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## **Error Elimination Based on Error Set**

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**Abstract:** This paper offers the concepts of error set and fuzzy error set, discusses various error set and fuzzy error set, and puts forward the relevant propositions and operations. The soundness and completeness for the propositions and operations are verified as well. Besides, operations of error set and the law they should satisfy are also explored. That transformation of error set consists of INVERSE, OR and PRODUCT and the transformation types include combination transformation, destruction transformation, increase transformation, similarity transformation are concluded. What's more, the application of error set transformation to data mining is expounded with cases.

**Key words:** Error set, fuzzy error set, error elimination, transformation

### **1. Introduction**

Depending on each other, the two aspects of a contradiction exist in the same system and can mutually transform under certain conditions (K.Z.Guo, S.Q.Zhang 1995). The contradiction of error and truth also follow the above rule without expectation. The truth can transform into the

error and vice versa on different premises like time, space, scientific fields and researching ends (Y.Q.Liu, K.Z.Guo 2000). Then what are their transforming methods and laws is worthy of our studying.

This article attempts to explicate the transforming methods and laws of errors through studying the change of error set (K.Z.Guo, S.Q.Zhang 2001). Errors usually result from mistakes of some factors or just a particular factor. While those errors, on the one hand, cause personal losses, on the other hand, leads to people's casualties, group's disintegration (S.Y.Liu, K.Z.Guo, D.C.Sun 2010), and the whole country's or even human being's destroy. All these errors constitute an error set. Therefore, the change of error set is not only in connection with the elements of error set but also with the discussion domain (Z.F.Jiang 1998), discerning rule and binary relation of error set.

In the field of fuzzy mathematics, the research of set mainly concentrates on the static form of fuzzy set and its effective forms of reasoning and rules (C.Y.Wang 1998). However, the dynamic changes of the fuzzy set are important parts of set research (H.B.Liu, K.Z.Guo 2010). In this article, the error set and dynamic error set are about to be expounded based on the error-eliminating theory. Besides, we will explicate the different forms of error set and fuzzy error set, with their relevant prepositions and operations available. What's more, That transformation of error set consists of INVERSE, OR and PRODUCT and the transformation types include combination transformation, destruction transformation, increase transformation, similarity transformation are concluded (K.Z.Guo, S.Q.Zhang 1991). Finally, the application of error set transformation to data mining is expounded with cases. In a word, focusing on error set, this paper is of important theoretical and practical significance for different fields.

## **2. Basic Definitions**

In universe, different events, people, and information form a complicated net of relations. Owing to the interaction and interplay among these actions, errors exist everywhere. The error set is used as a tool to describe the above phenomenon with the help of error theory. In the following part, classical error set, fuzzy error set and multivariate error set are explored based on error-eliminating theory. Moreover, the relevant propositions and operations with verification are expounded.

### ***The Definition of Error Set***

Suppose  $U$  is an object set,  $G$  is a group of rules to judge the error based on  $U$ . If  $E = \{(U(t), S(t), \bar{p}(t), T(t), L(t)), x(t) = f(G \neq u(t))) \mid (U(t), S(t), \bar{p}(t), T(t), L(t)) = u(t) \in U(t), f \subseteq U(t) \times \mathbb{R}, x(t) = f(G \neq u(t))\}$ , then  $E$  is an “error” set based on  $U$  for  $G$ , which can be called an error set.

$$U_C = \{u(t) \mid (u(t), x(t)) \in E, x(t) > 0\},$$

$$U_Z = \{u(t) \mid (u(t), x(t)) \in E, x(t) < 0\},$$

$$U_L = \{u(t) \mid (u(t), x(t)) \in E, x(t) = 0\},$$

$$U_K = \{u(t) \mid (u(t), x(t)) \in E, x(t) \geq 0, T(f(G \neq u(t))) < 0\},$$

$$U_{KH} = \{u(t) \mid (u(t), x(t)) \in E, x(t) \leq 0, T(f(G \neq u(t))) > 0\},$$

$$U_{KL} = \{u(t) \mid (u(t), x(t)) \in E, T(f(G \neq u(t))) = 0\},$$

$$U_H = U_Z - U_{KH}$$

$$U_S = U_E - U_K$$

are called the error domain, correctness domain, and critical domain of error set  $E$ ; the correctable domain, worsening domain, critical domain, good domain and bad domain of transformation  $T$  respectively.  $\mathbb{R}$  is the real number domain (K.Z.Guo 2008).

