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Computing the Special Determinants of Containing the Second Derivative

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Abstract.

In this paper The special determinants of contain the second derivative have been computed, which are high order determinants and their elements are consist of 0,1,2 order differentials. The elements of roads from 1 to n in the determinants are the 0 order differentials of $x_i(i,=0,1,2...n)$.The elements of roads from n+1 to 2n in the determinants are the 1 order differentials of $x_i(i,=0,1,2...n)$,and the elements of roads from 2n+1 to 3n in the determinants are the 2 order differentials of $x_i(i,=0,1,2...n)$.Because of the high order to compute them are difficulty. But the have been reduced to the sum of several determinants which are lowe-oder.The several determinants of low oder are easy to be computed .These results can be applied in interpolation method, which is important tool in modeling and simulation .

Key words:

Special determinants, computation, Interpolation,

1 Introduction

In papers [1-3], from the computation of special determinants, we have obtained a lot of results .Where the theorem 2.5 in paper [1] is an example that is following lemma

Lemma

$$D(3n+2)_{1+k,1} = -(x-x_0)(x_k-x_0)^2\prod_{\substack{i=1\\ i\neq k}}^{n}(x_i-x_0)[D(3n+1)_{k,1}$$

$$-\frac{1}{x_k-x_0}D(3n+1)_{n+1+k,1}+\frac{2}{(x_k-x_0)^2}D(3n+1)_{2n+1+k,1}] \quad . \qquad \textbf{(1.1)}$$

where $D(3n+2)_{1+k,1}$ are the cofactor of following determinants

$$D(3n+2)=\begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \mathrm{L} & x_0^{3n} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \mathrm{L} & x_1^{3n} \\ & & & & & \mathrm{L} & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \mathrm{L} & x_n^{3n} \\ 1 & 1 & x & x^2 & x^3 & \mathrm{L} & x^{3n} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \mathrm{L} & 3nx_1^{3n-1} \\ & & & & & \mathrm{L} & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \mathrm{L} & \frac{3n!}{(3n-2)!}x_n^{3n-1} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \mathrm{L} & \frac{3n!}{(3n-2)!}x_1^{3n-1} \\ & & & & & \mathrm{L} & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \mathrm{L} & \frac{3n!}{(3n-2)!}x_n^{3n-1} \end{vmatrix} \qquad \textbf{(1.2)}$$

and $D(3n+1)_{k,1}, D(3n+1)_{n+1+k,1}, D(3n+1)_{2n+1+k,1}$ are the cofactor of the following determinant

$D(3n+1)$ **:**

$$D(3n+1)=\begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \dots & x_m^{3n-1} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \dots & x_1^{3n-1} \\ & & & & & \dots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \dots & x_n^{3n-1} \\ 1 & 1 & x & x^2 & x^3 & \dots & x^{3n-1} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \dots & (3n-1)x_1^{3n-2} \\ & & & & & \dots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \dots & (3n-1)x_n^{3n-2} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \dots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-2} \\ & & & & & \dots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \dots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-2} \end{vmatrix} \qquad \textbf{(1.3)}$$

that have been applied in addition of several fractions, and simplified the computation. In this paper the determinants containing the second derivative is more complication but to obtain the interpolation, which is important to simulate engineering, the computation of the special determinants is necessary. Following is

the main results. But the more high-order determinants have not been calculated yet, and this article they have been done .

2.Main results

Theorem 2.1.

$$D(3n+3)_{1+k,1} = (-)^{n+1}(x-x_0)^2(x_k-x_0)^4\prod_{\substack{i=1\\ j\neq k}}^{n}(x_i-x_0)^2[D(3n+1)_{k,1}$$

$$-\frac{2}{x_k-x_0}D(3n+1)_{n+1+k,1}+\frac{3!}{(x_k-x_0)}D(3n+1)_{2n+1+k,1}]$$

(2.1).

