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The Solution of the More General Traveling Salesman Problem

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Abstract

In this paper, a more general traveling salesman problem of which the traditional traveling salesman problem is the special case has been discussed. We obtain simple method to solve which node is the best starting node of circuit to obtain the minimal total work.

Key Words: Traveling salesman problem, solution, graph theory, minimal total work.

1. Introduction

The traditional traveling salesman problem is the prolongation of the Hamiltonian problem. Let us take n cities v_1, v_2, $\cdots$, v_n and there is one road between each pair of them. Let the length of the roads be $d(v_i, v_j)$, such that:

$$d(v_i, v_j) \le d(v_i, v_k) + d(v_k, v_j), \forall i, j, k$$

The traveling salesman problem, or TSP for short, is to find the shortest way of visiting all of the cities and returning to the starting node. Each city is visited only once.

Paper [1] tell us if we give a n-order side weighted undirected complete graph, where the nodes are as cities and the sides are as the roads between cities, the traveling salesman problem is to find a Hamiltonian circuit in this graph to get the minimal sum of the weights of sides.

It is easy to find a Hamiltonian Circuit of which the sum of the weights of sides is the minimum by the method of exhaustion, but is not possible. Because in a n-order side weighted undirected complete graph, there are $\frac{(n-1)!}{2}$ Hamiltonian circuits. This is a very large number and it is often cited as the reason the TSP seems to be so difficult to solve. It is true that the rapidly growing value of $\frac{(n-1)!}{2}$ rules out the possibility of checking all tours one by one. For example, while n=25,there are $\frac{24!}{2}(\approx 3.1\times 10^{23})$ Hamiltonian circuits needed to be calculate. If it takes 1 nanosecond (10^{-9} second) per Hamiltonian circuit, it will takes us 1 millions of years to find the shortest one.

Yet no effective method is known for the TSP. There is a well-known algorithm called Nearest Neighbor Algorithm. It's simple: While a salesman is in a city, The next city he need to go to is the one, that is the nearest and he hasn't been to yet, and so on until all the cities are visited. The method of the Nearest Neighbor Algorithm is: In a n-order side weighted undirected complete graph, we can freely choose any node as the starting node x_1 of the Hamiltonian Circuit, and then find the nearest node to x_1 as the second node x_2. The nearest node to x_2 is as the third node x_3 and so on until all the nodes are chosen. In order to express convenient, we call the circuit, obtained by Nearest Neighbor Algorithm, the Nearest Neighbor Circuit. And we can call the Nearest Neighbor Circuit, with node v_i as its starting node, the v_i- Nearest Neighbor Circuit.

In the traditional traveling salesman problem the graphs it discussed are only sides weighted graphs. The graphs discussed here are complete weighted graphs, that all nodes and sides are weighted. In this paper, we find out that, not the same total work (For example: cost) maybe obtained by different starting nodes even in a same circuit and the shortest Hamiltonian circuit may not obtain the minimal total work. In this paper, we obtain simple method to solve which node is the best one as the starting and end node of the circuit to obtain the minimal total work. This is very important in simulation problem, for example in the urban planning administration or in the transportation company headquarters building planning. And it is the main result of this paper and the difference from the traditional traveling salesman problem.

2. Main Results

At first place we give the method:

Method 2.1. Consider in a n-order complete weighted undirected graph, there are n nodes $\left(v_1, v_2, \cdots, v_n\right)$. Let a_i means the weight of node v_i and k_{ij} means the weight of side between v_i and v_j, where $i, j \in 1,2,\cdots,n$. All the a_i and k_{ij} are known. Let $\overline{a}_{ij}$ means the j th weight node of the v_i- Nearest Neighbor Circuit. Let $\overline{k}_{ij}$ means the j th side weight of the v_i- Nearest Neighbor Circuit. We can call $\overline{a}_{ij}\overline{k}_{ij}$ the work of the j th node in the v_i- Nearest Neighbor Circuit.

