AMSE JOURNALS –2015-Series: Advances A; Vol. 35; N° 1; pp 48-64
Submitted Nov. 2014; Revised Jan. 2, 2015; Accepted June 15, 2015

The Computations of the Special Determinants and Their applications

Liang. J[1,2]

1. Depart of Fundamental Science,Gungdong University of Science & Technology
P.R.China Guangdong Xihul Road 523083 (E-maill Liangjp@126.com)
2. Institute of Applied Mathematics, Guangdong University of Technology,
P.R China Guang Zhou Huan shi domg Road 510075

Abstract

In this paper, the results of the computing the special determinants have been obtained which can be applied in the additions of sever fractions and continued fractions. Without the computing of the special determinants, the additions of the sever fractions, and continued fraction will be very difficult or even impossible. In fact, the additions of sever fractions and continued fraction are the key to induce the formula of interpolations, which are often applied in simulate engineering. For example, in the outline design, from some points given to approach the unknown curve, the interpolation is necessary. But in the process of interpolation induced, must need the additions of sever fractions. and continued fraction to finish the computation .We use computation of the special determinants that greatly simplify the process of the addition .In this paper, the results of the computing the special determinants and the application for simplify the additions of sever fractions and continued fraction have been given.

Keywords : Interpolation approach, Function. Determinants

1 Introduction

Some special determinants of the high-order determinant were discussed, whose elements of roads from 1 to n are the 0 order differentials of $x_i^j(i,j=0,1,2...n)$,the elements of roads from n+1 to 2n in the determinants are the 1 order differentials of $x_i^j(i,j=0,1,2...n)$,and elements of roads from 2n+1 to 3n in the determinants are the 2 order differentials of $x_i^j(i,j=0,1,2...n)$.These determinants have been reduced to be the lower-order determinants that greatly simplifies the complex operation ,containing sever fractions, and continued fractions. For example, following are the results obtained in papers [1-6] .

Lemma 1.1

In paper [6] there is a following result;

$$\frac{D(3n+4)_{1+k.1}}{D(3n+4)_{n+2.1}}=-(\frac{x-x_0}{x_k-x_0})^3[l_k(n)-\frac{3}{x_k-x_0}l_k^{'}(n)+\frac{2\cdot 3!}{(x_k-x_0)^2}l_k^{''}(n)]$$

$$=[1+3(x-x_k)\sum_{\substack{i=0\\i\neq k}}^{n}(\frac{1}{x_i-x_k})+2\cdot3(x-x_k)^2\sum_{\substack{i=0\\i\neq k}}^{n}(\frac{1}{x_i-x_k})^2$$

$$+3\cdot3(x-x_k)^2\sum_{\substack{i=1\\i\neq k}}^{n-1}(\frac{1}{x_i-x_k})\sum_{j=i+1}^{n}(\frac{1}{x_j-x_k})]\prod_{\substack{i=1\\i\neq k}}^{n}(\frac{x-x_i}{x_k-x_i})^3$$

$$=l_k(n+1) \qquad (1.1)$$

Where $\mathrm{D}(3\mathrm{n}+4)_{n=2,1}$ is the algebra cofactors dispelled the entries the n+2-th row、the 1st column in determinants D(3n+4):

$$\boldsymbol{D(3n+4)}=\begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+2} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+2} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+2} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n+2} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+2)\mathrm{x}_0^{3n+1} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+2)x_1^{3n+1} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+2)x_n^{3n+1} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_0 & \cdots & \frac{(3n+2)!}{3n!}x_0^{3n} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+2)!}{3n!}x_1^{3n} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+2)!}{3n!}x_n^{3n} \end{vmatrix}. \qquad \textbf{(1.2)}$$

and $D(3n+4)_{1+k,1}$ was dispelled the entries of the k-th row、the 1st column in determinant D(3n+4) too .And

$$l_k(n)=[1+3(x-x_k)\sum_{\substack{i=1\\i\neq k}}^{n}\frac{1}{x_i-x_k}+2\cdot 3(x-x_k)^2\sum_{\substack{i=1\\i\neq k}}^{n}\frac{1}{(x_i-x_k)^2}$$

$$+3g3(x-x_k)^2\sum_{\substack{i=1\\i\neq k}}^{n-1}\frac{1}{x_i-x_k}\sum_{j=i+1}^{n}\frac{1}{x_i-x_k}\prod_{\substack{i=1\\i\neq k}}^{n}(\frac{x-x_i}{x_k-x_i})^3 \qquad (1.3)$$

$$l'_k(n) = (x - x_k)[1 + 3(x - x_k)\sum_{\substack{i=1\\ i\neq k}}^{n}\frac{1}{x_i - x_k}]\prod_{\substack{i=1\\ i\neq k}}^{n}(\frac{x - x_i}{x_k - x_i})^3 \qquad (1.4)$$