$G(t) \neq u(t)$  includes:

- (a)  $u(t)$  is contradicted with  $G(t)$ .
- (b)  $u(t)$  definitely cannot be inferred from  $G(t)$ .
- (c)  $u(t)$  partially cannot be inferred from  $G(t)$ .
- (d)  $G(t)$  indefinitely cannot be inferred from  $u(t)$ .

In the above definition,  $f(G \neq u(t))$  is, in more common condition,  $f(u(t), G(t))$ .

Proposition 2.1.1 In  $U$ , if  $G_1 = G_2$ ,  $f_1 = f_2$ , when

$$E_1 = \{(U(t), S(t), \bar{p}(t), T(t), L(t)), x(t) = f_1(G \neq u(t))) \mid (U(t), S(t), \bar{p}(t), T(t), L(t)) = u(t) \in U(t), f_1 \subseteq U(t) \times \mathbb{R}, x(t) = f(G \neq u(t))\},$$

$$E_2 = \{(U(t), S(t), \bar{p}(t), T(t), L(t)), x(t) = f_2(G \neq u(t))) \mid (U(t), S(t), \bar{p}(t), T(t), L(t)) = u(t) \in U(t), f_2 \subseteq U(t) \times \mathbb{R}, x(t) = f(G \neq u(t))\},$$

then  $E_1 = E_2$ , vice versa.

Verification: since  $G_1 = G_2$  and  $f_1 = f_2$  for

$$\forall u(t) \in U, x(t) = f_1(G \neq u(t)) = f_2(G \neq u(t)) = y(t),$$

Therefore,  $\forall u(t) \in U$ , when  $(u(t), x(t)) \in E_1, (u(t), y(t)) \in E_2$  then  $x(t) = y(t)$ ,

thus,  $E_1 = E_2$ ,

On the other hand, if  $E_1 = E_2$ , then  $G_1 = G_2$  can be concluded from the definition.

If  $f_1 \Rightarrow f_2$  is true in  $U$ , then  $\exists u(t) \in U, f_1(G \neq u(t)) \neq f_2(G \neq u(t))$  is right.

Therefore, for  $(u(t),x(t))\in E_1,(u(t),y(t))\in E_2$ , then  $x(t)\neq y(t)$  is contradicted with  $E_1 = E_2$ , thus,  $f_1 = f_2$ .

### ***The Class of Error Set***

According to the features of the elements, error sets can be divided as follows (K.Z.Guo 2012):

(a) Classic error set

$$E = \{((U(t),S(t), \bar{p}(t),T(t), L(t)), x(t)= f(G \neq u(t))) | (U(t),S(t), \bar{p}(t),T(t),L(t))=u(t) \in U(t), f \subseteq U \times \{0,1\}, x(t)= f(G \neq u(t)))\}$$

(b) Fuzzy error set

$$E = \{((U(t),S(t), \bar{p}(t),T(t), L(t)), x(t)= f(G \neq u(t))) | (U(t),S(t), \bar{p}(t),T(t),L(t))=u(t) \in U(t), f \subseteq U \times [0,1], x(t)= f(G \neq u(t)))\}$$

(c) Error set with critical points

$$E = \{((U(t),S(t), \bar{p}(t),T(t), L(t)), x(t)= f(G \neq u(t))) | (U(t),S(t), \bar{p}(t),T(t),L(t))=u(t) \in U(t), f \subseteq U \times (-\infty, +\infty), x(t)= f(G \neq u(t)))\}$$

## **3. The Research of Fuzzy Error Set**

The following part mainly focuses on the definition, relation, and operation of error set.

Definition 3.1 Suppose  $U$  is an object set,  $S$  is a set of association rules in  $U$ , if  $\tilde{E} = \{(u, x) | u \in U, x = f(G \neq u), f \subseteq U \times [0, 1]\}$ , we call that  $\tilde{E}$  is a fuzzy error set for  $G$  in  $U$ .

### ***The Relation between Fuzzy Error Set***

(a) Equation

Definition 3.1.1 Suppose  $\tilde{E}_1 = \{(u, x) | u \in U, x = f(G \neq u), f \subseteq U \times [0, 1]\}$ ,  $\tilde{E}_2 = \{(u, y) | u \in U, y = f_2(G_2 \neq u), f_2 \subseteq U \times [0, 1]\}$ , if  $\forall u \in U$ , if  $\forall u \in U, x = y$ , and  $G_1 = G_2$ , then  $\tilde{E}_1$  and  $\tilde{E}_2$  is equivalent with rule  $G_1$  or  $G_2$ , denoted as  $\tilde{E}_1 = \tilde{E}_2$ .