Step 1 By Nearest Neighbor Algorithm, we let all the nodes v_i ,$i \in 1,2,\cdots,n$,as starting node separately, and find out all the $\overline{a}_{ij}$ and $\overline{k}_{ij}$, $j = 1,2,\cdots,n$,$i = 1,2,\cdots,n$. Therefor we can get n v_i ($i \in 1,2,\cdots,n$)-Nearest Neighbor Circuits and their total work:

$$\overline{a}_{11}\overline{k}_{11} + \overline{a}_{12}\overline{k}_{12} + \cdots + \overline{a}_{1n}\overline{k}_{1n} = \sum_{j=1}^{n} \overline{a}_{1j}\overline{k}_{1j},$$

$$\cdots\cdots\cdots$$

$$\overline{a}_{i1}\overline{k}_{i1} + \overline{a}_{i2}\overline{k}_{i2} + \cdots + \overline{a}_{in}\overline{k}_{in} = \sum_{j=1}^{n} \overline{a}_{ij}\overline{k}_{ij}, \tag{1}$$

$$\cdots\cdots\cdots$$

$$\overline{a}_{n1}\overline{k}_{n1} + \overline{a}_{n2}\overline{k}_{n2} + \cdots + \overline{a}_{nn}\overline{k}_{nn} = \sum_{j=1}^{n} \overline{a}_{nj}\overline{k}_{nj}.$$

Step 2 Compare (1). If $\min\left\{\sum_{j=1}^{n}\overline{a}_{1j}\overline{k}_{1j}, \sum_{j=1}^{n}\overline{a}_{2j}\overline{k}_{2j}, \cdots, \sum_{j=1}^{n}\overline{a}_{nj}\overline{k}_{nj}\right\} = \sum_{j=1}^{n}\overline{a}_{ij}\overline{k}_{ij}$, it shows that the v_i- Nearest Neighbor Circuit is the best Nearest Neighbor circuit whose total work is the minimum. And the minimal total work is $\sum_{j=1}^{n}\overline{a}_{ij}\overline{k}_{ij}$.

Example 2.1 Consider there is a transportation company has 6 goods distributing centers: v_1 , v_2 , v_3 , v_4 , v_5 , v_6 . The daily amounts of traffic of 6 goods distributing centers are known:

v_1 's quantity is 1, v_2 's quantity is 2, v_3 's quantity is 3, v_4 's quantity is 4, v_5 's quantity is 5, v_6 's quantity is 6. And the average unit transportation cost between each pair of goods distributing centers are known. Now the transportation company is going to decide one of the goods distributing centers as the main goods distributing center. Every day the fleet will start from this main goods distributing center, pass through all the other goods distributing centers once and only once, and come back to this main goods distributing center. Suppose the goods of the former goods distributing center will be unloaded in the later. Which goods distributing center should be decided as the main goods distributing center, such that the total transportation cost is the minimum?

The solution is: According to the question, construct a 6-order complete weighted undirected graph, as shown in the fig.1. Let the nodes weight means the goods' daily traffic amount of 6 goods distributing centers and the sides weight means the average unit transportation cost between goods distributing centers. Let $\overline{a}_{ij}$ means the j th node weight of the v_i- Nearest Neighbor Circuit, where $j,i=1,2,\cdots,6$. Let $\overline{k}_{ij}$ means the j th side weight of the v_i - Nearest Neighbor Circuit.