.

$$l''_k(n) = \frac{1}{2}(x - x_k)^2\prod_{\substack{i=1\\ i\neq k}}^{n}(\frac{x - x_i}{x_k - x_i})^3 \qquad (1.5)$$

Lemma 1.2

$$\frac{D(3n+1)_{k,1}}{D(3n+1)_{n+1,1}} = -l_k(n) \qquad (1.6)$$

Where $D(3n+1)_{k,1}$ $D(3n+1)_{n+1,1}$ are the algebra cofactors of determinants $D(3n+1)$ in following form:

$$D(3n+1) = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_m^{3n-1} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n-1} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n-1} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n-1} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n-1)x_1^{3n-2} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-2} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-2} \end{vmatrix}$$

(1.7)

and $l_k(n)$ is form (1.3).

Lemma 1.3.

$$\frac{D(3n+1)_{n+1+k}}{D(3n+1)_{n+1,1}}=-l_k'(n)\ . \qquad (1.8)$$

Where $D(3n+1)_{n+1+k,1}$, $D(3n+1)_{n+1,1}$ are the algebra cofactors of determinants $D(3n+1)$ in form (1.7).

Lemma 1.4.

$$\frac{D(3n+1)_{2n+1+k,1}}{D(3n+1)_{n+1,1}}=-l_k''(n) \qquad \textbf{(1.9)}$$

.

Where $D(3n+1)_{2n+1+k,1}$ $D(3n+1)_{n+1,1}$ are the algebra cofactors of the determinants in form (1.7) ,and $l_k'(n)$ is the form (1.4).

Lemma 1.5.

$$D(3n+4)_{1+k,1}=(-1)^{3n+1}2(x-x_0)^3(x_k-x_0)^6\prod_{i=1}^{n}(x_k-x_0)^9\left[D(3n+1)_{k,1}\right.$$

$$-\frac{3}{x_k - x_0} D(3n+1)_{n+1+k,1} + 2g3! D(3n+1)_{2n+1+k,1} \;\Big] \qquad \textbf{(1.10)}$$

Where $\boldsymbol{D(3n+4)_{1+k,1}}$ is the algebra cofactors of determinants $\boldsymbol{D(3n+4)}$ in form(1.2), and $\boldsymbol{D(3n+1)_{k,1}}$, $\boldsymbol{D(3n+1)_{n+1+k}}$ $\boldsymbol{D(3n+1)_{2n+1+k,1}}$ are the algebra cofactors of determinants in form (1.7)

Lemma 1.6.

$$\mathrm{D}(3\mathrm{n}+4)_{\mathrm{n}+2,1} = (-1)^{3\mathrm{n}+1} 2 \prod_{i=0}^{\mathrm{n}} (x_i - x_0)^9 D(3n+1)_{n+1.1} \qquad (1.11)$$

Where $\boldsymbol{D(3n+4)_{n+2,1}}$ is the algebra cofactors of determinants $\boldsymbol{D(3n+4)}$ in form(1.2), and $\boldsymbol{D(3n+1)_{n+1,1}}$ is the algebra cofactors of determinants in form (1.7)

2. Main results

Theorem 2.1

$$1 - \sum_{k=1}^{n} \left(\frac{x - x_0}{x_k - x_0}\right)^3 \left[l_k(n) - \frac{3}{x_k - x_0} l_k'(n) + \frac{2.3!}{(x - x_0)^2} l_k''(n)\right] = l_0(n+1). \qquad (2.1)$$

where $l_k(n)$, $l_k'(n)$, $l_k''(n)$ are the form (1.3),(1.4),(1.5) and $l_0(n+1)$ is the form (1.1),as $k = 0$:

$$l_0(n+1) = \quad [1+3(x-x_0)\sum_{i=1}^{n}(\frac{1}{x_i-x_0}) + 2g3(x-x_0)^2\sum_{i=1}^{n}(\frac{1}{x_i-x_0})$$

$$+3g3(x-x_0)^2\sum_{i=1}^{n-1}(\frac{1}{x_i-x_0})\sum_{j=i+1}^{n}(\frac{1}{x_j-x_0})]\prod_{i=1}^{n}(\frac{x-x_i}{x_0-x_i})^3. \qquad (2.2).$$