(b) Subset

Definition 3.1.2 Suppose  $U_1$  and  $U_2$  are subsets in  $U$ , and  $\tilde{E}_1 = \{(u, x) | u \in U_1, x = f(G \neq u), f \subseteq U_1 \times [0, 1]\}$ ,  $\tilde{E}_2 = \{(u, x) | u \in U_2, x = f(G \neq u), f \subseteq U_2 \times [0, 1]\}$ , if  $U_1 \subseteq U_2$  for association rule  $G$ ,

then  $\tilde{E}_1$  is them subset of  $\tilde{E}_2$ , so  $\tilde{E}_1 \subseteq \tilde{E}_2$ , or  $\tilde{E}_2 \supseteq \tilde{E}_1$ . According to the above definition, the following proposition can be obtained.

Proposition 3.1.1 Suppose  $\tilde{E}_1, \tilde{E}_2$ , and  $\tilde{E}_3$  are subsets for association rule G:

- (a)  $\tilde{E}_1 \subseteq \tilde{E}_1$ ,
- (b) if  $\tilde{E}_1 \subseteq \tilde{E}_2, \tilde{E}_1 \subseteq \tilde{E}_3$ , then  $\tilde{E}_2 \subseteq \tilde{E}_3$ .

Proposition 3.1.2 Suppose  $\tilde{E}_1, \tilde{E}_2$  are fuzzy sets for association rule  $G_1$  and  $G_2$ , then

$$\tilde{E}_1 = \{(u, x) | u \in U_1, x = f_1(G_1 \Rightarrow u), f_1 \subseteq U_1 \times [0, 1]\}$$

$$\tilde{E}_2 = \{(u, y) | u \in U_2, y = f_2(G_2 \Rightarrow u), f_2 \subseteq U_2 \times [0, 1]\}$$

If  $\forall u \in U, (u, x) \in \tilde{E}_1, (u, y) \in \tilde{E}_2$ , and  $x \leq y$ , then  $\tilde{E}_1 \leq \tilde{E}_2$  or  $\tilde{E}_2 \geq \tilde{E}_1$  for association rule in U.

Proposition 3.1.3 Suppose  $\tilde{E}_1, \tilde{E}_2, \tilde{E}_3$  are fuzzy subsets for association rule  $G_1, G_2, G_3$  in U

- (a)  $\tilde{E}_1 \subseteq \tilde{E}_1$ ,
- (b) if  $\tilde{E}_1 \subseteq \tilde{E}_2, \tilde{E}_1 \subseteq \tilde{E}_3$ , then  $\tilde{E}_2 \subseteq \tilde{E}_3$ .

### **The Operations of Fuzzy Error Set**

(a) The union of Fuzzy error set

If  $f(x, y, G_1, G_2) = 0$ , then there are association rule  $G_1, G_2$ :

Definition 3.2.1 suppose  $\tilde{E}_1$  and  $\tilde{E}_2$  are fuzzy sets for association rule  $G_1, G_2$  in U, and  $\tilde{E}_3 = \{(u, z) | (u, x) \in \tilde{E}_1, (u, y) \in \tilde{E}_2, z = \max(x, y)\}$ , then  $\tilde{E}_3 = \tilde{E}_1 \vee \tilde{E}_2$ ,  $\vee$  means union.

Proposition 3.2.1 Suppose  $\tilde{E}_1$ , and  $\tilde{E}_2$  are subsets for association rule  $G_1$  and  $G_2$ , then

- a.  $\tilde{E}_1 \vee \tilde{E}_1 = \tilde{E}_1$ ;
- b.  $\tilde{E}_1 \vee \tilde{E}_2 = \tilde{E}_2 \vee \tilde{E}_1$ ;
- c. if  $\tilde{E}_1 \leq \tilde{E}_2$ , then  $\tilde{E}_1 \leq \tilde{E}_1 \vee \tilde{E}_2 = \tilde{E}_2$ .

(b) The intersection of Fuzzy error set

Definition 3.2.2 suppose  $\tilde{E}_1$  and  $\tilde{E}_2$  are fuzzy sets for association rule  $G_1, G_2$  in  $U$  and  $\tilde{E}_3 = \{(u, z) | (u, x) \in \tilde{E}_1, (u, y) \in \tilde{E}_2, z = \min(x, y)\}$ , then  $\tilde{E}_3 = \tilde{E}_1 \wedge \tilde{E}_2$ ,  $\wedge$  means intersection.

