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Algebras of the Special Determinants

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Abstract. In this paper some special determinants have been computed, that is the key works are in simulate engineer, desire the out line from some given points to obtain the unknown curve. In this paper the continuous of computing special determines which are the determinants of order 3n, and consist of 0,1,2 order differentials. Namely ,: elements of roads from 1 to n in the determinants are the 0 oder differentials of $x_i^{\,j}(i,j=0,1,2...n)$.The elements of roads from n+1 to 2n in the determinants are the 1 order differentials of $x_i^{\,j}(i,j=0,1,2...n)$. The elements of roads from 2n+1 to 3n in the determinants are the 2 order differentials of $x_i^{\,j}(i,j=0,1,2...n)$ In one word, it is high-order determinants.

Key words : Interpolation, Approach,Function.Compute, Determinants

1 Introduction

Computation the special determinants of high- order is necessary to obtain the interpolation, which is important in simulation engineer. and the results of paper [1] have been applied in the Hirmit interpolation of two variables.But the special determinants is lower order than Paper [2].and the elements only contain one order differential. In papers [2]，the special determinants are (3n+3)-order and their elements are contain differential of two-order . From the computation of special determinants, a lot of results . have been obtained. Where the theorem 2.5 is an example

Lemma 1.1.

$$D(3n+3)_{1+k,1}=(-)^{n+1}(x-x_0)^2(x_k-x_0)^4\prod_{\substack{i=1\\ j\neq k}}^{n}(x_i-x_0)^2[D(3n+1)_{k,1} \tag{1.1}$$

Where $D(3n+3)_{k+1,1}$ **is the cofactor of the following determinants:**

$$\mathbf{D(3n+3)} = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+1} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+1} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+1} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n+1} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+1)x_0^{3n} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+1)x_1^{3n} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+1)x_n^{3n} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+1)!}{(3n-1)!}x_1^{3n-1} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+1)!}{(3n-1)!}x_n^{3n-1} \end{vmatrix}. \qquad (1.2)$$

and $D(3n+1)_{k,1}, D(3n+1)_{n+1+k,1}, D(3n+1)_{2n+1+k,1}$ are the cofactor of the following determinant

$\mathbf{D(3n+1)}$:

$$D(3n+1) = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_m^{3n-1} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n-1} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n-1} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n-1} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n-1)x_1^{3n-2} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-2} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-2} \end{vmatrix} \qquad (1.3)$$

From form 1.1, we can see the cofactor $D(3n+3)_{k+1,1}$ of the determinant $\mathbf{D(3n+1)}$ in form (1.3), The determinants , which is high order determinants, has been reduced to be the sum of several lower order determinants.But to obtain the interpolation , the computation of the determinants for the more high-order must be done. Following form is the result.

2.1 The main results

$$D(3n+4)_{1+k,1} = (-1)^{3n+1}2(x-x_0)^3(x_k-x_0)^6\prod_{\substack{i=1\\ i\neq k}}^{n}(x_i-x_0)^9\Big[D(3n+1)_{k,1}$$

$$-\frac{3}{x_k-x_0}D(3n+1)_{n+1+k} + \frac{2.3!}{(x_k-x_0)^2}D(3n+1)_{2n+1+k,1}\Big].$$

.

(2.1)

where $D(3n+4)_{n+3+k,1}$、$D(3n+4)_{n+2,1}$ are the cofactor of the determinants D(3n+4)

$$D(3n+4) = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n+2} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n+2} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n+2} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n+2} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n+2)x_0^{3n+1} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n+2)x_1^{3n+1} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n+2)x_n^{3n+1} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_0 & \cdots & \frac{(3n+2)!}{3n!}x_0^{3n} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & \cdots & \frac{(3n+2)!}{3n!}x_1^{3n} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n+2)!}{3n!}x_n^{3n} \end{vmatrix}.$$

(2.2)

