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A New Similarity Measure-Based MADM Method Under Dynamic Interval-valued Intuitionistic Fuzzy Environment

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Abstract

Similarity measure is an important information measure of interval-valued intuitionistic fuzzy (IVIF) sets. This article will put forward a new dynamic IVIF multi-attribute decision making (MADM) method based on a Ren and Wang's similarity measure. The new decision making method considers the impacts of membership degree, nonmembership degree and median point of IVIF sets. We develop a weighting method for the MADM problem for the case of attribute weights information completely unknown on the basis of the maximizing deviations method. Further, a new decision making method is developed based on the proposed similarity measures, and an application example proved the effectiveness and feasibility of the proposed methods.

Key words

Similarity measure; interval-valued intuitionistic fuzzy set; multi-attribute decision making method; maximizing deviations method

1. Introduction

With the increasing complexity and uncertainty of the social economic environment, there often exist different hesitancy degree or show a certain degree of lack of knowledge in the decision making process. Intuitionistic fuzzy (IF) sets can well describe the above mentioned situations [1]. In some situations, membership and non-membership degree are difficult to express by crisp numbers, and interval numbers can well describe these situations. For this reason, Atanassov and Gargov [2] extended the IF sets to the IVIF sets in 1989. By interval numbers depicting the membership and nonmembership degree, IVIF sets are more attractive than IF sets and can easily be quantified and execrated by decision-makers. IVIF sets are widely applied in management decision problems [3-4].

As the core problem of the IF set theory, similarity has been studied and widely applied in the fields such as pattern recognition, medical diagnosis, clustering analysis and MADM problems recent years. Similarity measure is an important tool for measuring the degree of resemblance between two fuzzy sets. Many similarity measures of fuzzy sets are investigated in the literatures. Li and Chen [5] firstly gave the definition of similarity measure between IF sets, and firstly used proposed simility measure in pattern recognition. Li and Chen's similarity measure only takes into the medians of two intervals, and thus it can easily be pointed out the counter-intuitive examples. Mitchell [6] proposed an improved similarity based on the Li and Chen's similarity measure from a statistical viewpoint. Baccour et al. [7] summarized the existed similarity measures and pointed out that each above similarity measure has drawbacks. More recently, Hwang and Yang [8] gave a new construction for similarity measures which can improve most existing similarity measures by considering lower, upper and middle fuzzy sets.

IVIF sets are an extension of IF sets, and has more interesting and application space than IF sets. There are still rare about how to use the proposed similarity measures to solve MADM problem. Under IVIF environment, this paper will develop a new decision making method based on the proposed similarity measures for the MADM problem under IVIF environment with unknown attribute weights information.

2. Preliminary knowledge

Atanassov and Gargov [2] proposed the definition of IVIF set as follows.

Definition 1 Let $X=\{x_1,x_2,\cdots,x_n\}$ be a finite universe of discourse, then we call $\tilde{U}=\{<x_j,\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j)>|\, x_j\in X\}$ an IVIF set. Here $\tilde{\mu}_{\tilde{U}}(x_j)$ and $\tilde{\upsilon}_{\tilde{U}}(x_j)$ are intervals, where $\tilde{\mu}_{\tilde{U}}(x_j)=[\mu^-_{\tilde{U}}(x_j),\mu^+_{\tilde{U}}(x_j)]$ and $\tilde{\upsilon}_{\tilde{U}}(x_j)=[\upsilon^-_{\tilde{U}}(x_j),\upsilon^+_{\tilde{U}}(x_j)]$. The element $<x_j,\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j)>$ is called an IVIF number [9]. Here $\tilde{\pi}_{\tilde{U}}(x_j)=[\pi^-_{\tilde{U}}(x_j),\pi^+_{\tilde{U}}(x_j)]$ is called the hesitation degree of an element x_j to U, where $\pi^-_{\tilde{U}}(x_j)=1-\mu^+_{\tilde{U}}(x_j)-\upsilon^+_{\tilde{U}}(x_j)$ and $\pi^+_{\tilde{U}}(x_j)=1-\mu^-_{\tilde{U}}(x_j)-\upsilon^-_{\tilde{U}}(x_j)$, for all $x_j\in X$. We briefly denote $<\tilde{\mu}_{\tilde{U}}(x_j),\tilde{\upsilon}_{\tilde{U}}(x_j)>$ by $\tilde{A}=<\tilde{\mu}_{\tilde{A}},\tilde{\upsilon}_{\tilde{A}}>$ or $\tilde{A}=<\tilde{\mu}_{\tilde{A}},\tilde{\upsilon}_{\tilde{A}},\tilde{\pi}_{\tilde{A}}>$, where