Proof. According to form (1.10), we have:

$$D(3n+4)_{1+k,1} = (-1)^{3n+1}2(x-x_0)^3(x_k-x_0)^6\prod_{i=1}^{n}(x_k-x_0)^9\Big[D(3n+1)_{k,1}$$

$$-\frac{3}{x_k-x_0}D(3n+1)_{n+1+k,1} + 2g3!D(3n+1)_{2n+1+k,1}$$

$$(2.3)$$

and

$$\frac{D(3n+4)_{1+k.1}}{D(3n+4)_{n+2.1}} = (\frac{x-x_0}{x_k-x_0})^3\Bigg[\frac{D(3n+1)_{k,1}}{D(3n+1)_{n+1,1}} - \frac{3}{x_k-x_0}\frac{D(3n+1)_{n+1+k,1}}{D(3n+1)_{n+1,1}}$$

$$+\frac{2.3!}{(x_k-x_0)^2}\frac{D(3n+1)_{n+1+k,1}}{D(3n+1)_{n+1,1}}\Bigg]. \qquad \mathbf{(2.4)}$$

See form (1.6),(1.8),(1.9),

$$\frac{D(3n+1)_{k,1}}{D(3n+1)_{n+1,1}} = -l_k(n).$$

$$\frac{D(3n+1)_{n+1+k}}{D(3n+1)_{n+1,1}} = -l_k'(n)$$

$$\frac{D(3n+1)_{2n+1+k,1}}{D(3n+1)_{n+1,1}} = -l_k''(n)$$

The right hand of form (2.4) is replaced by above forms, following has been established.

$$\frac{D(3n+4)_{1+k.1}}{D(3n+4)_{n+2.1}} = -(\frac{x-x_0}{x_k-x_0})^3[l_k(n) - \frac{3}{x_k-x_0}l_k^{'}(n) + \frac{2\cdot 3!}{(x_k-x_0)^2}l_k^{''}(n)] \quad (2.5)$$

and

$$1 - \sum_{k=1}^{n}(\frac{x-x_0}{x_k-x_0})^3 [l_k(n) - \frac{3}{x_k-x_0}l_k'(n) + \frac{2\cdot 3!}{(x_k-x_0)^2}l_k''(n)] \quad (2.6)$$

$$= 1 + \sum_{k=1}^{n}\frac{D(3n+4)_{1+k.1}}{D(3n+4)_{n+2.1}} = \frac{D(3n+4)_{n+2,1} + \sum_{k=1}^{n}D(3n+4)_{1+k.1}}{D(3n+4)_{n+2,1}}$$

$$= \frac{\sum_{k=1}^{n}D(3n+4)_{1+k.1} + D(3n+4)_{n+2,1}}{D(3n+4)_{n+2,1}}$$

$$= \frac{D(3n+4)}{D(3n+4)_{n+2,1}}. \quad (2.7)$$

that is the result of computing determinants D(3n+4) in form (2.7) : add-1 times the second column to the first column). .The determinant D(3n+4) in form(2.7) ,becomes:

$$D(3n+4) = \begin{vmatrix} -1 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+2} \\ 0 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+2} \\ & & & & & \cdots & \\ 0 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+2} \\ 0 & 1 & x & x^2 & x^3 & \cdots & x^{3n+2} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+2)x_0^{3n+1} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+2)x_1^{3n+1} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+2)x_n^{3n+1} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_0 & \cdots & \frac{(3n+2)!}{3n!}x_0^{3n} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+2)!}{3n!}x_1^{3n} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+2)!}{3n!}x_n^{3n} \end{vmatrix}.$$

$$= (-1)^{1+1}(-1)\begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+2} \\ 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n+2} \\ & & & & \cdots & \\ 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+2} \\ 1 & x & x^2 & x^3 & \cdots & x^{3n+2} \\ 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+2)\mathrm{x}_0^{3n+1} \\ 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+2)x_1^{3n+1} \\ & & & & \cdots & \\ 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+2)x_n^{3n+1} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_0 & \cdots & \frac{(3n+2)!}{3n!}x_0^{3n} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+2)!}{3n!}x_1^{3n} \\ & & & & \cdots & \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+2)!}{3n!}x_n^{3n} \end{vmatrix}.$$

$$= -D(3n+4)_{1,1} \tag{2.8}$$

Where we expands along the first column of D(3n+4) in form (2.4).