Proof: Begin from last column of $D(3n+4)_{n+2,1}$ $-x_0$ times the k-1-th column to the k-th column (k=3n+4,3n,.., 2,1), then expand the determinant along row 1 and then extract out the common factor $x_i - x_0$ from row i in $D(3n+4)_{n+2,1}$ (i=1,2,……,n). So above determinant becomes:

$$D(3n+4)_{1+k,1} = (-1)^{1+k+3n+8}2(x-x_0)^3(x_k-x_0)^6\prod_{\substack{i=1\\ i\neq k}}^{n}(x_i-x_0)^9 A \qquad \textbf{(2.3)}$$

where

$$D(3n+4)_{1+k,1} = \begin{vmatrix} 1 & x_0 & x_0^2 & x_0^3 & \dots & x_0^{3n+2} \\ 1 & x_1 & x_1^2 & x_1^3 & \dots & x_1^{3n+2} \\ & & & & \dots & \\ 1 & x_{k-1} & x_{k-1}^2 & x_{k-1}^3 & \dots & x_{k-1}^{3n+2} \\ 1 & x_{k+1} & x_{k+1}^2 & x_{k+1}^3 & \dots & x_{k+1}^{3n+2} \\ & & & & \dots & \\ 1 & x_n & x_n^2 & x_n^3 & \dots & x_n^{3n+2} \\ 1 & x & x^2 & x^3 & \dots & x^{3n+2} \\ 0 & 1 & 2x_0 & 3x_0^2 & \dots & (3n+2)x_0^{3n+1} \\ 0 & 1 & 2x_1 & 3x_1^2 & \dots & (3n+2)x_1^{3n+1} \\ & & & & \dots & \\ 0 & 1 & 2x_n & 3x_n^2 & \dots & (3n+1)_n^{3n+1} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{2!}x_0 & \dots & (3n+1)x_0^{3n+1} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{2!}x_1 & \dots & \frac{(3n+1)!}{3n!}x_1^{3n} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{2!}x_2 & \dots & \frac{(3n+1)!}{3n!}x_2^{3n} \\ 0 & 0 & & & \dots & \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{2!}x_n & & \frac{(3n+1)!}{3n!}x_n^{3n} \end{vmatrix} \qquad (2.4)$$

$$A = \begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 & L & x_1^{3n-1} \\ & & & & L & \\ 1 & x_{k-1} & x_{k-1}^2 & x_{k-1}^3 & L & x_{k-1}^{3n-1} \\ 1 & x_{k+1} & x_{k+1}^2 & x_{k+1}^3 & L & x_{k+1}^{3n-1} \\ & & & & L & \\ 1 & x_n & x_n^2 & x_n^3 & L & x_n^{3n-1} \\ 1 & x & x^2 & x^3 & L & x^{3n-1} \\ 0 & 1 & 2x_1 & 3x_1^2 & L & (3n-1)x_1^{3n-2} \\ & & & & L & \\ a & b & c & & L & d \\ & & & & L & \\ 0 & 1 & 2x_n & 3x_n^2 & L & (3n-1)x_n^{3n-2} \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_1 & L & \frac{(3n-1)!}{(3n-2)!}x_1^{3n-3} \\ & & & & L & \\ e & f & g & & L & h \\ & & & & L & \\ 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & L & \frac{(3n-1)!}{(3n-2)!}x_n^{3n-3} \end{vmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \leftarrow \textbf{\textit{row}} \quad \boldsymbol{n+k} \\ \\ \\ \leftarrow \textbf{\textit{row}} \quad \boldsymbol{2n+k} \\ \\ \\ \\ \\ \end{matrix} \qquad (2.5)$$

where

$$a = 3x_k^2 - 6x_k x_0 + 3x_0^2 = 3(x_k - x_0)^2.$$

$$b = 4x_k^3 - 9x_x^2 x_0 + 6x_k x_0^2 - x_0^3$$

$$= 3x_k(x_k - x_0) + (x_k - x_0)^2.$$

$$c = (3n+2)x_k^{3n+1} - 3(3n+1)x_k^{3n} x_0 + 3\times 3nx_k^{3n-1} x_0^2 - (3n-1)x_k^{3n-2} x_0^3.$$