$$\tilde{\mu}_{\tilde{A}}=[\mu^-_{\tilde{A}},\mu^+_{\tilde{A}}]\subset[0,1],\tilde{\upsilon}_{\tilde{A}}=[\upsilon^-_{\tilde{A}},\upsilon^+_{\tilde{A}}]\subset[0,1],\mu^+_{\tilde{A}}+\tilde{\upsilon}^+_{\tilde{A}}\le 1 \quad (1)$$

$$\tilde{\pi}_{\tilde{A}} = [\pi^-_{\tilde{A}}, \pi^+_{\tilde{A}}] \subset [0,1], \pi^-_{\tilde{A}} = 1 - \mu^+_{\tilde{A}} - \tilde{\upsilon}^+_{\tilde{A}}, \pi^+_{\tilde{A}} = 1 - \mu^-_{\tilde{A}} - \tilde{\upsilon}^-_{\tilde{A}} \tag{2}$$

Definition 2 [2] Let $\tilde{A}_i =< \tilde{\mu}_{\tilde{A}_i}, \tilde{\upsilon}_{\tilde{A}_i} > (i = 1,2)$ be two any IVIF numbers, then

(i) If $\mu^-_{\tilde{A}_1} \le \mu^-_{\tilde{A}_2}, \mu^+_{\tilde{A}_1} \le \mu^+_{\tilde{A}_2}$ and $\upsilon^-_{\tilde{A}_1} \ge \upsilon^-_{\tilde{A}_2}, \upsilon^+_{\tilde{A}_1} \ge \upsilon^+_{\tilde{A}_2}$, then $\tilde{A}_1$ is no larger than $\tilde{A}_2$, and noted by $\tilde{A}_1 \le \tilde{A}_2$;

(ii) If $\tilde{A}_1 \le \tilde{A}_2$ and $\tilde{A}_1 \ge \tilde{A}_2$, then $\tilde{A}_1$ is equal to $\tilde{A}_2$, and noted by $\tilde{A}_1 = \tilde{A}_2$.

By Definition 2, $\tilde{A}^* =< [1,1],[0,0] >$ is the largest IVIF number; $\tilde{A}^- =< [0,0],[1,1] >$ is the smallest IVIF number.

Ren and Wang [10] proposed a new similarity measure, which considers the impacts of membership degree, non-membership degree and median point of IVIF sets. The similarity measure is as follows:

Let $\tilde{A} = \{< x_i, [\mu^-_A(x_i), \mu^+_A(x_i)], [\upsilon^-_A(x_i), \upsilon^+_A(x_i)] > | x_i \in X\}$ be an IVIF set, then introduce an IVIF operator:

$$F_p(\tilde{A}) = \{< x, \mu^-_{\tilde{A}}(x) + p\Delta\mu_{\tilde{A}}(x), \upsilon^-_{\tilde{A}}(x) + p\Delta\upsilon_{\tilde{A}}(x) > | x \in X\} \tag{4}$$

where $\Delta\mu_{\tilde{A}}(x) = \mu^+_{\tilde{A}}(x) - \mu^-_{\tilde{A}}(x)$, $\Delta\upsilon_{\tilde{A}}(x) = \upsilon^+_{\tilde{A}}(x) - \upsilon^-_{\tilde{A}}(x)$. Ther $p \in [0,1]$ is called the attitude factor. Set $\bar{\mu}_{\tilde{A}}(x) = \mu^-_{\tilde{A}}(x) + p\Delta\mu_{\tilde{A}}(x)$, $\bar{\upsilon}_{\tilde{A}}(x) = \upsilon^-_{\tilde{A}}(x) + p\Delta\upsilon_{\tilde{A}}(x) >$,then the new information measure between $\tilde{A}$ and $\tilde{B}$ as follows [15]:

$$S_R(\tilde{A},\tilde{B}) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left[\frac{|\bar{\mu}_{\tilde{A}}(x_i) - \bar{\mu}_{\tilde{B}}(x_i)|}{\bar{\mu}_{\tilde{A}}(x_i) + \bar{\mu}_{\tilde{B}}(x_i)} + \frac{|\bar{\upsilon}_{\tilde{A}}(x_i) - \bar{\upsilon}_{\tilde{B}}(x_i)|}{3 - \bar{\upsilon}_{\tilde{A}}(x_i) - \bar{\upsilon}_{\tilde{B}}(x_i)} + |\bar{m}_{\tilde{A}}(x_i) - \bar{m}_{\tilde{B}}(x_i)|\right] \tag{5}$$

where $\bar{m}_{\tilde{A}}(x_i) = [\bar{\mu}_{\tilde{A}}(x_i) + 1 - \bar{\upsilon}_{\tilde{A}}(x_i)]/2$ and $\bar{m}_{\tilde{B}}(x_i) = [\bar{\mu}_{\tilde{B}}(x_i) + 1 - \bar{\upsilon}_{\tilde{B}}(x_i)]/2$.

If we consider the important degree of x_i, a weighted similarity measure between IFS $\tilde{A}$ and $\tilde{B}$ is proposed as follows:

$$S_{WR}(\tilde{A},\tilde{B}) = 1 - \frac{1}{2}\sum_{i=1}^{n} w_i\left[\frac{|\bar{\mu}_{\tilde{A}}(x_i) - \bar{\mu}_{\tilde{B}}(x_i)|}{1 + \bar{\mu}_{\tilde{A}}(x_i) + \bar{\mu}_{\tilde{B}}(x_i)} + \frac{|\bar{\upsilon}_{\tilde{A}}(x_i) - \bar{\upsilon}_{\tilde{B}}(x_i)|}{3 - \bar{\upsilon}_{\tilde{A}}(x_i) - \bar{\upsilon}_{\tilde{B}}(x_i)} + |\bar{m}_{\tilde{A}}(x_i) - \bar{m}_{\tilde{B}}(x_i)|\right] \tag{6}$$

where $w_i \in [0,1]$ $(i = 1,2,...,n)$ is the important degree of x_i, and $\sum_{i=1}^{n} w_i = 1$.

3. A new MADM method based on the proposed similarity

3.1. Discription of MADM probem

A MADM method is to find the best alternative from a set of m alternatives with respect to a set $O=\{o_1,o_2,\cdots,o_n\}$ of n attributes. Suppose that the ratings of alternatives A_i on attributes o_j are expressed with the IVIF number $\tilde{a}_{ij} =<[\mu_{ij}^-,\mu_{ij}^+],[\upsilon_{ij}^-,\upsilon_{ij}^+]>$ respectively, where $[\mu_{ij}^-,\mu_{ij}^+]$ and $[\upsilon_{ij}^-,\upsilon_{ij}^+]$ express the membership (satisfactory) and non-membership (non-satisfactory) degree of the alternative A_i on the attribute o_j with respect to the fuzzy concept "excellence" given by the decision maker.Thus, a MADM problem can be modeled by decision matrix:

$$D=(\tilde{a}_{ij})_{m\times n}=(<[\mu_{ij}^-,\mu_{ij}^+],[\upsilon_{ij}^-,\upsilon_{ij}^+]>)_{m\times n}=\begin{matrix} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix}\begin{matrix} o_1 & o_2 & \cdots & o_n \\ \left(\begin{matrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{matrix}\right)\end{matrix}$$

Let $\boldsymbol{W}=(w_1,w_2,...,w_n)^T$ be the weight vector of all attributes, where $0\le w_j\le 1$ ($j=1,2,\cdots,m$) is weight of each attribute $o_j\in O$, and $\sum_{j=1}^{n} w_j=1$. The attribute weights information is usually unknown or partially known due to the insufficient knowledge or limitation of time of decision makers in the decision making process. Therefore, for the case of weights completely unknown, this paper will develop a new weighting method as follows:

Let $d(A,B)=1-S_R(A,B)$, then it can be easily shown that $d(A,B)$ is a distance measure between two IVIF sets A and B. Motivated by maximizing deviations method [11], we can determine the weights as follows:

$$w_j=\frac{\sum_{i=1}^{m}\sum_{k=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} d(\tilde{a}_{ij},\tilde{a}_{kj})},\quad j=1,2,...,n \tag{7}$$