$$D(3n+4) = -D(3n+4)_{1,1} \tag{2.9}$$

The right hand-side in form (2.3) is replaced by above form, and notice form (1.1);

$$1-\sum_{k=1}^{n}\left(\frac{x-x_0}{x_k-x_0}\right)^3\left[l_k(n)-\frac{3}{x_k-x_0}l'_k(n)+\frac{2.3!}{(x_k-x_0)^2}l''_k(n)\right]$$

$$= -\frac{D(3n+4)_{1,1}}{D(3n+4)_{n+2,1}} = -\frac{D(3n+4)_{1+0.1}}{D(3n+4)_{n+2.1}}. \tag{2.10}$$

Consider form (1.1)

$$\frac{D(3n+4)_{1+k,1}}{D(3n+4)_{n+1,1}} = -[1+3(x-x_k)\sum_{\substack{i=0\\i\neq k}}^{n}(\frac{1}{x_i-x_k}) + 2\cdot 3(x-x_k)^2\sum(\frac{1}{x_i-x_k})^2$$

$$+3\cdot 3(x-x_k)^2\sum_{\substack{i=0\\i\neq k}}^{n-1}(\frac{1}{x_i-x_k})\sum_{\substack{j=i+1\\j\neq k}}^{n}\frac{1}{x_j-x_k}]\prod_{\substack{i=0\\i\neq k}}^{n}(\frac{x-x_i}{x_k-x_i})^3$$

$$= -l_k(n+1).$$

$$\frac{D(3n+4)_{1+0}}{D(3n+4)_{n+1,1}} = -[1+3(x-x_0)\sum_{i=1}^{n}(\frac{1}{x_i-x_0}) + 2\cdot 3(x-x_0)^2\sum_{i=1}^{n}(\frac{x-x_i}{x_0-x_i})^2.$$

$$+3\cdot 3(x-x_0)^2\sum_{i=1}^{n-1}(\frac{1}{x_i-x_0})\sum_{j=i+1}^{n}(\frac{1}{x_j-x_0})]\prod_{i=1}^{n}(\frac{x-x_i}{x_0-x_i})^3$$

$$= -l_0(n+1)$$

As $k=0.$

From form (2.10) we have :

$$1-\sum_{k=1}^{n}(\frac{x-x_0}{x_k-x_0})^3[l_k(n) - \frac{3}{x_k-x_0}l'_k(n) + \frac{2.3!}{(x_k-x_0)^2}l''_k(n)] = l_0(n+1)$$

It means theorem is correct.

3. Application.

To simplify, the following determinants, in which the element contains no derivative;

$$D = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 \\ 1 & 1 & x_1 & x_1^2 & x_1^3 \\ 1 & 1 & x_2 & x_2^2 & x_2^3 \\ 1 & 1 & x_3 & x_2^2 & x_3^3 \\ 1 & 1 & x & x^2 & x^3 \end{vmatrix} \qquad (3.1)$$

And its algebra cofactors:

$$D_{1,1} = \begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x & x^2 & x^3 \end{vmatrix} \qquad (3.2)$$

$$D_{5.1} = \begin{vmatrix} 1 & x_0 & x_0^2 & x_0^3 \\ 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^2 \end{vmatrix} \qquad (3.3)$$

They were computed:

$$D_{1,1} = (-1)^{1+1}(x_1 - x)(x_2 - x)(x_3 - x)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} \qquad (3.4)$$

$$D_{5.1} = (-1)^{5+1}(x_1 - x_0)(x_2 - x_0)(x_3 - x_0)\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} \qquad (3.5)$$

And

$$\frac{D_{3.1}}{D_{5.1}} = -\frac{(x_1 - x)(x_2 - x)(x_3 - x)}{(x_1 - x_0)(x_2 - x_0)(x_3 - x_0)} \qquad (3.6)$$