$$= \frac{3!}{1!} x_k^{3n-1}(x_k - x_0) + \frac{3!}{1!}(3n-1)x_k^{3n-2}(x_k - x_0)^2 + \frac{(3n-1)!}{(3n-3)!} x_k^{3n-3}(x_k - x_0)^3$$

$$d = \frac{3!}{1!} x_k - 3\times\frac{2!}{0!} = \frac{3!}{1!}(x_k - x_0).$$

$$e = \frac{4!}{2!} x_k^2 - 2\times\frac{3!}{1!} x_k x_0 + 2\times\frac{2!}{0!}\frac{3!}{1!} x_k x_0 - \frac{3!}{1!} x_k x_0 + 3\times\frac{2!}{0!} x_0^2$$

$$= \frac{3!}{1!}(x_k - x_0)^2 + \frac{3!}{1!} x_k(x_k - x_0) \ .$$

$$f = (3n+2)x_k^{3n+1} - 2(3n+1)x_k^{3n} x_0 + 3nx_k^{3n-1} x_0^2 - (3n+1)x_k^{3n} x_0$$

$$+2\times 3nx_k^{3n-1} x_0^2 - (3n-1)x_k^{3n-2} x_0^3$$

$$= x_k^{3n+1} + (3n+1)x_k^{3n+1} - 3(3n+1)x_k^{3n} x_0 + 9nx_k^{3n-1} x_0^2 - (3n-1)x_k^{3n-2} x_0^3$$

$$= \frac{3!}{1!} x_k^{3n-1}(x_k - x_0) + \frac{3!}{1!}\frac{(3n-1)!}{(3n-2)!} x_k^{3n-2}(x_k - x_0)^2 + \frac{(3n-1)!}{(3n-3)!} x_k^{3n-3}(x_k - x_0)^3$$

According to the determinants property the determinants A ,can be rewritten to be

$$A = D_1 + D_2 + D_3 + D_4 + D_5 + D_6. \qquad \textbf{(2.6)}$$

Where

$$
\mathbf{D_1} = \begin{vmatrix}
1 & x_1 & \cdots & x_1^{3n-1} \\
 & & \cdots & \\
1 & x_{k-1} & \cdots & x_{k-1}^{3n-1} \\
1 & x_{k+1} & \cdots & x_{k+1}^{3n-1} \\
 & & \cdots & \\
1 & x_n & \cdots & x_n^{3n-1} \\
1 & x & \cdots & x^{3n-1} \\
0 & 1 & \cdots & (3n-1)x_1^{3n-2} \\
 & & \cdots & \\
3(x_k - x_0)^2 & 3x_k(x_k - x_0)^2 & \cdots & 3x_k^{3n-1}(x_k - x_0)^2 \\
 & & \cdots & \\
0 & 1 & \cdots & (3n-1)x_n^{3n-2} \\
0 & 0 & \cdots & \dfrac{(3n-1)!}{(3n-3)!}x_k^{3n-3} \\
 & & \cdots & \\
\dfrac{3!}{1!}(x_k - x_0) & \dfrac{3!}{1!}x_k(x_k - x_0) & \cdots & \dfrac{3!}{1!}x_k^{3n-1}(x_k - x_0) \\
 & & \cdots & \\
0 & 0 & \cdots & \dfrac{(3n-1)!}{(3n-3)!}x_k^{3n-3}
\end{vmatrix}
\begin{matrix}
\\ \\ \\ \\ \\ \\ \\ \\ \\
\leftarrow Row(n+k) \\
\\ \\ \\ \\
\leftarrow Row(2\mathrm{n}+\mathrm{k}) \\
\\ \\
\end{matrix}
$$

