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Computation of the Special Determinants

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Abstract. In this paper the special determinants have been computed, in which the results can be applied to simplify the addition of sever fractions and continued fraction. The computation of addition for sever faction and continued fraction is necessary to induce the interpolations, which are often use in simulate engineering for instance the design of outline for the satellite, the plane, the spacecraft. But the additions of sever factions and continued fractions are very complex because of their excessive number of continued fraction .According to the general addition, this work cannot be done. In this paper, the results of the sever fraction and continued fraction have been obtained, through computing the special determinants

Keywords : Interpolation approach, Function. Determinants

1 Introduction

In the paper [1-7], some special determinants have been computed. There are some results, for example, below:

Lemma 1

$$D(2n+3)_{k,1} = (-1)^{n+5}(x_k - x_0)^2(x - x_0)^2 \prod_{i=1, i\neq k}^{n}(x_i - x_0)^4 \Big[D(2n+1)_{k,1}$$

$$-\frac{1}{x_k - x_0} D(2n+1)_{n+k+1,1}\Big] \qquad (1.1)$$

Where $D(2n+3)_{k,1}$, is the algebra cofactor of the following is determinates:

$$(2n+3) = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & \dots & x_0^{2n+1} \\ 1 & 1 & x_1 & x_1^2 & \dots & x_1^{2n+1} \\ & & \dots & \dots & & \\ 1 & 1 & x_n & x_n^2 & \dots & x_n^{2n+1} \\ 1 & 1 & x & x^2 & \dots & x^{2n+1} \\ 0 & 0 & 1 & 2x_0 & \dots & (2n+1)x_0^{2n} \\ 0 & 0 & 1 & 2x_1 & \dots & (2n+1)x_1^{2n} \\ & & \dots & \dots & & \\ 0 & 0 & 1 & 2x_n & \dots & (2n+1)x_n^{2n} \end{vmatrix} \qquad (1.2)$$

And $D(2n+1)_{k,1}$, $D(2n+1)_{n+k+1,1}$ is the algebra cofactors of the following determinants:

$$D(2n+1)=\begin{vmatrix} 0 & 1 & x_1 & x_1^2 & \dots & x_1^{2n-1} \\ 1 & 1 & x_2 & x_2^2 & \dots & x_2^{2n-1} \\ & & \dots & \dots & & \\ 1 & 1 & x_n & x_n^2 & \dots & x_{2n}^{2n-1} \\ 1 & 1 & x & x^2 & \dots & x^{2n-1} \\ 0 & 0 & 1 & 2x_1 & \dots & (2n-1)x_1^{2n-2} \\ 0 & 0 & 1 & 2x_2 & \dots & (2n-1)x_2^{2n-2} \\ & & \dots & \dots & & \\ 0 & 0 & 1 & 2x_n & \dots & (2n-1)x_n^{2n-2} \end{vmatrix} \qquad (1.3)$$

That can be seen the high-order determinants has been $D(3n+1)_{1,1}$ $D(2n+1)_{1,1}$ $D(2n+1)_{n+1+k,1}$ reduced to be lower-order determinants ,which is used to simplify the operation of continued fraction, and then the interpolation can be derived, which is useful for simulation engineer. To obtain more interpolation, the more determinants need to be computed. But papers [8-15] have not discussed them, until now. Following section is about the main results of this paper.

2. Main results.

Main results of this paper are as follows:

Theorem 2.1

$$D(3n+3)_{1,1}=(-1)^{3n+1}(x-x_1)^3(x_0-x_1)^3\prod_{i=2}^{n}(x-x_i)^9[D(3n)_{1,1}-\frac{3}{(x_0-x_1)}D(3n)_{n+2,1}]$$

Where $D(3n+3)_{1,1}$ $D(3n+3)_{n+2,1}$ are the algebra cofactors of the following determinants (1.8),

$$D(3n+3)=\begin{vmatrix}0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+1}\\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+1}\\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+1}\\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n+1}\\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+1)x_0^{3n}\\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+1)x_1^{3n}\\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+1)x_n^{3n}\\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1}\\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1}\end{vmatrix}. \qquad (2.1)$$

And $D(3n)_{n+2,1}$, $D(3n)_{1,1}$ are algebra cofactors of following determinants :

$$D(3n)=\begin{vmatrix}0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n-2}\\ 1 & 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n-2}\\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n-2}\\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n-2}\\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n-2)x_0^{3n-3}\\ 0 & 0 & 1 & 2x_2 & 3x_2^2 & \cdots & (3n-2)x_2^{3n-3}\\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n-2)x_n^{3n-3}\\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & \cdots & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4}\\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4}\end{vmatrix} \qquad (2.2)$$