Similarly,

$$\frac{D_{2.1}}{D_{5.1}} = -\frac{(x_0 - x)(x_2 - x)(x_3 - x)}{(x_0 - x_1)(x_2 - x_1)(x_3 - x_1)} \tag{3.7}$$

$$\frac{D_{3.1}}{D_{5.1}} = -\frac{(x_0 - x)(x_1 - x\ )(x_3 - x\ )}{(x_0 - x_2)(x_1 - x_2)(x_3 - x_2)} \tag{3.8}$$

$$\frac{D_{4.1}}{D_{5.1}} = -\frac{(x_0 - x\ )(x_1 - x\ )(x_2 - x\ )}{(x_0 - x_3)(x_1 - x_3)(x_2 - x_3)} \tag{3.9}$$

The results can be used in addition of several fractions;

$$1 - \frac{(x_0 - x)(x_2 - x)(x_3 - x)}{(x_0 - x_1)(x_2 - x_1)(x_3 - x_1)} - \frac{(x_0 - x)(x_1 - x)(x_3 - x)}{(x_0 - x_2)(x_1 - x_2)(x_3 - x_2)}$$

$$-\frac{(x_0 - x)(x_1 - x)(x_2 - x)}{(x_0 - x_3)(x_1 - x_3)(x_2 - x_3)}. \tag{3.10}$$

Above form was replaced by form (3.6),(3.7),(3.8),(3.9).that result in

$$1 + \frac{D_{2.1}}{D_{5.1)}} + \frac{D_{3.1}}{D_{5.1}} + \frac{D_{4.1}}{D_{5.1}} = \frac{D_{2.1} + D_{3.1} + D_{4.1} + D_{5.1}}{D_{5.1}} \tag{3.11}$$

Because the numerical of determinants (3.1) is

$$D = 0 \times D_{1.1} + 1 \times D_{1.2} + 1 \times D_{1.3} + 1 \times D_{1.4} + D_{1.5} \tag{3.12}$$

The elements of column 1 in determinants (3.1) were subtracted column 2.this result is:

$$D = \begin{vmatrix} -1 & 1 & x_0 & x_0^2 & x_0^3 \\ 0 & 1 & x_1 & x_1^2 & x_1^3 \\ 0 & 1 & x_2 & x_2^2 & x_2^3 \\ 0 & 1 & x_3 & x_2^2 & x_3^3 \\ 0 & 1 & x & x^2 & x^3 \end{vmatrix} \tag{3.13}$$

And then expand the first column, we have:

$$D = \begin{vmatrix} -1 & 1 & x_0 & x_0^2 & x_0^3 \\ 0 & 1 & x_1 & x_1^2 & x_1^3 \\ 0 & 1 & x_2 & x_2^2 & x_2^3 \\ 0 & 1 & x_3 & x_2^2 & x_3^3 \\ 0 & 1 & x & x^2 & x^3 \end{vmatrix} = -D_{1.1} \qquad (3.14).$$

Compare (3.11), (3.14) two types:

$$D_{1,2} + D_{1.3} + D_{1.4} + D_{1.5} = -D_{1,1} \qquad (3.15)$$

And

$$1 + \frac{D_{2.1}}{D_{5.1}} + \frac{D_{3.1}}{D_{5.1}} + \frac{D_{4.1}}{D_{5.1}} = \frac{D}{D_{5.1}} = \frac{-D_{1.1}}{D_{5.1}} = \frac{(x_1 - x)(x_2 - x)(x_3 - x)}{(x_1 - x_0)(x_2 - x_0)(x_3 - x_0)}$$

It means:

$$1 - \frac{(x_0 - x\ )(x_2 - x)(x_3 - x)}{(x_0 - x_1)(x_2 - x_1)(x_3 - x_1)} - \frac{(x_0 - x)(x_1 - x)(x_3 - x)}{(x_0 - x_2)(x_1 - x_2)(x_3 - x_2)} - \frac{(x_0 - x)(x_1 - x)(x_2 - x)}{(x_0 - x_3)(x_1 - x_3)(x_2 - x_3)}$$

$$= \frac{(x_1 - x)(x_2 - x)(x_3 - x)}{(x_1 - x_0)(x_2 - x_0)(x_3 - x_0)}$$

An example

It is meaningful to the interpolation method and is important tool in modeling and simulation .For example in simulation engineering, the following interpolation