（2.7）

$$
D_2 = \begin{vmatrix}
 & & & * & \\
3(x_k - x_0)^2 & 3x_k(x_k - x_0)^2 & & \mathrm{L} & 3x_k^{3n-1}(x_k - x_0)^2 \\
 & & & \mathrm{L} & \\
0 & 1 & 2x_k & \mathrm{L} & (3n-1)x_k^{3n-2} \\
 & & & * &
\end{vmatrix}
\begin{matrix}
\\
\leftarrow Row(n+k) \\
\\
\leftarrow Row(2n+k) \\
\\
\end{matrix}
$$

$$
D_3 = \left| \begin{array}{cccc}
 & & * & \\
3(x_k - x_0)^2 & 3x_k(x_k - x_0)^2 & \mathrm{L} & 3x_k^{3n-1}(x_k - x_0)^2 \\
 & & \mathrm{L} & \\
0 & 0 & \mathrm{L} & \frac{(3n-1)!}{(3n-3)!} x_k^{3n-2}(x_k - x_0)^3 \\
 & & * &
\end{array} \right| \begin{array}{l} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{array}
$$

$$
D_4 = \left| \begin{array}{cccc}
 & & * & \\
0 & (x_k - x_0)^3 & \mathrm{L} & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\
 & & \mathrm{L} & \\
\frac{3!}{1!}(x_k - x_0) & \frac{3!}{1!}x_k(x_k - x_0) & \mathrm{L} & \frac{3!}{1!}x_k^{3n-1}(x_k - x_0 \\
 & & * &
\end{array} \right| \begin{array}{l} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{array}
$$

$$
D_5 = \left| \begin{array}{cccc}
 & & * & \\
0 & (x_k - x_0)^3 & \mathrm{L} & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\
 & & \mathrm{L} & \\
0 & \frac{3!}{1!}(x_k - x_0)^2 & \mathrm{L} & \frac{3!}{1!}\frac{(3n-1)!}{(3n-3)!} x_k^{3n-2}(x_k - x_0)^2 \\
 & & * &
\end{array} \right| \begin{array}{l} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{array}
$$

$$D_6 = \begin{vmatrix} & & * & \\ 0 & (x_k - x_0)^3 & \cdots & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\ & & \cdots & \\ 0 & \frac{3!}{1!}(x_k - x_0)^2 & \cdots & \frac{3!}{1!}\frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}(x_k - x_0)^2 \\ & & * & \end{vmatrix} \begin{matrix} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{matrix}$$

and only the entries of the (n+k)-th row and the $(2n+k)$ -th row in determinants $\mathbf{D}_{2,,}$, D_3, D_4, D_5, D_6. have been written and the rest of entries of each row are as the same with the corresponding entries of each row in determinant D_1,and have been noted by$*$.

For D_1,put forward the common factor $\frac{3!}{1!}(x_k - x_0)$ in $Row(n+k), Row(2n+k)$,we have

$$D_1 = 3\frac{3!}{1!}(x_k - x_0)^3 \begin{vmatrix} 1 & x_1 & \cdots & x_1^{3n-1} \\ & & \cdots & \\ 1 & x_{k-1} & \cdots & x_{k-1}^{3n-1} \\ 1 & x_{k+1} & \cdots & x_{k+1}^{3n-1} \\ & & \cdots & \\ 1 & x_n & \cdots & x_n^{3n-1} \\ 1 & x & \cdots & x^{3n-1} \\ 0 & 1 & \cdots & (3n-1)x_1^{3n-2} \\ & & \cdots & \\ 1 & x_k & \cdots & x_k^{3n-1} \\ & & \cdots & \\ 0 & 1 & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3} \\ & & \cdots & \\ 1 & x_k & \cdots & x_k^{3n-1} \\ & & \cdots & \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3} \end{vmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \leftarrow Row(n+k) \\ \\ \\ \\ \\ \leftarrow Row(2n+k) \\ \\ \\ \end{matrix}$$