Proof : According to the definition of the algebra cofactor $D(3n+3)_{1,1}$ is the form below:

$$D(3n+3)_{1,1} = (-1)^{1+1} \begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+1} \\ 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n+1} \\ & & & & \cdots & \\ 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+1} \\ 1 & x & x^2 & x^3 & \cdots & x^{3n+1} \\ 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+1)x_0^{3n} \\ 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+1)x_1^{3n} \\ & & & & \cdots & \\ 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+1)x_n^{3n} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1} \\ & & & & \cdots & \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1} \end{vmatrix}. \qquad (2.3)$$

Beginning the last column in form (2.3) add $-x_1$ times the k-1-th column to the k-th column (k=3n+2, 3n+1, 3n... 2, 1)

$$D(3n+3)_{1,1}=\begin{vmatrix}
1 & 0 & 0 & 0 & \ldots & 0\\
1 & x_2-x_1 & x_2^2-x_2x_1 & x_2^3-x_2^2x_1 & L & x_2^{3n+1}-x_2^{3n}x_1\\
1 & x_3-x_1 & x_3^2-x_3x_1 & x_3^3-x_3^2x_1 & L & x_3^{3n+1}-x_3^{3n}x_1\\
 & & & & L & \\
 & & & & L & \\
1 & x_n-x_1 & x_n^2-x_2x_1 & x_n^3-x_n^2x_1 & L & x_n^{3n+1}-x_n^{3n}x_1\\
1 & x-x_1 & x^2-xx_1 & x^3-xx_1 & L & x^{3n+1}-x^{3n}x_1\\
0 & 1 & 2x_0-x_1 & 3x_0^2-2x_0x_1 & L & (3n+1)x_0^{3n}-3nx_0^{3n-1}x_1\\
0 & 1 & 2x_1-x_1 & 3x_1^2-2x_1x_1 & L & (3n+1)x_1^{3n}-3nx_1^{3n-1}x_1\\
0 & 1 & 2x_2-x_1 & 3x_2^2-2x_2x_1 & L & (3n+1)x_2^{3n}-3nx_2^{3n-1}x_1\\
 & & & & L & \\
0 & 1 & 2x_n-x_1 & 3x_n^2-2x_nx_1 & L & (3n+1)x_n^{3n}-3nx_n^{3n-1}x_1\\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1-\frac{2!}{01}x_1 & L & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1}-\frac{3n!}{(3n-2)!}x_1^{3n-2}x_1\\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2-\frac{2!}{0!}x_1 & L & \frac{(3n+1)!}{(3n-1)!}x_2^{3n-1}-\frac{3n!}{(3n-2)!}x_2^{3n-2}x_1\\
 & & & & L & \\
 & & & & L & \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n-\frac{2!}{0!}x_1 & L & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1}-\frac{3n!}{(3n-2)!}x_n^{3n-2}x_1
\end{vmatrix}$$

(2.4)

Expand the determinant along rows 1 and then extract out the common factor $x_i - x_0$ from rows i in $D(3n+3)_{1,1}$, (i=1,2,...,n) , So above determinant becomes :

$$D(3n+1)_{1,1} = (-1)^{1+1}(x-x_1)\prod_{i=2}^{n}(x_i - x_1)A \qquad (2.5)$$

Where A is following determinants:

$$A=\begin{vmatrix}
1 & x_2 & x_2^2 & \cdots & x_2^{3n} \\
1 & x_3 & x_3^2 & \cdots & x_3^{3n} \\
 & & & \cdots & \\
 & & & \cdots & \\
1 & x_n & x_n^2 & \cdots & x_n^{3n} \\
1 & x & x^2 & \cdots & x^{3n+1}-x^{3n}x_1 \\
1 & 2x_0-x_1 & 3x_0^2-2x_0x_1 & \cdots & (3n+1)x_0^{3n}-3nx_0^{3n-1}x_1 \\
1 & 2x_1-x_1 & 3x_1^2-2x_1x_1 & \cdots & (3n+1)x_1^{3n}-3nx_1^{3n-1}x_1 \\
1 & 2x_2-x_1 & 3x_2^2-2x_2x_1 & \cdots & (3n+1)x_2^{3n}-3nx_2^{3n-1}x_1 \\
 & & & \cdots & \\
1 & 2x_n-x_1 & 3x_n^2-2x_nx_1 & \cdots & (3n+1)x_n^{3n}-3nx_n^{3n-1}x_1 \\
0 & \frac{2!}{0!} & \frac{3!}{0!}x_2-\frac{2!}{0!}x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1}-\frac{3n!}{(3n-2)!}x_1^{3n-2}x_1 \\
0 & \frac{2!}{0!} & \frac{3!}{1!}-\frac{2!}{0!}x_2 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_2^{3n-1}-\frac{3n!}{(3n-2)!}x_2^{3n-2}x_1 \\
 & & & \cdots & \\
 & & & \cdots & \\
0 & \frac{2!}{0!} & \frac{3!}{1!}-\frac{2!}{0!}x_n & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1}-\frac{3n!}{(3n-2)!}x_n^{3n-2}x_1
\end{vmatrix} \qquad (2.6).$$