Where $D(3n+3)_{k+1,1}$ the cofactor of the following is determinates:

$$D(3n+3)=\begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+1} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+1} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+1} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n+1} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+1)x_0^{3n} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+1)x_1^{3n} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+1)x_n^{3n} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1} \end{vmatrix}.$$

（**2.2**）

And $D(3n+1)_{k,1}$ $D(3n+1)_{2n+1+k,1}$ $D(3n+1)_{n+1+k,1}$ **are the cofactors of the determinants**

$D(3n+1)$ in **form (1.3).**

Proof: According to the definition of cofactor, we have:

$$D(3n+3)_{1+k,1}=(-1)^{k+2}\begin{vmatrix}
1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+1} \\
1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+1} \\
 & & & & \cdots & \\
1 & x_{k-1} & x_{k-1}^2 & x_{k-1}^3 & \cdots & x_{k-1}^{3n+1} \\
1 & x_{k+1} & x_{k+1}^2 & x_{k+1}^3 & \cdots & x_{k+1}^{3n+1} \\
 & & & & \cdots & \\
1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+1} \\
1 & x & x^2 & x^3 & \cdots & x^{3n+1} \\
0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+1)x_0^{3n} \\
0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+1)x_1^{3n} \\
 & & & & \cdots & \\
0 & 1 & 2x_k & 3x_k^2 & \cdots & (3n+1)x_k^{3n} \\
 & & & & \cdots & \\
0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+1)x_n^{3n} \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1} \\
 & & & & \cdots & \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_k & \cdots & \frac{(3n+1)!}{(3n-1)!}x_k^{3n-1} \\
 & & & & \cdots & \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1}
\end{vmatrix} \qquad (2.3)$$

Similarly paper [1] ,delete the element about x_0 in the first row, the first column. Beginning from last column of $D(3n+3)_{k+1,1}$, $-\mathbf{x_0}$ times the $(i-1)$-th column to the i-th column ($i=3n,3n-1,\ldots,2,1.$),and then expand the determinant along row 1 and then extract out the common factor x_i-x_0 from row i in $D(3n+3)_{k+1,1}$ $(i=1,2,\ldots,n)$ So above determinant becomes:

$$D(3n+3)_{1+k,1} = (-)^{n+k+3}(x-x_0)^2(x_k - x_0)^4 \prod_{i \neq k}^{n}(x-x_0)^6 A$$

$$A = \begin{vmatrix}
1 & x_1 & x_1^2 & x_1^3 & \mathrm{L} & x_1^{3n-1} \\
 & & & & \mathrm{L} & \\
1 & x_{k-1} & x_{k-1}^2 & x_{k-1}^3 & \mathrm{L} & x_{k-1}^{3n-1} \\
1 & x_{k+1} & x_{k+1}^2 & x_{k+1}^3 & \mathrm{L} & x_{k+1}^{3n-1} \\
 & & & & \mathrm{L} & \\
1 & x_n & x_n^2 & x_n^3 & \mathrm{L} & x_n^{3n-1} \\
1 & x & x^2 & x^3 & \mathrm{L} & x^{3n-1} \\
0 & 1 & 2x_1 & 3x_1^2 & \mathrm{L} & (3n-1)x_1^{3n-2} \\
 & & & & \mathrm{L} & \\
a & b & c & & \mathrm{L} & d \\
 & & & & \mathrm{L} & \\
0 & 1 & 2x_n & 3x_n^2 & \mathrm{L} & (3n-1)x_n^{3n-2} \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \mathrm{L} & \frac{(3n-1)!}{(3n-2)!}x_1^{3n-3} \\
 & & & & \mathrm{L} & \\
e & f & g & & \mathrm{L} & h \\
 & & & & \mathrm{L} & \\
0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \mathrm{L} & \frac{(3n-1)!}{(3n-2)!}x_n^{3n-3}
\end{vmatrix}$$