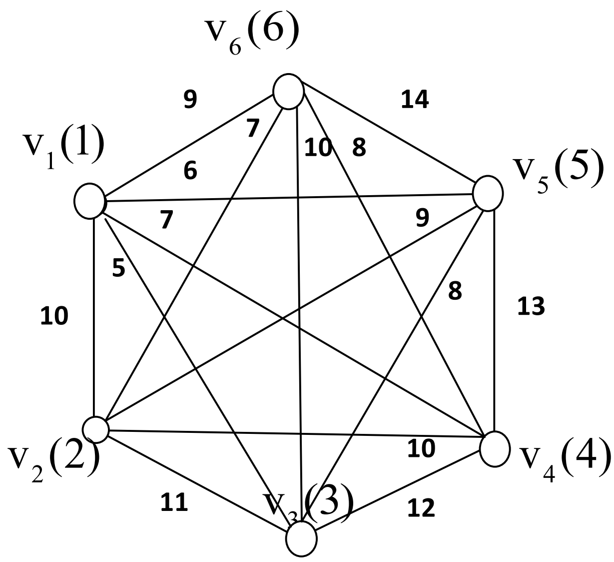

Fig.1. 6-order Complete Weighted Undirected Graph

Step 1. By Nearest Neighbor Algorithm, we let all the nodes as starting node separately, and find out all the $\bar{k}_{ij}$ and $\bar{a}_{ij}$, $j = 1,2,\cdots,6$, $i = 1,2,\cdots,6$. So we can get 6 v_i - Nearest Neighbor Circuit, where $i = 1,2,\cdots,6$, and their total work (the total transportation cost):

1) Let the node v_1 as the starting node. Because of $\min\{10,5,7,6,9\} = 5$, so the nearest node to v_1 is v_3. Then v_3 is as the second node and side $v_1 v_3$ is as the first side of the v_1 - Nearest Neighbor Circuit, as shown in the Fig.2. Therefore, we can get $\bar{a}_{11} = 1$ and $\bar{k}_{11} = 5$.

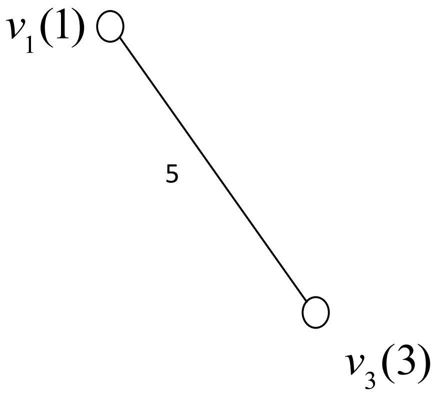

Fig.2. $v_1 \underset{5}{-} v_3$ of the v_1 - Nearest Neighbor Circuit

To the node v_3, Because of $\min\{11,10,8,12\} = 8$, so the nearest node is v_5, besides v_1. Then v_5 is as the third node and side $v_3 v_5$ is as the second side of the v_1 - Nearest Neighbor Circuit, as shown in the Fig.3. Therefore, we can get $\bar{a}_{12} = 3$ and $\bar{k}_{12} = 8$.

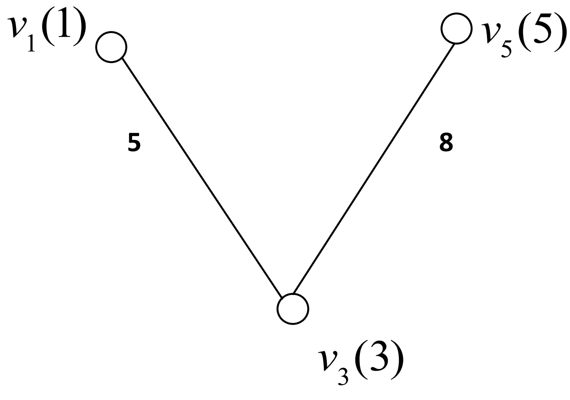

Fig.3. $v_1 \underset{5}{-} v_3 \underset{8}{-} v_5$ Of the v_1 - Nearest Neighbor Circuit

To the node v_5, Because of $\min\{14,9,13\} = 9$, so the nearest node is v_2, besides v_1 and

v_3. Then v_2 is as the fourth node and side v_5v_2 is as the third side of the v_1 - Nearest Neighbor Circuit, as shown in the Fig.4. Therefor, we can get $\bar{a}_{13} = 5$ and $\bar{k}_{13} = 9$.

