 0.7938 | (1.0,1.5) | 0.7939 |
| 0.7967 | 2.5 | 0.7926 | (1.0,2.0) | 0.7916 |
| 0.7967 | 3.0 | 0.7922 | (2.0,1.0) | 0.7956 |

**Example 2** (Estimation based on record value)

In order to compare the Bayes estimators with the maximum likelihood estimator and the minimum variance unbiased estimator, the example adopted from the Nelson [15] is used. To illustrate the test of the voltage strength of a certain electronic insulating liquid, the breakdown time of 19 samples of this kind of insulating liquid is measured under 34KV voltage, and the time distribution of the compressive strength of Nelson experimental electronic insulation is usually fitted by exponential distribution. As is known to all, an integral part of the observed value of an exponential data constitutes the geometric data. The first 6 record values $\underline{x} = (x_{U(1)}, x_{U(2)}, x_{U(3)}, x_{U(4)}, x_{U(5)}, x_{U(6)}) = (4, 8, 31, 33, 36, 72)$ from the geometric distribution were observed in the first samples from the geometric distribution. The maximum likelihood estimate of the parameters is: $\hat{R}_{MLE} = 0.9178$ , the minimum variance unbiased estimator of the parameter is: $\hat{R}_U = 0.9306$ , the Bayes estimate of the parameter is shown in Table 3.

**Table3**. Bayes estimates of geometric distribution reliability under different prior parameters

| Prior parameter $(a, b)$ | (0,0) | (0.5,0.5) | (0.5,1.0) | (1.0,1.0) | (1.0,1.5) | (1.5,1.5) |
|---|---|---|---|---|---|---|
| $\hat{R}_{BS}$ | 0.9178 | 0.9122 | 0.9060 | 0.9067 | 0.9007 | 0.9013 |
| $\hat{R}_{BP}$ | 0.9195 | 0.9139 | 0.9079 | 0.9085 | 0.9026 | 0.9032 |

It has been observed from Table1 to Table3 that there is very little change in values of the Bayesian estimates of the reliability $R$. We have considered two priors with the sole intension of providing alternatives to the practitioner. We believe these are quite flexible and capable of modeling a wide variety of prior information. Based on record values, the Bayes estimation of the reliability of the square error loss and precautionary loss function are derived in the case of a given geometric distribution. From table 3 we can see the hyper parameters $(a, b)$ the value of impact on the results of Bayes is not great, obtained under the weighted square loss function of the Bayes estimates closer to the reliability of the minimum variance unbiased estimation, so in the application, we propose to use Bayes estimation $\hat{R}_{BP}$ .

## 5. Conclusion

A Bayesian approach for the estimation in Geometric distribution has been presented. Based on complete sample and record values, this paper studies the Bayes estimation of the parameter of Geometric distribution under a precautionary loss function, which is a asymmetric loss function particularly suitable to be used in estimating reliability and failure rate occurred in engineering fields. Two prior distributions, namely, the quasi-prior and Beta prior distributions are used as the prior distribution of the unknown parameter. To illustrate the results, two numerical simulations are used to analyze. The results obtained in this paper can be applied to estimate the reliability parameter of geometric model in practical problems. The methods can also be similarly to extend to other distributions, such as binomial distribution, negative binomial distribution.

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