$=0,$

(2.8)

That result of the elements in row n+k,and in 2n+k are same.

$$D_2 = \begin{vmatrix} & & * & \\ 3(x_k-x_0)^2 & 3x_k^2(x_k-x_0)^2 & L & 3x_k^{3n-1}(x_k-x_0)^2 \\ & & L & \\ 0 & \frac{3!}{1!}(x_k-x_0)^2 & L & \frac{3!}{1!}\frac{(3n-1)!}{(3n-2)!}x_k^{3n-2}(x_k-x_0)^2 \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{matrix}$$

$$= \begin{vmatrix} \mathbf{1} & \mathbf{x_1} & \cdots & \mathbf{x_1^{3n-1}} \\ & & \cdots * & \\ \mathbf{1} & \mathbf{x_{k-1}} & \cdots & \mathbf{x_{k-1}^{3n-1}} \\ \mathbf{1} & \mathbf{x_{k+1}} & \cdots & \mathbf{x_{k+1}^{3n-1}} \\ & & \cdots & \\ \mathbf{1} & \mathbf{x_n} & \cdots & \mathbf{x_n^{3n-1}} \\ \mathbf{1} & \mathbf{x} & \cdots & \mathbf{x^{3n-1}} \\ \mathbf{0} & \mathbf{1} & \cdots & \mathbf{(3n-1)x_1^{3n-2}} \\ & & \cdots & \\ \mathbf{3(x_k-x_0)^2} & \mathbf{3x_k(x_k-x_0)^2} & \cdots & \mathbf{3x_k^{3n-1}(x_k-x_0)^2} \\ & & \cdots & \\ \mathbf{0} & \mathbf{1} & \cdots & \mathbf{(3n-1)x_n^{3n-2}} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}} \\ & & \cdots & \\ \mathbf{0} & \mathbf{\frac{3!}{1!}(x_k-x_0)^2} & \cdots & \mathbf{\frac{3!}{1!}\frac{(3n-1)!}{(3n-2)!}x_k^{3n-2}(x_k-x_0)^2} \\ & & \cdots & \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}} \end{vmatrix} \begin{matrix} \\ \\ \leftarrow \mathbf{Row(k)} \\ \\ \\ \\ \\ \\ \\ \leftarrow \mathbf{Row(n+k)} \\ \\ \\ \\ \\ \leftarrow \mathbf{Row(2n+k)} \\ \\ \\ \end{matrix}$$

Put forward the common factor $3(x_k - x_0)^2$ in $Row(n+k)$, and $\frac{3!}{1!}(x_k - x_0)^2$ in $Row(2n+k)$, we have

$$D_2 = 3\times\frac{3!}{1!}(x_k - x_0)^4 \begin{vmatrix} 1 & x_1 & \cdots & x_1^{3n-1} \\ & & \cdots & \\ 1 & x_{k-1} & \cdots & x_{k-1}^{3n-1} \\ 1 & x_{k+1} & \cdots & x_{k+1}^{3n-1} \\ & & \cdots & \\ 1 & x_n & \cdots & x_n^{3n-1} \\ 1 & x & \cdots & x^{3n-1} \\ 0 & 1 & \cdots & (3n-1)x_1^{3n-2} \\ & & \cdots & \\ 1 & x_k & \cdots & x_k^{3n-1} \\ & & \cdots & \\ 0 & 1 & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_1^{3n-3} \\ & & \cdots & \\ 0 & 1 & \cdots & \frac{(3n-1)!}{(3n-2)!}x_k^{3n-3} \\ & & \cdots & \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-3} \end{vmatrix} \begin{matrix} \\ \\ Row(k-1) \\ \\ \\ \\ \\ \\ \\ \leftarrow Row(n+k) \\ \\ \\ \\ \\ \leftarrow Row(2n+k) \\ \\ \\ \end{matrix} \quad (2.6)$$