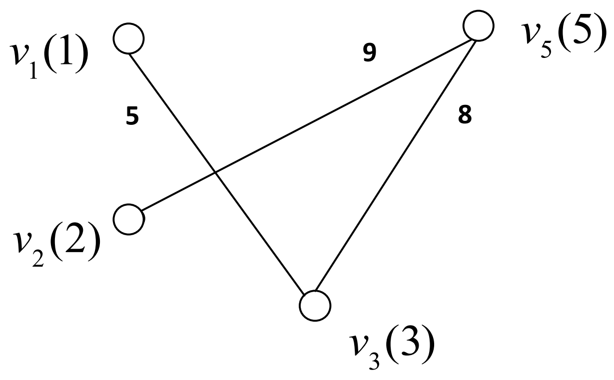

Fig.4. $v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2$ of the v_1 - Nearest Neighbor Circuit

To the node v_2, Because of $\min\{10, 7\} = 7$, so the nearest node is v_6, besides v_1 , v_3 and v_5. Then v_6 is as the fifth node and side v_2v_6 is as the fourth side of the v_1 - Nearest Neighbor Circuit, as shown in the Fig.5. Therefor, we can get $\bar{a}_{14} = 2$ and $\bar{k}_{14} = 7$.

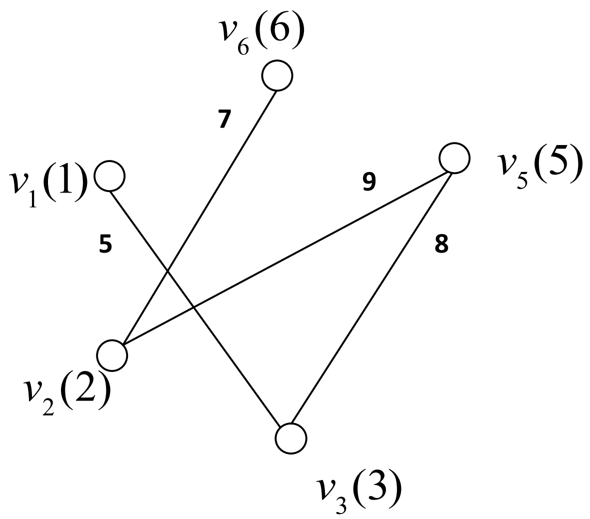

Fig.5. $v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6$ of the v_1 - Nearest Neighbor Circuit

To the node v_6, the nearest node is v_4, besides v_1 , v_3 , v_5 and v_2. So v_4 is as the sixth node and side v_6v_4 is as the fifth side of the v_1 - Nearest Neighbor Circuit, as shown in the

Fig.6. Therefor, we can get $\overline{a}_{15} = 6$ and $\overline{k}_{15} = 8$.

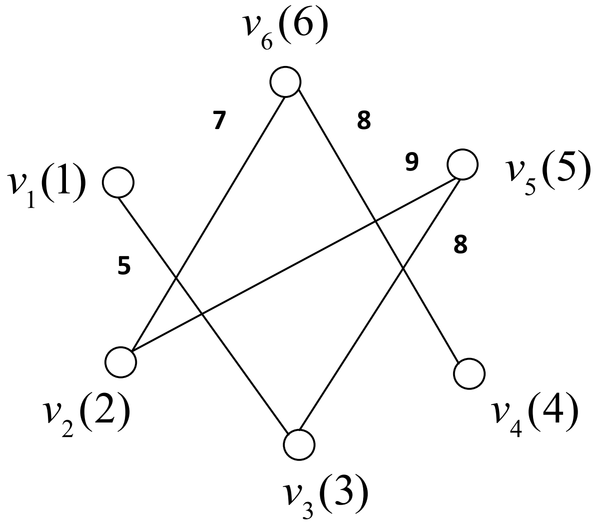

Fig.6. $v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4$ of the v_1 - Nearest Neighbor Circuit

Link v_4 to v_1. So the side v_4v_1 is as the sixth side of the v_1 - Nearest Neighbor Circuit. And the v_1 - Nearest Neighbor Circuit is finished like that: $v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4 \underset{7}{-} v_1$, as shown in the Fig.7. Therefore, we can get $\overline{a}_{16} = 4$, $\overline{k}_{16} = 7$ and total work (the total transportation cost) of the v_1 - Nearest Neighbor Circuit is:

$$\overline{a}_{11}\overline{k}_{11} + \overline{a}_{12}\overline{k}_{12} + \cdots + \overline{a}_{16}\overline{k}_{16} = \sum_{j=1}^{6} \overline{a}_{1j}\overline{k}_{1j}$$