Inform (2.6), line $n+i$ ($i=3,4,...,n-1.$) elements minus the line i $(i=1,2,...,n-1.)$ elements, and exact out the common factors, determinants (2.6) became :

$$A=\prod_{i=2}^{n}(x_i-x_1)B \qquad (2.7)$$

$$
B=\begin{vmatrix}
1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n} \\
 & & & & \cdots & \\
1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n} \\
1 & x & x^2 & x^3 & \cdots & x^{3n} \\
1 & 2x_0-x_1 & 3x_0^2-2x_0x_1 & 4x_0^3-3x_0^2x_1 & \cdots & (3n+1)x_0^{3n}-3nx_0^{3n-1}x_1 \\
1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n} \\
0 & 1 & 2x_2 & 3x_2^2 & \cdots & 3nx_2^{3n-1} \\
 & & & & \cdots & \\
0 & 1 & 2x_n & 3x_n^2 & \cdots & 3nx_n^{3n-1} \\
0 & \frac{2!}{0!} & \frac{3!}{1!}x_1-\frac{2!}{0!}x_1 & \frac{4!}{2!}x_1^2-\frac{3!}{1!}x_1x_1 & \cdots & \frac{(3n+1)!}{(3n-1)}x_1^{3n-2}-\frac{3n!}{(3n-2)!}x_1^{3n-3}x_1 \\
 & & & & \cdots & \\
0 & \frac{2!}{0!} & \frac{3!}{1!}x_2-\frac{2!}{0!}x_1 & \frac{4!}{2!}x_2^2-\frac{3!}{1!}x_2x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_2^{3n-2}-\frac{3n!}{(3n-2)!}x_2^{3n-3}x_1 \\
0 & \frac{2!}{0!} & \frac{3!}{1!}x_n-\frac{2!}{0!}x_1 & \frac{4!}{2!}x_n^2-\frac{3!}{1!}x_nx_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-2}-\frac{3n!}{(3n-2)!}x_n^{3n-1}x_1
\end{vmatrix}
$$

The elements in rows 2n+2+i (i=1,2,...n) minus two times of the elements in rowsn+2+i (i=12....,n).

$$
B=\prod_{i=2}^{n}(x_i-x_1)\begin{vmatrix}
1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n} \\
 & & & & \cdots & \\
1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n} \\
1 & x & x^2 & x^3 & \cdots & x^{3n} \\
1 & 2x_0-x_1 & 3x_0^2-2x_0x_1 & 4x_0^3-3x_0^2x_1 & \cdots & (3n+1)x_0^{3n}-3nx_0^{3n-1}x_1 \\
1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n} \\
0 & 1 & 2x_2 & 3x_2^2 & \cdots & 3nx_2^{3n-1} \\
 & & & & \cdots & \\
0 & 1 & 2x_n & 3x_n^2 & \cdots & 3nx_n^{3n-1} \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{3n!}{(3n-2)!}x_1^{3n-2} \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & \cdots & \frac{3n!}{(3n-2)!}x_2^{3n-2} \\
 & & & & \cdots & \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{3n!}{(3n-2)!}x_n^{3n-1}
\end{vmatrix} \qquad (2.8)
$$