(**2.5**)

and

$$a = 2x_k - 2x_0 = 2(x_k - x_0) = 2(x_k - x_0) + 0$$

$$b = 3x_k^2 - 4x_k x_0 + x_0^2 = 2x_k(x_k - x_0) + (x_k - x_0)$$

$$c = 4x_k^3 - 6x_k^2 x_0 + 2x_k x_0^2 = 2x_k^2(x_k - x_0) + 2x_k(x_k - x_0)^2$$

$$d = (3n+1)x_k^{3n} - 3nx_k^{3n-1}x_0 - 3nx_k^{3n-1}x_0 + (3n-1)x_k^{3n-1}x_0^2$$

$$= (3n-1)x_k^{3n} + 2x_k^{3n} - 6nx_k^{3n-1}x_0 + (3n-1)x_k^{3n-2}x_0^2$$

$$= (3n-1)x_k^{3n} + 2x_k^{3n} - (6n-2)x_k^{3n-1}x_0 - 2x_k^{3n-1}x_0 + (3n-1)x_k^{3n-2}x_0^2$$

$$= (3n-1)x_k^{3n} - (6n-2)x_k^{3n-1}x_0 + (3n-1)x_k^{3n-2}x_0^2 + 2x_k^{3n} - 2x_k^{3n-1}x_0$$

$$= (3n-1)x_k^{3n-2}(x_k - x_0)^2 + 2x_k^{3n-1}(x_k - x_0)$$

$$= 2x_k^{3n-1}(x_k - x_0) + (3n-1)x_k^{3n-2}(x_k - x_0)^2$$

$$e = \frac{2!}{0!} = \frac{2!}{0!} + 0$$

$$f = \frac{3!}{1!}x_k - 2\times\frac{2!}{0!}x_0 = \frac{2!}{0!}x_k + 2\times\frac{2!}{0!}(x_k - x_0)$$

$$g = \frac{4!}{2!}x_k^2 - 2\times\frac{3!}{1!}x_k x_0 + \frac{2!}{0!}x_0^2 = \frac{2!}{0!}[x_k^2 + 4x_k(x_k - x_0) + (x_k - x_0)^2]$$

$$h = \frac{(3n+1)!}{(3n-1)!}x_k^{3n-1} - 2\frac{3n!}{(3n-2)!}x_k^{3n-2}x_0 + \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}x_0^2$$

$$= \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}\left[\frac{(3n+1)3n}{(3n-1)(3n-2)}x_k^2 - 2\times\frac{3n}{3n-2}x_k x_0 + x_0^2\right]$$

$$= \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}\left[\frac{9n^2+3n-9n^2+9n-2}{(3n-1)(3n-2)}x_k^2 + x_k^2 - 2x_k x_0 - 2\frac{3n-3n+2}{3n-2}x_k x_0 + x_0^2\right]$$

$$= \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}\left[\frac{12n-2}{(3n-1)(3n-2)}x_k^2 + x_k^2 - 2x_k x_0 - 2\frac{3n-3n+2}{3n-2}x_k x_0 + x_0^2\right]$$

$$\mathbf{= \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}\left[\frac{12n-2}{(3n-1)(3n-2)}x_k^2 - 2\frac{2}{3n-2}x_k x_0 + (x_k - x_0)^2\right]}$$

$$\mathbf{= \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}\left[\frac{12n-2}{(3n-1)(3n-2)}x_k^2 - \frac{4}{3n-2}x_k^2 + \frac{4}{3n-2}x_k^2\right.}$$

$$\mathbf{\left. - \frac{4}{3n-2}x_k x_0 + (x_k - x_0)^2\right]}$$

$$= \frac{(3n-1)!}{(3n-3)!}\left[\frac{2x_k^2}{(3n-1)(3n-2)} + \frac{4x_k}{3n-2}(x_k - x_0) + (x_k - x_0)^2\right]$$

$$= 2x_k^{3n-1} + 4\frac{(3n-1)!}{(3n-2)!}x_k^{3n-2}(x_k - x_0) + \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}(x_k - x_0)^2$$