That is

$$1 \times 5 + 3 \times 8 + 5 \times 9 + 2 \times 7 + 6 \times 8 + 4 \times 7 = 164 \qquad (2)$$

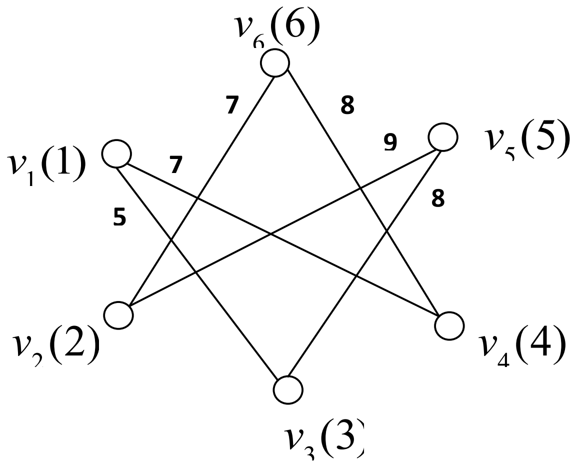

Fig.7. $v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4 \underset{7}{-} v_1$ of the v_1 - Nearest Neighbor Circuit

2) In the same way, let the node v_2 as the starting node. We can get the v_2 - Nearest Neighbor Circuit like that: $v_2 \underset{7}{-} v_6 \underset{8}{-} v_4 \underset{7}{-} v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2$, as shown in the Fig.8. Therefor, we can get $\overline{a}_{21} = 2$, $\overline{k}_{21} = 7$, $\overline{a}_{22} = 6$, $\overline{k}_{22} = 8$, $\overline{a}_{23} = 4$, $\overline{k}_{23} = 7$, $\overline{a}_{24} = 1$, $\overline{k}_{24} = 5$, $\overline{a}_{25} = 3$, $\overline{k}_{25} = 8$, $\overline{a}_{26} = 5$, $\overline{k}_{26} = 9$ and total work (the total transportation cost) of the v_2 - Nearest Neighbor Circuit is:

$$\overline{a}_{21}\overline{k}_{21} + \overline{a}_{22}\overline{k}_{22} + \cdots + \overline{a}_{26}\overline{k}_{26} = \sum_{j=1}^{6} \overline{a}_{2j}\overline{k}_{2j}$$

That is

$$2 \times 7 + 6 \times 8 + 4 \times 7 + 1 \times 5 + 3 \times 8 + 5 \times 9 = 164 \qquad (3)$$

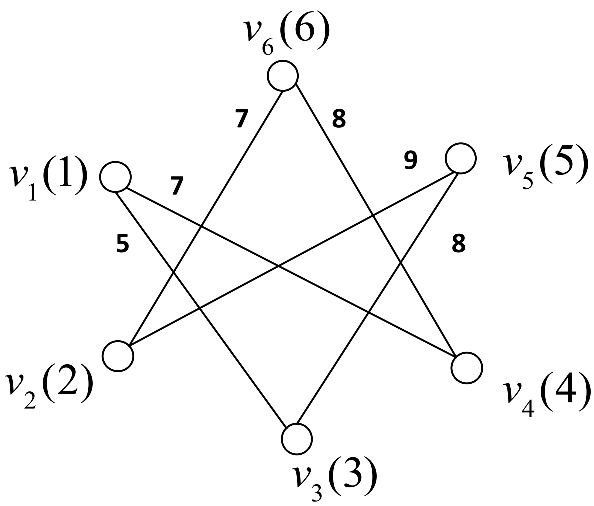

Fig.8. $v_2 \underset{7}{-} v_6 \underset{8}{-} v_4 \underset{7}{-} v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2$ of the v_2 - Nearest Neighbor Circuit