Proposition 3.2.2 Suppose  $\tilde{E}_1, \tilde{E}_2$ , and  $\tilde{E}_3$  are subsets for association rule  $G_1, G_2, G_3$  then

- a.  $\tilde{E}_1 \wedge \tilde{E}_1 = \tilde{E}_1$ ;
- b.  $\tilde{E}_1 \wedge \tilde{E}_2 = \tilde{E}_2 \wedge \tilde{E}_1$ ;
- c.  $\tilde{E}_1 \wedge (\tilde{E}_2 \wedge \tilde{E}_3) = (\tilde{E}_1 \wedge \tilde{E}_2) \wedge \tilde{E}_3$

## 4. The Types and Operating Laws of Error Set's Transformation

### *The Transforming PRODUCT, OR, INVERSE*

(a) The Transforming PRODUCT

Definition 4.1.1 If the set  $E$  acts on  $T_1$  and then on  $T_2$ , the relation between  $T_1$  and  $T_2$  is called the product of  $T_1$ , which is denoted as  $(T_1 \wedge T_2)(E)$ , namely  $(T_1 \wedge T_2)(E) = T_1(T_2(E))$ .

(b) The Transforming OR

Definition 4.1.2 For a set, if either  $T_1$  or  $T_2$  acts on  $E$ , the relation between  $T_1$  and  $T_2$  is called the OR of  $T_1$  and  $T_2$ , which is denoted as  $(T_1 \vee T_2)(E)$ , namely,  $(T_1 \vee T_2)(E) = T_1(E) \vee T_2(E)$ .

(c) The Transforming INVERSE

Definition 4.1.3 If  $T_1(E_1) = E_2, T_2(E_2) = E_1$ , then the relation between  $T_1$  and  $T_2$  is called the INVERSE of  $T_1$  and  $T_2$ , which can be denoted as  $T_1 = T_2^{-1}$  or  $T_2 = T_1^{-1}$ .

### *Combination Transformation*

Definition 4.2.1 If  $E_{i1}, E_{i2}, \dots, E_{ini} (n_i \geq 2)$  and  $T(E_{g1}, E_{g2}, \dots, E_{gng}) = E_i$ , then  $T$  is the combination transformation of  $E$ , which is denoted as  $T_{zu}$ .

In the definition, if  $E_i, i \in \{1, 2, \dots, n, \dots\}$  all belong to  $E$ , then for  $E$  there is no discussion domain transformation in  $T_{zu}$ . If not all  $E_i, i \in \{1, 2, \dots, n, \dots\}$  belong to  $E$ , then for  $E$  there are possible discussion domain transformation, rule transformation, and binary relation transformation in  $T_{zu}$ .

In fact, combination transformation  $T_{zu}$  is the inverse operation of decomposition transformation  $T_f$ . Thus, we can also define (a) element combination transformation  $T_{fys}^{-1}$ ; (b)

discussion domain combination transformation  $T_{f_y}^{-1}$ ; (c) binary relation combination transformation  $T_{f_{gx}}^{-1}$ ; (d) comprehensive combination transformation  $T_{f_{zh}}^{-1}$ .

The proposition 4.2.1 Suppose  $E$  is an error set based on  $U$  for the rule  $G$ , and  $E_1$  and  $E_2$  are two error sets. If  $E_{i1}, E_{i2}, \dots, E_{ini} \in E_1, E_{g1}, E_{g2}, \dots, E_{gng} \in E_2, \exists E_{ik1} \neq E_{gk2}, k_1 \in \{1, 2, \dots, n_j\}, k_2 \in \{1, 2, \dots, n_g\}, T_{f_{ys}}^{-1}(E_{i1}, E_{i2}, \dots, E_{ini}) = E_i \neq T_{f_{ys}}^{-1}(E_{g1}, E_{g2}, \dots, E_{gng}) = E_g$ , then  $T_{f_{ys}}^{-1}(E_1 \cap E_2) = T_{f_{ys}}^{-1}(E_1) \cap T_{f_{ys}}^{-1}(E_2)$ .