According to the property of determinants form (2.5) can be rewritten to be

$$D(3n+3)_{1+k,1} = (-1)^{n+k+3}(x - x_0)^2 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^6 (D_1 + D_2 + D_3 + D_4 + D_5 + D_6) \tag{2.6}$$

where

$$D_1 = 3(x_k - x_0)^2 \begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{3n} \\ & & & \cdots & \\ 1 & x_{k-1} & x_{k-1}^2 & \cdots & x_{k-1}^{3n-1} \\ 1 & x_{k+1} & x_{k+1}^2 & \cdots & x_{k+1}^{3n-1} \\ & & & \cdots & \\ 1 & x_n & x_n^2 & \cdots & x_n^{3n-1} \\ 1 & x & x^2 & \cdots & x^{3n-1} \\ 1 & x_1 & x_1^2 & \cdots & x_1^{3n-1} \\ & & & \cdots & \\ 1 & x_k & x_k^2 & \cdots & x_k^{3n-1} \\ & & & \cdots & \\ 0 & 1 & 2x_n & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \frac{2!}{0!} & \cdots & \frac{(3n-1)!}{(3n-3)!}x_1^{3n-2} \\ & & & \cdots & \\ \frac{2!}{0!} & \frac{2!}{0!}x_k & \frac{2!}{0!}x_k^2 & \cdots & \frac{2!}{0!}x_k^{3n} \\ & & & \cdots & \\ 0 & 0 & \frac{2!}{0!} & \cdots & \frac{(3n-1)!}{(3n-3)!}x_1^{3n-3} \end{vmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \leftarrow Row(n+k) \\ \\ \\ \\ \\ \leftarrow Row(2n+k) \\ \\ \\ \end{matrix}$$

$$D_2 = 2\frac{2!}{0!}(x_k - x_0)^2 \begin{vmatrix} & & & * & \\ 1 & x_k & x_k^2 & L & x_k^{3n-1} \\ & & & L & \\ 0 & 1 & 2x_k & L & (3n-1)x_k^{3n-2} \\ & & & * & \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \\ \\ \end{matrix}$$

$$D_3 = 2(x_k - x_0)^3 \begin{vmatrix} & & & * & \\ 1 & x_k & x_k^2 & L & x_k^{3n-1} \\ & & & L & \\ 0 & 1 & \frac{2!}{0!} & L & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-2} \\ & & & * & \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \\ \\ \end{matrix}$$

$$D_4 = \frac{2!}{0!}(x_k - x_0)^2 \begin{vmatrix} & & & * & \\ 0 & 1 & 2x_k & L & (3n-1)x_k^{3n-2} \\ & & & L & \\ 1 & x_k & x_k^2 & L & x_k^{3n-1} \\ & & & * & \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \\ \\ \end{matrix}$$

$$D_5 = 2\frac{2!}{0!}(x_k - x_0)^3 \begin{vmatrix} & & & * & \\ 0 & 1 & 2x_k & L & (3n-1)x_k^{3n-2} \\ & & & L & \\ 0 & 1 & 2x_k & L & (3n-1)x_k^{3n-2} \\ & & & * & \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \\ \\ \end{matrix}$$

$$D_6 = (x_k - x_0)^4 \begin{vmatrix} & & & * & \\ 0 & 1 & 2x_k & \mathrm{L} & (3n-1)x_k^{3n-2} \\ & & & \mathrm{L} & \\ 0 & 0 & \frac{2!}{0!} & \mathrm{L} & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3} \\ & & & * & \end{vmatrix} \begin{matrix} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{matrix}$$

and only the entries of the (n+k)-th row and the $(2n+k)$ -th row in determinants $\mathbf{D}_{2„}$, D_3, D_4, D_5, D_6. have been written and the rest of entries of each row are as the same with the corresponding entries of each row in determinant D_1,and have been noted by$*$.