3) In the same way, let the node v_3 as the starting node. We can get the v_3 - Nearest Neighbor Circuit like that: $v_3 \underset{5}{-} v_1 \underset{6}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4 \underset{12}{-} v_3$, as shown in the Fig.9. Therefor, we can get $\overline{a}_{31} = 3$, $\overline{k}_{31} = 5$, $\overline{a}_{32} = 1$, $\overline{k}_{32} = 6$, $\overline{a}_{33} = 5$, $\overline{k}_{33} = 9$, $\overline{a}_{34} = 2$, $\overline{k}_{34} = 7$, $\overline{a}_{35} = 6$, $\overline{k}_{35} = 8$, $\overline{a}_{36} = 4$, $\overline{k}_{36} = 12$ and total work (the total transportation cost) of the v_3 - Nearest Neighbor Circuit is:

$$\overline{a}_{31}\overline{k}_{31} + \overline{a}_{32}\overline{k}_{32} + \cdots + \overline{a}_{36}\overline{k}_{36} = \sum_{j=1}^{6} \overline{a}_{3j}\overline{k}_{3j}$$

That is
$$3 \times 5 + 1 \times 6 + 5 \times 9 + 2 \times 7 + 6 \times 8 + 4 \times 12 = 176 \qquad (4)$$

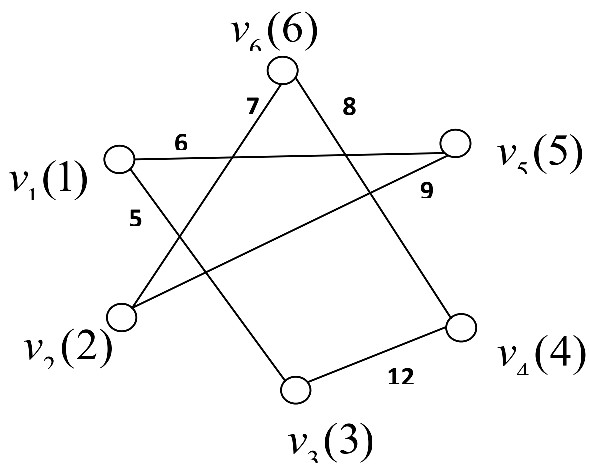

Fig.9. $v_3 \underset{5}{-} v_1 \underset{6}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4 \underset{12}{-} v_3$ of the v_3 - Nearest Neighbor Circuit

4) In the same way, let the node v_4 as the starting node. We can get the v_4 - Nearest Neighbor Circuit like that: $v_4 \underset{7}{-} v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4$ as shown in the Fig.10.

Therefor, we can get: $\overline{a}_{41} = 4$, $\overline{k}_{41} = 7$, $\overline{a}_{42} = 1$, $\overline{k}_{42} = 5$, $\overline{a}_{43} = 3$, $\overline{k}_{43} = 8$, $\overline{a}_{44} = 5$, $\overline{k}_{44} = 9$, $\overline{a}_{45} = 2$, $\overline{k}_{45} = 7$, $\overline{a}_{46} = 6$, $\overline{k}_{46} = 8$. And total work (the total transportation cost) of the v_4 - Nearest Neighbor Circuit is:

$$\overline{a}_{41}\overline{k}_{41} + \overline{a}_{42}\overline{k}_{42} + \cdots + \overline{a}_{46}\overline{k}_{46} = \sum_{j=1}^{6} \overline{a}_{4j}\overline{k}_{4j}$$

That is

$$4\times7+1\times5+3\times8+5\times9+2\times7+6\times8=164 \qquad (5)$$

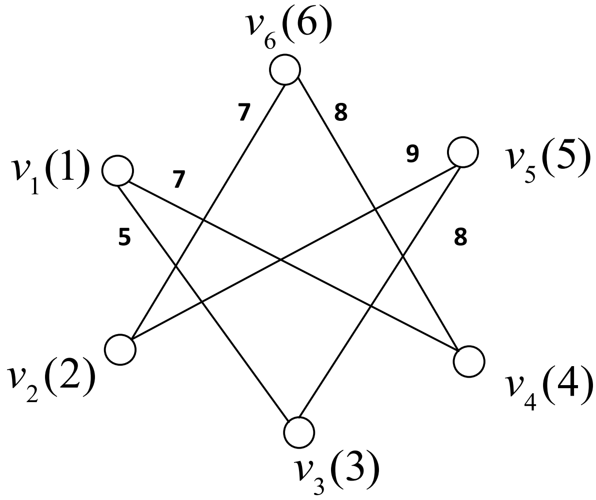

Fig.10. $v_4 \underset{7}{-} v_1 \underset{5}{-} v_3 \underset{8}{-} v_5 \underset{9}{-} v_2 \underset{7}{-} v_6 \underset{8}{-} v_4$ of the v_4 - Nearest Neighbor Circuit