Here $\tilde{a}_{ij} =<[\mu_{ij}^-,\mu_{ij}^+],[\upsilon_{ij}^-,\upsilon_{ij}^+]>$ is the element of decision making matrix $D=(\tilde{a}_{ij})_{m\times n}$, and $d(\tilde{a}_{ij},\tilde{a}_{kj})$ is the distance of IVIF numbers $\tilde{a}_{ij}$ and $\tilde{a}_{kj}$, which has the following form:

$$d(\tilde{a}_{ij},\tilde{a}_{kj})=1-S_R(\tilde{a}_{ij},\tilde{a}_{kj})=\frac{1}{2}\sum_{j=1}^{n} w_j\left(\frac{|\bar{\mu}_{ij}-\bar{\mu}_{kj}|}{\bar{\mu}_{ij}+\bar{\mu}_{kj}}+\frac{|\bar{\upsilon}_{ij}-\bar{\upsilon}_{kj}|}{3-\bar{\upsilon}_{ij}-\bar{\upsilon}_{kj}}+|\bar{m}_{ij}-\bar{m}_{kj}|\right) \tag{8}$$

where $\bar{\mu}_{ij}=(1-p)\mu_{ij}^-+p\mu_{ij}^+$, $\bar{\mu}_{kj}=(1-p)\mu_{kj}^-+p\mu_{kj}^+$, $\bar{\upsilon}_{ij}=(1-p)\upsilon_{ij}^-+p\upsilon_{ij}^+$, $\bar{\upsilon}_{kj}=(1-p)\upsilon_{kj}^-+p\upsilon_{kj}^+$, $\bar{m}_{ij}=(\bar{\mu}_{ij}+1-\bar{\upsilon}_{ij})/2$, $m_{kj}=(\bar{\mu}_{kj}+1-\bar{\upsilon}_{kj})/2$.

3.2 A novel MADM method based on the proposed similarity

In this subsection, we put forward the new MADM method based on the above-mentioned work. The specific calculation steps are given as follows:

Step 1. Calculate the attribute weights according to Eq. (7);

Step 2. Determine the positive ideal solution (PIS) of the IVIF MADM problem as $A^* = (\tilde{a}_1^*, \tilde{a}_2^*, \cdots, \tilde{a}_n^*)$, where $\tilde{a}_j^* = <[1,1],[0,0]>$ ($j = 1,2,\cdots,n$).

Step 3. According to the weighted similarity measure defined in Eq. (6), the similarity measure between alternative A_i with PIS is calculated as follows:

$$S_i^+ = \sum_{j=1}^{n} w_j S_R(\tilde{a}_{ij}, \tilde{a}_j^+)$$

$$= 1 - \frac{1}{2}\sum_{j=1}^{n} w_j \left[\frac{1-(1-p)\mu_{ij}^- - p\mu_{ij}^+}{2+(1-p)\mu_{ij}^- + p\mu_{ij}^+} + \frac{(1-p)\upsilon_{ij}^- + p\upsilon_{ij}^+}{3-(1-p)\upsilon_{ij}^- - p\upsilon_{ij}^+} + 1 - \frac{(1-p)(\mu_{ij}^- + \upsilon_{ij}^-) + 1 - p(\mu_{ij}^+ + \upsilon_{ij}^+)}{2} \right]$$

Step 4. Rank the alternatives according to the similarity measures S_i^+. The larger the value of S_i^+ with respect to the better alternative A_i

4. Numerical example

In order to illustrate the effectiveness and practicability of the proposed MADM method, an example of a risk investment decision-making problem adopted from Liu et al. [12] is used to be analyzed. Suppose that a company is prepared to use a large amount of money for project investment, through the preliminary market survey and analysis, selected 5 alternative investment enterprises. There are five parallel alternatives A_1, A_2, A_3, A_4, A_5 to be selected. The evaluation attributes are the risk analysis (o_1), the growth analysis (o_2), social and political impact analysis (o_3) and the environmental impact analysis (o_4).