For the determinants D_1 the entries of row $(2n+k)$ are $\frac{2!}{0!}$ times the entries $(n+k)$, so

$D_1 = 0.$

For the determinants D_2, using exchange the entries of neighboring two rows, and then move the entries of the $(n+k)$-th row to the k -th row and move the entries of the $(2n+k)$-th row to the $(n+k)$-th row, we have:

$$D_2 = 2 \times \frac{2!}{0!}(x_k - x_0)^2(-1)^{2n-1}\left|D(3n+1)_{2n+1+k,1}\right|$$

where $\left|D(3n+1)_{2n+1+k,1}\right|$ is the minor determinant of $D(3n+1)$in form (2.3).

Similarly, exchange the entries of neighboring two rows, we move the entries of the $(n+k)$-th row to the k -th row. in D_3,.we get:

$$D_3 = 2(x_k - x_0)^3(-1)^n\left|D(3n+1)_{n+1+k,1}\right|$$

Consider determinants D_4, the entries of neighboring two rows are exchanged from the $(2n+k)$-th row to k -th row,. Then the determinants D_4 has following form:

$$D_4 = \frac{2!}{0!} \times (x_k - x_0)^2 (-1)^{2n+1} \left| D(3n+1)_{2n+1+k,1} \right|$$

In D_5. The entries of the $(n+k)$ -th row and $(2n+k)$ -th row are same . So $D_5 = 0$..

According to the definition of cofactor .

$$D_6 = (x_k - x_0)^4 \left| D(3n+1)_{1+k,1} \right|$$

Consider the results of the D_1, D_2, D_3, D_4, D_5, D_6 , we have

$$D = (-1)^{n+k+3}(x - x_k)^2 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^6 [0 + 2\frac{2!}{0!}(x_k - x_0)^2 (-1)^{2n+1} \left| D(3n+1)_{2n+1+k,1} \right|$$

$$+2(x_k - x_0)^3 (-1)^n \left| D(3n+1)_{n+1+k,1} \right| + \frac{2!}{0!}(x_k - x_0)^2 (-1)^{2n+1} \left| D(3n+1)_{2n+1+k,1} \right|$$

$$+0 + (x_k - x_0)^4 \left| D(3n+1)_{1+k,1} \right|]$$

$$= (-)^{n+1}(x - x_k)^2 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^6 [3\frac{2!}{0!}(x_k - x_0)^2 (-1)^{2n+2+k} \left| D(3n+1)_{2n+1+k,1} \right|$$

$$-2(x_k - x_0)^3 (-1)^{n+2+k} \left| D(3n+1)_{n+1+k,1} \right| + (x_k - x_0)^4 (-1)^{k+2} \left| D(3n+1)_{1+k,1} \right|$$

$$= (-)^{n+1}(x - x_0)^2 (x_k - x_0)^4 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^6 [D(3n+1)_{1+k,1} - \frac{2!}{(x_k - x_0)} D(3n+1)_{n+1+k,1}$$

$$+ \frac{3!}{(x_k - x_0)^2} D(3n+1)_{2n+1+k,1}]$$

That is the form (2.1).

Similarly, we can get following theorems:

Theorem 2.2

$$D(3n+3)_{1,1} = (-1)^{3n} 2(x - x_1)^3 (x_0 - x_1) \sum \prod_{i=2}^{n} (x_i - x_1)^9 [D(3n)_{1,1} - \frac{3}{x_0 - x_1} D(3n)_{n+2,1}] \tag{2.7}$$

Where $D(3n+3)_{11}$ is the cofactor of $D(3n+3)$ in form (2.4), and $D(3n)_{11}$, $D(3n)_{n+2,1}$, are the cofactors of following determinant $D(3n)$:

$$D(3n)=\begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & L & x_0^{3n-2} \\ 1 & 1 & x_2 & x_2^2 & x_2^3 & L & x_2^{3n-2} \\ & & & & & L & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & L & x_n^{3n-2} \\ 1 & 1 & x & x^2 & x^3 & L & x^{3n-2} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & L & (3n-2)x_0^{3n-3} \\ 0 & 0 & 1 & 2x_2 & 3x_2^2 & L & (3n-2)x_2^{3n-2} \\ & & & & & L & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & L & (3n-2)x_n^{3n-3} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & L & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ & & & & & L & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & L & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \end{vmatrix}.$$

where each item of $\boldsymbol{x_1}$ has disappeared comparing with **D(3n+3)** in form (2.5)