Verification: If  $E_{i1}, E_{i2}, \dots, E_{ini} \subseteq E_1 \cap E_2$ , then

$$E_{i1}, E_{i2}, \dots, E_{ini} \subseteq E_1, E_{i1}, E_{i2}, \dots, E_{ini} \subseteq E_2,$$

$$\therefore T_{f_{ys}}^{-1}(E_1 \cap E_2) \subseteq T_{f_{ys}}^{-1}(E_1) \cap T_{f_{ys}}^{-1}(E_2).$$

$$\text{If } E_i \in T_{f_{ys}}^{-1}(E_1) \cap T_{f_{ys}}^{-1}(E_2),$$

$$\text{then } E_i \in T_{f_{ys}}^{-1}(E_1), E_i \in T_{f_{ys}}^{-1}(E_2).$$

$$\text{Suppose } \{E_{i1}, E_{i2}, \dots, E_{ini}\} \subseteq E_1, \{E_{g1}, E_{g2}, \dots, E_{gng}\} \subseteq E_2,$$

$$\text{then } T_{f_{ys}}^{-1}(E_{i1}, E_{i2}, \dots, E_{ini}) = E_i, T_{f_{ys}}^{-1}(E_{g1}, E_{g2}, \dots, E_{gng}) = E_j,$$

$$\therefore n_i = n_g, E_{ik} = E_{gk}, k_1 = 1, 2, \dots, n_j,$$

$$\text{Otherwise, } \exists E_{ik1} \neq E_{gk2},$$

$$E_{ik1} \in \{E_{i1}, E_{i2}, \dots, E_{ini}\} \subseteq E_1, E_{ik2} \in \{E_{g1}, E_{g2}, \dots, E_{gng}\} \subseteq E_2,$$

$$\text{Then } T_{f_{ys}}^{-1}(E_{i1}, E_{i2}, \dots, E_{ini}) \neq T_{f_{ys}}^{-1}(E_{g1}, E_{g2}, \dots, E_{gng})$$

$$\therefore (E_{i1}, E_{i2}, \dots, E_{ini}) = (E_{g1}, E_{g2}, \dots, E_{gng}) \subseteq (E_1 \cap E_2)$$

$$\therefore T_{f_{ys}}^{-1}(E_1) \cap T_{f_{ys}}^{-1}(E_2) = T_{f_{ys}}^{-1}(E_1 \cap E_2)$$

$$\therefore T_{f_{ys}}^{-1}(E_1 \cap E_2) = T_{f_{ys}}^{-1}(E_1) \cap T_{f_{ys}}^{-1}(E_2).$$

### ***Deconstruction Transformation***

Definition 4.3.1 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $E$  based on  $U$  for discerning rules  $G(t)$ . If  $T_h((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t))) = ((\Phi, \Phi, \Phi, \Phi, \Phi), \Phi = \Phi(\Phi, \Phi))$ , then  $T_h$  is the deconstruction decomposition based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$ , which is denoted as  $T_h$ .

The meaning of deconstruction:  $T_h(\text{deconstruction transforming words}) \rightarrow \{\text{kill, die out, destroy, disappear, dismiss, sell out, abandon, dismissory, move out...}\}$

Definition 4.3.2 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hly}((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t))) = ((\Phi, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$ , then  $T_{hly}$  is

the deconstruction transformation based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , and is denoted as  $T_{hly}$ .

The meaning of discussion domain destruction:  $T_{hl}(\text{discussion domain destruction}) \rightarrow$  discussion domain nonexistence  $\rightarrow$  no discussion domain to discuss or no need to discuss in certain discussion domain.

Definition 4.3.3 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $E$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hsw}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, \Phi, \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , then  $T_{hsw}$  is the destruction transformation of error objects based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hsw}$ .

The meaning of object destruction:  $T_{hsw}(\text{object destruction}) \rightarrow$  object nonexistence  $\rightarrow$  no object to discuss or no need to discuss the object

Definition 4.3.4 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hjk}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(t), \Phi, T(t), L(t)), x(t)=f(u(t), G(t)))$ , then  $T_{hk}$  is the destruction transformation of error space based on  $U$  for the rule  $G(t)$  and  $A((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hjk}$ .

The meaning of space destruction:  $T_{hjk}(\text{space destruction}) \rightarrow$  space nonexistence  $\rightarrow$  no space to discuss or no need to discuss in the space

Definition 4.3.5 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{htx}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(t), \bar{p}, \Phi, L(t)), x(t)=f(u(t), G(t)))$ , then  $T_{htx}$  is the destruction transformation of error characteristics based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{htx}$ .

The meaning of characteristic destruction:  $T_{htx}(\text{characteristic destruction}) \rightarrow$  characteristic nonexistence  $\rightarrow$  no characteristic to discuss or no need to discuss the characteristic

Definition 4.3.6 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hlz}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(t), \bar{p}, T(t), \Phi), x(t)=f(u(t), G(t)))$ , then  $T_{hlz}$  is deconstruction transformation of error magnitude based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hlz}$ .