5) In the same way, let the node v_5 as the starting node. We can get the v_5 - Nearest Neighbor Circuit like that: $v_5 \underset{6}{-} v_1 \underset{5}{-} v_3 \underset{10}{-} v_6 \underset{7}{-} v_2 \underset{10}{-} v_4 \underset{13}{-} v_5$, as shown in the Fig.11. Therefore, we can get $\bar{a}_{51}=5$, $\bar{k}_{51}=6$, $\bar{a}_{52}=1$, $\bar{k}_{52}=5$, $\bar{a}_{53}=3$, $\bar{k}_{53}=10$, $\bar{a}_{54}=6$, $\bar{k}_{54}=7$, $\bar{a}_{55}=2$, $\bar{k}_{55}=10$, $\bar{a}_{56}=4$, $\bar{k}_{56}=13$ and total work (the total transportation cost) of the v_5 - Nearest Neighbor Circuit is:

$$\bar{a}_{51}\bar{k}_{51}+\bar{a}_{52}\bar{k}_{52}+\cdots+\bar{a}_{56}\bar{k}_{56}=\sum_{j=1}^{6}\bar{a}_{5j}\bar{k}_{5j}$$

That is

$$5\times6+1\times5+3\times10+6\times7+2\times10+4\times13=179 \qquad (6)$$

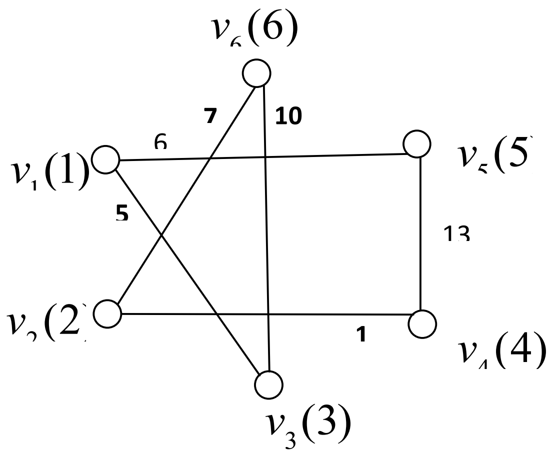

Fig.11. $v_5 \underset{6}{-} v_1 \underset{5}{-} v_3 \underset{10}{-} v_6 \underset{7}{-} v_2 \underset{10}{-} v_4 \underset{13}{-} v_5$ of the v_5-Nearest Neighbor Circuit

6) In the same way, let the node v_6 as the starting node. We can get the v_6-Nearest Neighbor Circuit like that: $v_6 \underset{7}{-} v_2 \underset{9}{-} v_5 \underset{6}{-} v_1 \underset{5}{-} v_3 \underset{12}{-} v_4 \underset{8}{-} v_6$, as shown in the Fig.12. Therefor, we can get $\overline{a}_{61} = 6$,$\overline{k}_{61} = 7$,$\overline{a}_{62} = 2$,$\overline{k}_{62} = 9$,$\overline{a}_{63} = 5$,$\overline{k}_{63} = 6$,$\overline{a}_{64} = 1$,$\overline{k}_{64} = 5$,$\overline{a}_{65} = 3$,$\overline{k}_{65} = 12$, $\overline{a}_{66} = 4$, $\overline{k}_{66} = 8$ and total work (the total transportation cost) of the v_6-Nearest Neighbor Circuit is:

$$\overline{a}_{61}\overline{k}_{61} + \overline{a}_{62}\overline{k}_{62} + \cdots + \overline{a}_{66}\overline{k}_{66} = \sum_{j=1}^{6} \overline{a}_{6j}\overline{k}_{6j}$$

That is

$$6 \times 7 + 2 \times 9 + 5 \times 6 + 1 \times 5 + 3 \times 12 + 4 \times 8 = 163 \qquad (7)$$