Repeat the method obtained form (2.8). In determinants of form (2.8), the elements in rows n+2,and rows 2n+2, are deleted, and exact out the common factors the result is

$$B = (-1)^{3n+5}(x - x_1)^3 \prod_{i=2}^{n}(x_i - x_1)^9 C \qquad (2.9)$$

Where C is

$$C = \begin{vmatrix} 1 & x_2 & x_2^2 & \mathrm{L} & x_2^{3n-2} \\ 1 & x_3 & x_3^2 & \mathrm{L} & x_3^{3n-2} \\ & & & \mathrm{L} & \\ & & & \mathrm{L} & \\ 1 & x_n & x_n^2 & \mathrm{L} & x_n^{3n-2} \\ 1 & x & x^2 & \mathrm{L} & x^{3n-2} \\ C_1 & C_2 & C_3 & \mathrm{L} & C_4 \\ 0 & 1 & 2x_2 & \mathrm{L} & (3n-2)x_1^{3n-3} \\ 0 & 1 & 2x_3 & \mathrm{L} & (3n-2)x_2^{3n-3} \\ & & & \mathrm{L} & \\ 0 & 1 & 2x_n & \mathrm{L} & (3n-2)x_n^{3n-3} \\ 0 & 0 & \frac{2!}{0!} & \mathrm{L} & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ 0 & 0 & \frac{2!}{0!} & \mathrm{L} & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ & & & \mathrm{L} & \\ & & & \mathrm{L} & \\ 0 & 0 & \frac{2!}{0!} & \mathrm{L} & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \end{vmatrix} \qquad (2.10)$$

and

$$C_1 = 3x_0^2 - 6x_0x_1 + 3x_1^2 = 3(x_0 - x_1)^2 = 0 + 3(x_0 - x_1)^2.$$

$$C_2 = 4x_0^3 - 9x_0^2x_1 + 6x_0x_1^2 - x_1^3 = 3x_0(x_0 - x_1)^2 + (x_0 - x_1)^3$$

$$C_3 = 3x_0^2(x_0 - x_1)^2 + 2x_0(x_0 - x_1)^3.$$

$$C_4 = 3x_0^{3n-2}(x_0 - x_1)^2 + (3n-2)x_0^{3n-3}(x_0 - x_1)^3 \qquad (2,11)$$

Considering the nature of the determinants, the determinants can be reduced to the following two determinants

where

$$C = \bar{C}_1 + \bar{C}_2.$$

$$\bar{C}_1 = \begin{vmatrix} 1 & x_2 & x_2^2 & \cdots & x_2^{3n-2} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{3n-2} \\ & & & \cdots & \\ & & & \cdots & \\ 1 & x_n & x_n^2 & \cdots & x_n^{3n-2} \\ 1 & x & x^2 & \cdots & x^{3n-2} \\ 3(x_0 - x_1)^2 & 3x_0(x_0 - x_1)^2 & 3x_0^2(x_0 - x_1)^2 & \cdots & 3x_0^{3n-2}(x_0 - x_1)^2 \\ 0 & 1 & 2x_2 & \cdots & (3n-2)x_1^{3n-3} \\ 0 & 1 & 2x_3 & \cdots & (3n-2)x_2^{3n-3} \\ & & & \cdots & \\ 0 & 1 & 2x_n & \cdots & (3n-2)x_n^{3n-3} \\ 0 & 0 & \frac{2!}{0!} & \cdots & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ 0 & 0 & \frac{2!}{0!} & \cdots & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ & & & \cdots & \\ & & & \cdots & \\ 0 & 0 & \frac{2!}{0!} & \cdots & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \end{vmatrix}$$

$$= 3(x_0 - x_1)^2 C^* \qquad (2.12).$$

where

$$C^{*} = \begin{vmatrix} 1 & x_2 & x_2^2 & x_2^3 & \dots & x_2^{3n-2} \\ & & & & \dots & \\ 1 & x_n & x_n^2 & x_n^3 & \dots & x_n^{3n-2} \\ 1 & x & x^2 & x^3 & \dots & x^{3n-2} \\ 1 & x_0 & x_0^2 & x_0^3 & \dots & x_0^{3n-2} \\ 0 & 1 & 2x_2 & 3x_n^2 & \dots & (3n-2)x_n^{3n-3} \\ & & & & \dots & \\ 0 & 1 & 2x_n & 3x_n^2 & \dots & (3n-2)x_n^{3n-3} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & \dots & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ & & & & \dots & \\ 0 & 0 & \frac{2!}{0!} & \frac{31}{1!}x_1 & \dots & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \end{vmatrix} \qquad (2.13)$$