According to the above four evaluation attributes, the expert group evaluated the performance of five selected companies in the last 3 years, and constructed the evaluation values of IVIF decision-making matrix $D(t) = (\tilde{a}_{ij}(t))_{m\times n} = (<[\mu_{ij}^-(t), \mu_{ij}^+(t)],[\upsilon_{ij}^-(t), \upsilon_{ij}^+(t)]>)_{m\times n}$ ($t = 1,2,3$). The weights of each time period are respectively $t_1 = 0.2000, t_2 = 0.3000, t_3 = 0.5000$, and suppose that the attribute weights are unknown. Using statistical methods, the rating $\tilde{a}_{ij}$ of the alternative A_i on attribute o_j in the t time period can be obtained. Here $\tilde{a}_{ij}(t) = <[\mu_{ij}^-(t), \mu_{ij}^+(t)],[\upsilon_{ij}^-(t), \upsilon_{ij}^+(t)]>$, and the membership degree $[\mu_{ij}^-(t), \mu_{ij}^+(t)]$ (i.e. $[\mu_{ij}^-(t), \mu_{ij}^+(t)]$ means the satisfactory degree) and non-membership degree $[\upsilon_{ij}^-(t), \upsilon_{ij}^+(t)]$ (i.e. $[\upsilon_{ij}^-(t), \upsilon_{ij}^+(t)]$ means the non-satisfactory degree). IVIF information decision matrixes of each time period are shown in Table 1-3.

Table 1. IVIF information decision matrix ($t = 1$)

o_1	o_2	o_3	o_4

A_1	<[0.5,0.6],[0.3,0.4]>	<[0.5,0.6],[0.2,0.3]>	<[0.2,0.3],[0.6,0.7]>	<[0.1,0.2],[0.7,0.8]>
A_2	<[0.6,0.7],[0.2,0.3]>	<[0.7,0.8],[0.1,0.2]>	<[0.7,0.8],[0.1,0.2]>	<[0.3,0.4],[0.4,0.5]>
A_3	<[0.5,0.6],[0.3,0.4]>	<[0.4,0.5],[0.3,0.4]>	<[0.5,0.6],[0.2,0.3]>	<[0.6,0.7],[0.2,0.3]>
A_4	<[0.8,0.9],[0.0,0.1]>	<[0.5,0.6],[0.3,0.4]>	<[0.2,0.3],[0.4,0.5]>	<[0.2,0.3],[0.5,0.6]>
A_5	<[0.6,0.7],[0.2,0.2]>	<[0.3,0.4],[0.4,0.5]>	<[0.7,0.8],[0.0,0.1]>	<[0.5,0.6],[0.3,0.4]>

Table 2. IVIF information decision matrix ($t = 2$)

	o_1	o_2	o_3	o_4
A_1	<[0.3,0.4],[0.3,0.5]>	<[0.4,0.5],[0.2,0.3]>	<[0.1,0.2],[0.5,0.6]>	<[0.0,0.1],[0.6,0.7]>
A_2	[<0.6,0.7],[0.2,0.3]>	<[0.6, 0.7], [0.0,0.1]>	<[0.5, 0.6], [0.0,0.1]>	<[0.3, 0.4],[0.4,0.5]>
A_3	<[0.5,0.6],[0.2, 0.3]>	<[0.3, 0.4], [0.3,0.4]>	<[0.4, 0.5], [0.1,0.2]>	<[0.5, 0.6], [0.1,0.2]>
A_4	<[0.7,0.8],[0.0, 0.1]>	<[0.5, 0.6], [0.1,0.2]>	<[0.2, 0.3], [0.3,0.4]>	<[0.1, 0.2], [0.5,0.6]>
A_5	<[0.5,0.6],[0.1, 0.2]>	<[0.3, 0.4], [0.2,0.3]>	<[0.6, 0.8], [0.0,0.1]>	<[0.4, 0.5], [0.2,0.3]>