Theorem 2.3

$$D(3n+3)_{n+2,1}=(-1)^{n+1}\prod_{i=1}^{n}(x_i-x_0)^6 D(3n+1)_{n+1,1} \qquad (2.8)$$

where, $\boldsymbol{D(3n+3)_{n+2,1}}$ is the cofactor of determinant $\boldsymbol{D(3n+3)}$ in form (2.2) deleted the entries of the (n+2)-th row and the 1st column, and $\boldsymbol{D(3n+1)_{1,1}}$ is the cofactor of determinant $\boldsymbol{D(3n+3)}$ in form (2.2)

3 Conclusion

The main results of this paper are the theorems 2.1 2.2 2.3 . They are the computation of high order determinants and their elements are consist of 0,1,2 order differentials whose elements of roads from 1 to n+1 in the determinants are the 0 order differentials of $\boldsymbol{x_i(i=0,1,2...)}$..and the elements of roads from n+2 to 2n+2 in the determinants are the 1 order differentials of $\boldsymbol{x_i(i=0,1,2...)}$.,and the elements of roads from 2n+3 to 3n+3 in the determinants are the 2 order differentials of $\boldsymbol{x_i(i=0,1,2...)}$..Because of the high order ,to compute them are difficulty. But the have been reduced to the sum of several determinants which are lowe-oder. The several determinants of low order are easy to be computed .These results can be applied in interpolation method, which is important tool in modeling and simulation .For example the design the out line of air plane ,bigger ship and satellite need interpolation But the solution of the problem have not been seen in papers [4-12]. The theorems 2.1 2.2 2.3 are the basic of solution of the problem. Subject the limit of paper page , the more results will appear in our other papers.

References

1 Liang. J.(2011) " A simplification for several fration forms".Advances in modeling . A-general methematics. Vol 48.No 1, PP 1-16..

2..Liang.J(2011) "Special determints and thier computations". Advances in modelling. A-general methematics. Vol 48.No 2, PP 27-41.

3..Liang. J. (2011)"Uniqueness of interpolation polynimial for two variables". Advances in modelling. A-general methematics. Vol 48.No 2, PP 42-56.

4. C. Bennett and R. Shapely.(1998) " Interpolation of Operators". Academic press, New-York, MR89e: 46001.

5. J. Bergh, J, L. O strum.(1976) "Interpolation spaces. an Introduction, springer-Verlag", New York, MR58:2349.

6..J.H. Bramble(1995) Interpolation between Soboles spaces in Lipschitz domains with an application to multigrin theory" Math. Comp., 64:1359-1365 MR95m:46042. ,

7.J.H. Bramble and X.Zhang.(2000) "The analysis of multigrin methods, in: Handbook for Numerical Analysis", Viol, Ⅶ, 173-415, P. Ciarlet and J.L.Lions, eds, North Holland, Amsterdam, MR2001m: 65183.

8.R.B.Kellogg.(1971) " Interpolation between subspaces of a Hilbert space, Technical note BN-719, Institutor for Fluid Dynamics and Applied Mathematics", University of Maryland, USA, College park.

10 L.R.Scott and S.Zhang,(1990) " Finite element interpolation of non-smooth functions satisfying boundary conditions", Math. Comp. 54,483-493.MR 90j:65021.

11. L.B. Wahlbin,(1981) " A quasi-optimal estimate in piecewise polynomial Galekin approximation of parabolic problems, in Numerical Analysis", Berlin-New York, ,PP,230-245. MR83f:65157.

12.C.Bacuta, J.H.Bramble, J.Pasciak. (2001_)"New interpolation results and application to finite elements methods for elliptic boundary value problems". East-West J. Numer. Math.9:179-198,