The meaning of magnitude deconstruction:  $T_{hlz}(\text{magnitude deconstruction}) \rightarrow$  magnitude nonexistence  $\rightarrow$  no magnitude to discuss or no need to discuss the magnitude

Definition 4.3.7 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hCz}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(t), \bar{p}, T(t), L(t)), \Phi=f(u(t), G(t)))$ , then  $T_{hCz}$  is the destruction transformation of error value based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hCz}$

The meaning of error value destruction:  $T_{hCz}$  (error value destruction)  $\rightarrow$  error value nonexistence  $\rightarrow$  no error value to discuss

Definition 4.3.8 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hhs}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(t), \bar{p}, T(t), L(t)), x(t)=\Phi(u(t), G(t)))$ , then  $T_{hhs}$  is the destruction transformation of functions based on  $U$  for the rule  $G(t)$  and  $A((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hhs}$ .

The meaning of function destruction:  $T_{hhs}$ (function destruction)  $\rightarrow$  function nonexistence  $\rightarrow$  no function to discuss or no need to discuss the function

Definition 4.3.9 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the error logic variable based on  $U$  for discerning rule  $G(t)$ . If  $T_{hgz}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), \Phi))$ , then  $T_{hgz}$  is the destruction transformation of error rules based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hgz}$ .

The meaning of rule destruction:  $T_{hgz}$ (rule destruction)  $\rightarrow$  rule nonexistence  $\rightarrow$  no rule to utilize or no need to discuss the rule

Definition 4.3.10 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hsj}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((U, S(\Phi), \bar{p}, T(\Phi), L(\Phi)), x(\Phi)=f(u(\Phi), G(\Phi)))$ , then  $T_{hsj}$  is the time destruction transformation based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hsj}$ .

The meaning of time destruction:  $T_{hsj}$  (time destruction)  $\rightarrow$  time nonexistence  $\rightarrow$  no time to make use of or no need to discuss during that period

Definition 4.3.11 Suppose  $((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T_{hqb}((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))) = ((\Phi, \Phi, \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$  or  $((U, \Phi, \Phi, T(t), L(t)), x(t)=f(u(t), G(t))) = \dots = ((\Phi, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), \Phi)) = \dots = ((\Phi, \Phi, \Phi, \Phi,$

$\Phi$ ),  $\Phi=f((\Phi,\Phi),\Phi)$ ), then  $T_{hqb}$  is the complete destruction transformation based on  $U$  for the rule  $G(t)$  and  $((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{hqb}$ .

The meaning of the complete destruction transformation:  $T_{hqb}$ (above two or all elements destruction)  $\rightarrow$  above two or all elements nonexistence or no need to discuss above two or all elements The proposition 4.3.1  $T_h(C)=\Phi$ . (the verification is omitted), the discussion of the change of transformation  $T_h^{-1}$  is left out.

### ***Increase Transformation***

Definition 4.4.1 Suppose  $((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T((U, S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))=\{((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))), ((U_1, S_1(t), \bar{p}_1, T_1(t), L_1(t)), x_1(t)=f_1(u_1(t), G_1(t))), ((U_2, S_2(t), \bar{p}_2, T_2(t), L_2(t)), x_2(t)=f_2(u_2(t), G_2(t))), \dots, ((U_n, S_n(t), \bar{p}_n, T_n(t), L_n(t)), x_n(t)=f_n(u_n(t), G_n(t)))\}$ , then  $T$  is the error increase transformation based on  $U$  for the rule  $G(t)$  and  $((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , which is denoted as  $T_{zj}$ .

In  $T_{zj} ((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))=\{((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t))), ((U_1, S_1(t), \bar{p}_1, T_1(t), L_1(t)), x_1(t)=f_1(u_1(t), G_1(t))), ((U_2, S_2(t), \bar{p}_2, T_2(t), L_2(t)), x_2(t)=f_2(u_2(t), G_2(t))), \dots, ((U_n, S_n(t), \bar{p}_n, T_n(t), L_n(t)), x_n(t)=f_n(u_n(t), G_n(t)))\}$ , if  $((U_1, S_1(t), \bar{p}_1, T_1(t), L_1(t)), x_1(t)=f_1(u_1(t), G_1(t))) \in U_1(t)$ ,  $((U_2, S_2(t), \bar{p}_2, T_2(t), L_2(t)), x_2(t)=f_2(u_2(t), G_2(t))) \in U_2(t), \dots, ((U_n, S_n(t), \bar{p}_n, T_n(t), L_n(t)), x_n(t)=f_n(u_n(t), G_n(t))) \in U_n(t)$ , and  $U(t) \rightarrow U(t) \cup U_1(t) \cup U_2(t) \cup \dots \cup U_n(t)$ , and in  $U_1(t), U_2(t), \dots, U_n(t)$ , at least one  $U_i(t) \neq \Phi$ , then  $T_{zj}$  is the discussion domain increase of  $((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$  and is denoted as  $T_{zjly}$ .