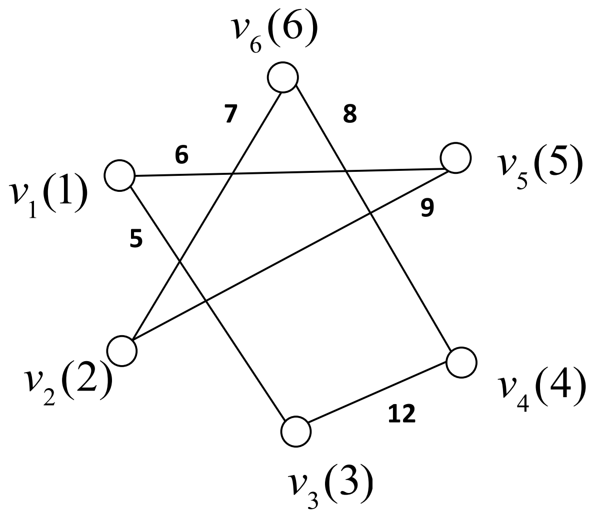

Fig.12. $v_6 \underset{7}{-} v_2 \underset{9}{-} v_5 \underset{6}{-} v_1 \underset{5}{-} v_3 \underset{12}{-} v_4 \underset{8}{-} v_6$ of the v_6-Nearest Neighbor Circuit

We can see that, in fact, the v_1-Nearest Neighbor Circuit, the v_2-Nearest Neighbor Circuit and the v_4-Nearest Neighbor Circuit are the same circuit although by different starting nodes v_1, v_2, v_4. The v_3-Nearest Neighbor Circuit and the v_6-Nearest Neighbor Circuit are the same circuit although by different starting nodes v_3, v_6, but their total work (total transportation cost) $\sum_{j=1}^{n} \bar{a}_{3j}\bar{k}_{3j} = 176, \sum_{j=1}^{n} \bar{a}_{6j}\bar{k}_{6j} = 163$ are different.

Step 2 Compare (2)-(7). Because of $\min\{(2)-(7)\} = \sum_{j=1}^{n} \bar{a}_{6j}\bar{k}_{6j} = 163$, so the v_6-Nearest Neighbor Circuit is the best circuit whose total work (total transportation cost) is the minimum and we should choose goods distributing center v_6 as the main goods distributing center, such that the total work (the total transportation cost) is the minimum.

In fact, in the traditional traveling salesman problem, the shortest Hamiltonian Circuit obtained by Nearest Neighbor Algorithm is the circuit with v_1 or v_2 or v_3 as starting node, not v_6.

The main result of this example 2.1 is: 1) We find out the best Nearest Neighbor Circuit whose total work is the minimum; 2) The starting node and the end node of the best Nearest Neighbor Circuit has been solved, that is, which goods distributing center should be decided as the main goods distributing center has been solved. This is very important in simulation problem.

3. Conclusions

1) In method 2.1, when the nodes weight and sides weight are known, we let all the nodes as starting node separately and find out all the $\bar{a}_{ij}$ and $\bar{k}_{ij}$, $j = 1,2,\cdots,n$, $i = 1,2,\cdots,n$ by Nearest Neighbor Algorithm. Then we can get n Nearest Neighbor Circuit and their total work. By compare total work, the best Nearest Neighbor Circuit whose total work is the minimum is solved.

2) In example 2.1,we find out that, not the same total work may be obtain by different starting nodes even in a same circuit and the shortest Hamiltonian circuit may not obtain the minimal total work.

3) In paper [1], the graphs it discussed are only sides weighted graphs. The graphs discussed in this paper are complete weighted graphs, that all nodes and sides are weighted; In paper [1], the traditional traveling salesman problem is to find the shortest Hamiltonian Circuit such that the sum of the weights of sides is the minimum. In this paper, a more general traveling salesman problem has been discussed to find the best Nearest Neighbor Circuit such that the total work is the minimum.

4) The Nearest Neighbor Algorithm is the special case of method 2.1, when $a_1 = a_2 = \cdots = a_n = 1$.

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