Table 3. IVIF information decision matrix ($t = 3$)

	o_1	o_2	o_3	o_4
A_1	<[0.3,0.4],[0.5,0.6]>	<[0.5,0.5],[0.4,0.5]>	<[0.1, 0.2],[0.7,0.7]>	<[0.0,0.1],[0.8,0.9]>
A_2	<[0.5,0.6],[0.3,0.4]>	<[0.6,0.7],[0.2,0.3]>	<[0.6, 0.6], [0.3,0.4]>	<[0.3, 0.4], [0.5,0.6]>
A_3	<0.4, 0.5],[0.4, 0.5]>	<[0.4,0.5],[0.5,0.5]>	<[0.4, 0.5], [0.3, 0.4]>	<[0.5, 0.6], [0.3,0.4]>
A_4	<[0.7,0.8],[0.1,0.2]>	<[0.5,0.6],[0.4,0.4]>	<[0.2, 0.3], [0.5, 0.6]>	<[0.1, 0.2], [0.6,0.7]>
A_5	<[0.5,0.6],[0.3,0.3]>	<[0.2,0.3],[0.4,0.6]>	<[0.6, 0.7], [0.1, 0.2]>	<[0.4, 0.5], [0.3,0.4]>

The calculation steps of the proposed method are given as follows:

Step 1. First, we use the UDIFWA operator [13] to set up the uncertain intuitionistic fuzzy matrix $D(t)$ ($t = 1,2,3$) to be a comprehensive decision matrix D, as shown in Table 4.

Table 4. Comprehensive decision matrix D

	o_1	o_2	o_3	o_4

A_1	<[0.3456,0.4467], [0.3873,0.5238]>	<[0.4719,0.5218], [0.2828,0.3873]>	<[0.1210,0.2211], [0.6136, 0.6684]>	<[0.0209,0.1210], [0.7145,0.8152]>
A_2	<[0.5528,0.6536], [0.2449,0.3464]>	<[0.6224,0.7234], [0, 0.1990]>	<[0.5962,0.6518], [0, 0.2297]>	<[0.3000,0.4000], [0.4472, 0.5477]>
A_3	<[0.4523,0.5528], [0.3067, 0.4102]>	<[0.3716,0.4719], [0.3873,0.4472]>	<[0.4215,0.5218], [0.1990,0.3067]>	<[0.5218,0.6224], [0.1990,0.3067]>
A_4	<[0.7234,0.8259], [0, 0.1414]>	<[0.5000,0.6000], [0.2491, 0.3249]>	<[0.2000,0.3000], [0.4102, 0.5123]>	<[0.1210,0.2211], [0.5477, 0.6481]>
A_5	<[0.5218,0.6224], [0.1990,0.2449]>	<[0.2517,0.3519], [0.3249, 0.4699]>	<[0.6224,0.7551], [0, 0.1414]>	<[0.4215,0.5218], [0.2656,0.3669]>

Setp 2. Set $p = 0.1$, then according to the Eq. (9), the attribute weights vector is obtained as $\boldsymbol{W} = (w_1, w_2, w_3, w_4)^T = (0.1800, 0.1792, 0.3307, 0.3101)^T$

Step 3. The PIS (A^*) is $A^* = (\tilde{a}_1^*, \tilde{a}_2^*, \tilde{a}_3^*, \tilde{a}_4^*) = (<[1,1],[0.0]>, \ldots, <[1,1],[0.0]>)$

Step 4. According to Eq. (11), the similarity measures S_i^* of each alternative from PIS are calculated as: $S_1^* = 0.3611, S_2^* = 0.6901, S_3^* = 0.6408, S_4^* = 0.5235$ and $S_5^* = 0.6853$.

Step 5. Therefore, the ranking order is $A_2 \succ A_5 \succ A_3 \succ A_4 \succ A_1$, and A_2 is the desirable alternative. This result is in agreement with Liu et al [12].The attitude factor p reflects the attitude of the decision maker to recognize the interval number, and it is more consistent with the objective reality.

5. Conclusions.

In this paper, the similarity measure of IVIF sets is analyzed and studied. Firstly, we construct a new similarity calculating formula, and then basis on this simliarity, a new decision making method is put forward for the MADM problem with attribute values expressed by IVIF numbers and completely unknown information of attribute weights. The advantages of the proposed decision making method are as follows:

(1) Similarity measure can be used to measure the similarity between sets, and it can be used to avoid the deficiency of intuitionistic fuzzy additive operation to some extent;

(2) A new method for determining the weights of attributes based on the maximum deviation method is proposed. An example of risk project investment illustrated the proposed method is feasible and effective;

(3) The proposed similarity measure can be applied to pattern recognition, medical diagnosis and cluster analysis field, the proposed MADM method can be applied to such as venture investment project selection, site selection, emergency management and decision.

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