$$\bar{C}_2 = \begin{vmatrix} 1 & x_2 & x_2^2 & L & x_2^{3n-2} \\ 1 & x_3 & x_3^2 & L & x_3^{3n-2} \\ & & & L & \\ & & & L & \\ 1 & x_n & x_n^2 & L & x_n^{3n-2} \\ 1 & x & x^2 & L & x^{3n-2} \\ 0 & (x_0-x_1)^3 & 2x_0(x_0-x_1)^3 & L & (3n-2)x_0^{3n-3}(x_0-x_1)^3 \\ 0 & 1 & 2x_2 & L & (3n-2)x_1^{3n-3} \\ 0 & 1 & 2x_3 & L & (3n-2)x_2^{3n-3} \\ & & & L & \\ 0 & 1 & 2x_n & L & (3n-2)x_n^{3n-3} \\ 0 & 0 & \frac{2!}{0!} & L & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ 0 & 0 & \frac{2!}{0!} & L & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ & & & L & \\ & & & L & \\ 0 & 0 & \frac{2!}{0!} & L & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \end{vmatrix}$$

$$= (x_0 - x_1)^3 C^{**} \tag{2.14}$$

$$C^{**} = \begin{vmatrix} 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n-2} \\ & & & & \cdots & \\ 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n-2} \\ 1 & x & x^2 & x^3 & \cdots & x^{3n-2} \\ 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n-2)x_0^{3n-3} \\ 0 & 1 & 2x_2 & 3x_2^2 & \cdots & (3n-2)x_2^{3n-3} \\ & & & & \cdots & \\ 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n-2)x_n^{3n-3} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & \cdots & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ & & & & \cdots & \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \end{vmatrix} = D(3n)_{1,1} \tag{2.15}$$

where $D(3n)_{1,1}$ is the algebra cofactor. In form (2.12), the way between two line elements with the exchange, of the elements of line n+1 to 1 lines. This result is

$$C^{*} = (-1)^n \begin{vmatrix} 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n-1} \\ 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^{3n-1} \\ & & & & \cdots & \\ 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n-1} \\ 1 & x & x^2 & x^3 & \cdots & x^{3n-1} \\ 0 & 1 & 2x_2 & 3x_2^2 & \cdots & (3n-1)x_2^{3n-2} \\ & & & & \cdots & \\ 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_2^{3n-3} \\ & & & & & \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-3} \end{vmatrix}$$

$$= -D(3n)_{n+2,1} \tag{2.16}$$

$D(3n)_{n+1,1}$ is the algebra cofactor of determinants (2.2). See forms (2.10),(2.11),(2.12),)2.13),(2.14),(2.15),(2.16),,following is correct.

$$C = (x_0 - x_1)^3 D(3n)_{1,1} - 3(x_0 - x_1)^2 D(3n)_{n+2,1} \qquad (2.17)$$

From forms (2.5),(2.7),(2.9),(2.10),and (2.17) ,form (2.5) can be written to be

$$D(3n+3)_{1,1} = (-1)^{3n+5}(x-x_1)^3 \prod_{i=2}^{n}(x_i - x_1)^9[(x_0 - x_1)^3 D_{1,1} - 3(x_0 - x_1)^2 D_{n+2,1}]$$

$$= (-1)^{3n+1}(x-x_1)^3(x_0 - x_1)^3 \prod_{i=2}^{n}(x - x_i)^9[D(3n)_{1,1} - \frac{3}{(x_0 - x_1)} D(3n)_{n+2,1}]$$

It is theorem (2.1).

An example

It is meaningful to the interpolation method and is important tool in modeling and simulation .For example in simulation engineering, the following interpolation:

$$L_n(x) = \sum_{k=0}^{n} y_k \prod_{\substack{i=0 \\ i \neq k}}^{n} \left(\frac{x - x_i}{x_k - x_i}\right) \qquad (2.18)$$

can be used to design the satellite . If following data were given:

$A_0(x_0, y_0), A_1(x_1, y_1), \ldots, A_n(x_n, y_n)$.

The outline of satellite simulation must pass the above data. But to obtain the interpolation (3.16) .the computation of the special determinants is necessary. The more points (data) the more we can approach the outline of the satellite. But the solution of the problem for the above interpolations has not been seen in papers [7-14]. The theorems 2.1 are the basic f solution of the problem. Subject to the limit of paper pages, more results of application in interpolation will appear in other papers.

Conclusion

Theorem 2.1 is main result of this paper , in which it can be seen the high order determinants D(3n+3),was reduced to be addition of lower order determinants D(3n),The lower order determinants D(3n) can be expressed as multiplication of many continued fraction , which is very useful to obtain the interpolation. As we know interpolation is important in simulation technology. For example, in the satellite design, shipbuilding engineering projects plane and other exterior design simulation has been widely used. Due to the limited number of pages by paper, the computation of the lower order determinants D(3n) ,will be published in another article .

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