,Swap adjacent two row elements method row n+k, and 2n+k for row k and n+k respectively, and notice $\frac{(3n-1)!}{(3n-2)!} = 3n-1$. The determinants (2.6) becomes:

$$D_2 = 3\times\frac{3!}{1!}(x_k - x_0)^4 \times(-1)^{2n-1}\times\begin{vmatrix} 1 & x_1 & \cdots & x_1^{3n-1} \\ & & \cdots & \\ 1 & x_k & \cdots & x_k^{3n-1} \\ & & \cdots & \\ 1 & x_n & \cdots & x_n^{3n-1} \\ 1 & x & \cdots & x^{3n-1} \\ 0 & 1 & \cdots & x_1^{3n-1} \\ & & \cdots & \\ 0 & 1 & \cdots & (3n-1)x_k^{3n-2} \\ & & \cdots & \\ 0 & 1 & \cdots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_1^{3n-2} \\ & & \cdots & \\ 0 & 1 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_{k-1}^{3n-2} \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_{k+1}^{3n-3} \\ & & \cdots & \\ 0 & 0 & \cdots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-3} \end{vmatrix} \begin{matrix} \\ \\ \leftarrow Row(\mathrm{k}) \\ \\ \\ \\ \\ \\ \leftarrow Row(n+k) \\ \\ \\ \\ \\ \leftarrow Row(2n+k-1) \\ \\ \\ \\ \end{matrix}$$

$$= 3\times\frac{3!}{1!}(x_k - x_0)^4 \times(-1)^{2n-1}\, x\left|D(3n+1)_{2n+1+k\ \ 1}\right| \qquad \textbf{(2.7)}$$

Similarly to obtain form (2.7), Put forward the common factor 3 $(x_k - x_0)^2$ in $Row(n+k)$,and $(\mathbf{x}_k - x_0)^3$ in row(2n+k) ,and then,swap adjacent two row elements method row n+k, for row k, we have:

$$D_3 = \begin{vmatrix} & & * & \\ 3(x_k - x_0)^2 & 3x_k(x_k - x_0)^2 & L & 3x_k^{3n-1}(x_k - x_0)^2 \\ & & L & \\ 0 & 0 & L & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-2}(x_k - x_0)^3 \\ & & * & \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \\ \\ \end{matrix}$$

$$= 3(x_k - x_0)^5 \times(-1)^n \times\left|D(3n+1)_{n+1+k\ \ 1}\right|. \qquad (2.8)$$

Similarly, put forward the common factor $(x_k - x_0)^3$ in $row(n+k)$, and $\frac{3!}{1!}(x_k - x_0)$ In row (2n+k) and then swap adjacent two row elements method row 2n+k, for row k respectively,

$$D_4 = \begin{vmatrix} & & * & \\ 0 & (x_k - x_0)^3 & \cdots & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\ & & \cdots & \\ \frac{3!}{1!}(x_k - x_0) & \frac{3!}{1!}x_k(x_k - x_0) & \cdots & \frac{3!}{1!}x_k^{3n-1}(x_k - x_0 \\ & & * & \end{vmatrix} \begin{matrix} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{matrix}$$

$$= \frac{3!}{1!}(x_k - x_0)^4(-1)^{2n}\left|D(3n+1)_{2n+1+k\ 1}\right| \qquad (2.9)$$

Because the elements in row(2n+k) are $\frac{3!}{1!}\frac{1}{(x_k - x_0)}$ times of the elements in row(n+k),

$$D_5 = \begin{vmatrix} & & * & \\ 0 & (x_k - x_0)^3 & \cdots & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\ & & \cdots & \\ 0 & \frac{3!}{1!}(x_k - x_0)^2 & \cdots & \frac{3!}{1!}\frac{(3n-1)!}{(3n-2)!}x_k^{3n-2}(x_k - x_0)^2 \\ & & * & \end{vmatrix} \begin{matrix} \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \end{matrix}$$