$$L_n(x) = \sum_{k=0}^{n} y_k \prod_{\substack{i=1 \\ i \neq k}}^{n} \left(\frac{x - x_i}{x_k - x_i}\right) \qquad (3.16)$$

can be used to design the satellite . If following dada were given:

$$A_0(x_0, y_0), A_1(x_1, y_1), ..., A_n(x_n, y_n).$$

The outline of satellite simulation must pass the above data. But to obtain the interpolation (3.16) .the computation of the special determinants is necessary. The more points (data) the more we can approach the outline of the satellite. But the solution of the problem for the above interpolations has not been seen in papers [7-14]. The theorems 2.1 are the basic of solution of the problem. Subject to the limit of paper pages, more results of application in interpolation will appear in other papers.

Conclusion

The main results of this paper are the theorems 2.1 . They are the computation of high order determinants and their elements are consist of 0,1,2 order differentials whose elements of roads from 1 to n+1 in the determinants are the 0 order differentials of $x_i(i = 0,1,2...)$.and the elements of roads from n+2 to 2n+2 in the determinants are the 1 order differentials of $x_i(i = 0,1,2...)$,and the elements of roads from 2n+3 to 3n+3 in the determinants are the 2 order differentials of $x_i(i = 0,1,2...)$.Because of the high order ,to compute them are difficulty. But they have been reduced to the sum of several determinants which are lower-order. The several determinants of low order are easy to be computed, which is applied in A few continued fractions of complex operations. It is meaningful to the interpolation method, which is important tool in modeling and simulation .For example the design the outline of air plane, bigger ship and satellite need interpolation. But the solution of the problem has not been seen in papers [7-14]. The theorems 2.1 are the basic of solution of the problem. Subject the limit of paper page, the more results of application in interpolation will appear in our other papers.

. References

1 Liang. J. “A simplification for several fraction forms”. AMSE journals, Series Advances A-general mathematics. Vol 48.No 1, PP 1-16 (2001)

2. Liang. J “Special determinants and their computation”. AMSE Journals, Series Advances A, vol. 48. N° 2, PP 27-41. (2011)

3. Liang. J. “Uniqueness of interpolation polynomial for two variables”. AMSE Journals, Series Advances A, vol. 48. N° 2 PP 42-56 (2011)

4. Liang. J. “The interpolation formula of Hermitic in two variables”. AMSE Journals, Series Advances A, vol. 48. N° 2, PP 1-20 (2012)

5. Liang. J. “Computing the special determinants”, AMSE Journals, Series Advances A, vol. 48. N° 1, PP 1-11 (2013)

6. Liang J. “The results of computing the special determinant”, AMSE Journals, Series Advances A, vol. 50 (2014).

7. C. Bennett and R. Shapely “Interpolation of operators”. Academic press, New-York, MR89e: 46001 (1998)

8. J. Bergh, J, Ł. O strum. “Interpolation spaces. An Introduction”, Springer-Verlag, New York, MR58:2349 (1976)

9. J.H. Bramble Interpolation between Soboles spaces in Lipschitz domains with an application to multigrin theory" Math. Comp., 64:1359-1365 MR95m:46042 (1995)

10. J.H. Bramble and X. Zhang. "The analysis of multigrin methods, in: Handbook for Numerical Analysis", Viol, Ⅶ, 173-415, P. Ciarlet and J.L.Lions, eds, North Holland, Amsterdam, MR2001m: 65183. (2000)

11.R.B. Kellogg. "Interpolation between subspaces of a Hilbert space, Technical note BN-719, Institutor for Fluid Dynamics and Applied Mathematics", University of Maryland, College park (1971)

12 L.R. Scott and S. Zhang, "Finite element interpolation of non-smooth functions satisfying boundary conditions", Math. Comp. 54,483-493.MR 90j:65021 (1990)

13. L.B. Wahlbin, "A quasi-optimal estimate in piecewise polynomial Galekin approximation of parabolic problems in Numerical Analysis", Berlin-New York, PP, 230-245. MR83f:65157 (1981)

14..C. Bacuta, J.H. Bramble, J. Pasciak. "New interpolation results and application to finite elements methods for elliptic boundary value problems". East-West J. Numer. Math.9:179-198 (2001)