In the above condition,  $U(t) \rightarrow U(t) \cup U_1(t) \cup U_2(t), \dots, \cup U_n(t)$  performs increase transformation on the discussion domain  $U$  of the object  $u(t)$  to achieve the expected goal. For instance, considering the effect of pan-the Delta of the Pearl River in Guangxi Province, the discussion domain is increased to pan-the Delta of the Pearl River from Guangxi Province when discussing the economic problem in Guangxi Province.

### ***Similarity Transformation***

Definition 4.5.1 Suppose  $((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))$ , the rule  $G(t)$  to discern errors is the element of error set  $C$  based on  $U$  for discerning rule  $G(t)$ . If  $T((U,S(t), \bar{p}, T(t), L(t)), x(t)=f(u(t), G(t)))=((U,S(t), \bar{p}, T(t), L(t)))', x(t)'=f(u'(t), G'(t))$ , then  $T_x$  is the similarity transformation based on  $U$  for the rule  $G(t)$ , which is denoted as  $T_x$ .

In  $((U, S(t), \bar{p}, T(t), L(t))', x(t)' = f(u(t)', G'(t)))$ , if  $((U, S(t), \bar{p}, T(t), L(t))', x'(t) = f(u'(t), G'(t))) = ((U', S(t), \bar{p}, T(t), L(t)), x'(t) = f(u'(t), G(t)))$ , then  $T_x$  is the similarity transformation of discussion domain based on  $U$  for the rule  $G(t)$  and  $((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$ , which is denoted as  $T_{xly}$ .

In the above situation, if  $U_1(t) = kU_2(t)$ ,  $k > 0$ , the exchange of  $U_1(t)$  and  $U_2(t)$  is performing the similarity transformation of discussion domain on the object  $u(t)$  to achieve the expected goal. Here,  $U_1(t) = kU_2(t)$  means that the measure or potential of  $U_1(t)$  is  $k$  times of the measure or potential of  $U_2(t)$ . For example, when discussing the domestic human resources, the discussion domain Shanxi province  $U_1(t)$  and the discussion domain China  $U_2(t)$  are two similar discussion domains. And the two can be exchanged.

## 5. The Application of Error Transformation to Data Mining

In the following part, the author will take the example of customer resources management of some supermarket in Guangzhou city to explain the meaning of all domains in an error set.

Suppose that the discussion domain  $U$  is all the customers of the supermarket in Guangzhou city, any customer  $u \in U$ ,  $G(t)$  is a group of rules to judge whether customers satisfy the requirements to become VIPs, then the error set of discussion domain  $U$  is  $C = \{((U(t), S(t), \bar{p}(t), T(t), L(t)), x(t) = f(G \neq u(t))) \mid (U(t), S(t), \bar{p}(t), T(t), L(t)) = u(t) \in U(t), f \subseteq U(t) \times R, x(t) = f(G \neq u(t))\}$ , among which the elements in the discussion domain  $U$  can be changeable and  $R$  is the real number domain.

(a) when  $T$  is not changed, all the customers who satisfy the requirements to be VIPs in Guangzhou is the correct domain of error set  $C$ , which can be denoted as  $U_z = \{u(t) \mid (u(t), x(t)) \in C, x(t) < 0\}$ .

All the customers who don't satisfy the requirements to be VIPs in Guangzhou is the error domain of error set  $C$ , which can be denoted as  $U_c = \{u(t) \mid (u(t), x(t)) \in C, x(t) > 0\}$ .

The customer who either satisfy the requirements or not to be VIPs in Guangzhou city is the critical domain of error set  $C$ , which is denoted as  $U_L = \{u(t) \mid (u(t), x(t)) \in C, x(t) = 0\}$ .

For instance, the customers who satisfy the requirements to be VIPs but don't apply for the VIP card are classified into VIP or non-VIP customers in practical.

(b) If the rule  $G(t)$  and discussion domain  $U$  keep stable, the change of error set consists of conducting promotion or give rewards to customers who satisfy the requirements to be VIPs. Consequently partial customers will increase consumption amount to meet the requirements to be

VIPs. Then the newly increased VIP customers belong to the changeable domain of transformation  $T$ , which is denoted as  $U_K = \{u(t) | (u(t), x(t)) \in C, x(t) \geq 0, T(f(G \neq u(t))) < 0\}$ .

If the company examines and verifies VIPs every year, then the consuming amount of some customers would not satisfy the standards. Those original VIPs who are eliminated later belong to the worsening domain, which is denoted as  $U_{KH} = \{u(t) | (u(t), x(t)) \in C, x(t) \leq 0, T(f(G \neq u(t))) > 0\}$ .

The original VIPs who keep their VIP identification after examination form the good domain, which can be denoted as  $U_H = U_Z - U_{KH}$ .

The original non-VIPs who are still not VIPs after taking some action compose the bad domain, which is denoted as  $U_S = U_C - U_K$ .

The original customers, VIPs or not, who are still in critical states form the critical domain, which can be denoted as  $U_{KL} = \{u(t) | (u(t), x(t)) \in C, T(f(G \neq u(t))) = 0\}$ .

How to deal with these customers depends on practical situation. But most companies are willing to take those customers as VIPs.

(c) If both the discussion domain and its customers keep unchanged, and the rule  $G(t)$  to judge customers is changed, then  $(u, x) = ((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$  in error set, that's to say, the rule  $G(t)$  of  $u = (U, S(t), \bar{p}, T(t), L(t)), x = x(t) = f(u(t), G(t))$  is changed.

Suppose  $T$  transformation is to increase or reduce the requirements for the consuming amount of money in order to change the restrictions of consuming amount of the rule  $G(t)$ , then the correctable domain  $U_K$  represents the customers who are not original VIPs but upgrade to be VIPs after changing the consuming amount standards; the worsening domain  $U_{KH}$  represents the customers who are original VIPs but are eliminated after the change; the good domain  $U_H$  represents the customers who keep their VIP identification after the change; the bad domain  $U_S$  represents the VIPs who are originally not VIPs are still non-VIPs after the change.

(d) If transformation  $T$  is to change the discussion domain  $U$ , then in error set  $(u, x) = ((U, S(t), \bar{p}, T(t), L(t)), x(t) = f(u(t), G(t)))$ , namely,  $U$  in  $u = (U, S(t), \bar{p}, T(t), L(t)), x = x(t) = f(u(t), G(t))$  is changed.

When customers belong to the union between the new discussion domain  $U_1$  and the original discussion domain  $U$ , the requirements of VIPs are not changed; When customers lie out the original discussion domain  $U$ , it is necessary to stipulate again VIP requirements, which can be the same as before or not.

Suppose  $U_2$  is to enlarge the customer district. For instance, originally, only the residents in Guangzhou City are qualified to become VIPs, namely, discussion domain = {all customers in Guangzhou City who buy the company's products}. However, the district are extended to Guangdong Province, namely,  $U_2 = \{all customers in Guangdong Province who buy the$

company's products}. After the transformation of the discussion domain  $U$  in error set, then the correctable domain  $U_K$  represents the customers outside Guangzhou City who become VIPs after the change; the worsening domain  $U_{KH}$  represents the original VIPs in Guangzhou city who degenerate to non-VIPs because of the entrance of customers outside Guangzhou City and the restrictions of quota of people; the good domain  $U_H$  represents the original VIPs maintain their VIP identification after the change; the bad domain  $U_S$  represents the original non-VIPs are still not VIPs after the change.

From the above demonstration, we can conclude that error set can quantifiedly express the change of objects, by which classification of objects can be done. This advantage enables decision makers to have a grasp of situations and know what to do and to mine usable information in data bank by taking some measure such as transformation.

## 6. Discussion and Conclusion

This article has studied the related operations of error set and its laws through comprehensive mathematical theoretical knowledge. That the operations of error set include INVERSE, OR and PRODUCT, and the basic transformations include combination transformation, transformation destruction, increase transformation, similarity transformation. The methods and laws of error transformation can be found out in order to establish efficient models to predict, prevent and eliminate errors with the help of the transformation of error set. In the future study we will concentrate our efforts on establishing the error case data bank in all fields as well as the error-eliminating and error-preventing expert systems to support making decisions.

The error-eliminating theory have increasingly attracted the attention of academia and industry, especially in the fields of management and decision-making. But, what we have done is not enough. So, we call for more scholars from all over the world to do research about error-eliminating theory. Only in this way, can the theory enjoy wider value of applications in more fields.

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