$$D_5 = 0 \quad . \qquad (2.10)$$

$$D_6 = \begin{vmatrix} & & * & \\ 0 & (x_k - x_0)^3 & \cdots & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\ & & \cdots & \\ 0 & \frac{3!}{1!}(x_k - x_0)^2 & \cdots & \frac{3!}{1!}\frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}(x_k - x_0)^2 \\ & & * & \end{vmatrix} \begin{matrix} \\ \leftarrow Row(n+k) \\ \\ \leftarrow Row(2n+k) \\ \\ \end{matrix}$$

$$= \begin{vmatrix} 1 & x_1 & \dots & x_1^{3n-1} \\ & & \dots & \\ 1 & x_{k-1} & \dots & x_{k-1}^{3n-1} \\ 1 & x_{k+1} & \dots & x_{k+1}^{3n-1} \\ & & \dots & \\ 1 & x_n & \dots & x_n^{3n-1} \\ 1 & x & \dots & x^{3n-1} \\ 0 & 1 & \dots & (3n-1)x_1^{3n-2} \\ & & \dots & \\ 0 & (x_k - x_0)^3 & \dots & (3n-1)x_k^{3n-2}(x_k - x_0)^3 \\ & & \dots & \\ 0 & 1 & \dots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \dots & \frac{(3n-1)1}{(3n-3)!}x_1^{3n-3} \\ & & \dots & \\ 0 & 0 & \dots & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3}(x_k - x_0)^3 \\ & & \dots & \\ 0 & 0 & \dots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-3} \end{vmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \leftarrow Row(n+k) \\ \\ \\ \\ \\ \leftarrow Row(2n+k) \\ \\ \\ \end{matrix}$$

When the common factor $(x_k - x_0)^3$ in row(n+k) is putted forward, we get:

$$D_6 = (x_k - x_0)^6 \begin{vmatrix} 1 & x_1 & \dots & x_1^{3n-1} \\ & & \dots & \\ 1 & x_{k-1} & \dots & x_{k-1}^{3n-1} \\ 1 & x_{k+1} & \dots & x_{k+1}^{3n-1} \\ & & \dots & \\ 1 & x_n & \dots & x_n^{3n-1} \\ 1 & x & \dots & x^{3n-1} \\ 0 & 1 & \dots & (3n-1)x_1^{3n-2} \\ & & \dots & \\ 0 & 1 & \dots & (3n-1)x_k^{3n-2} \\ & & \dots & \\ 0 & 1 & \dots & (3n-1)x_n^{3n-2} \\ 0 & 0 & \dots & \frac{(3n-1)1}{(3n-3)!}x_1^{3n-3} \\ & & \dots & \\ 0 & 0 & \dots & \frac{(3n-1)!}{(3n-3)!}x_k^{3n-3} \\ & & \dots & \\ 0 & 0 & \dots & \frac{(3n-1)!}{(3n-3)!}x_n^{3n-3} \end{vmatrix} = (x_k - x_0)^6 \left| D\ (3n+1)_{1+k\ 1} \right|$$

(2.11)

Above the results have been computed for the determinants .That results in:

$$D(3n+4)_{1+k\ 1} = (-1)^{1+k+3n+8}(x - x_0)^3 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^9 \Big[0$$

$$+3 \times \frac{3!}{1!}(x_k - x_0)^4 \times (-1)^{2n-1} \times \left| D(3n+1)_{2n+1+k,1} \right| + 3(x_k - x_0)^5 \times (-1)^n \left| D(3n+1)_{n+1+k,1} \right|$$

$$+ \frac{3!}{1!}(x_k - x_0)^4 \times (-1)^{2n} \left| D(3n+1)_{2n+1+k,1} \right| + 0 + (x_k - x_6)^6 \left| D(3n+1)_{k+11} \right| \Big].$$

$$= (-1)^{3n+1}(x - x_0)^3 (x_k - x_0)^6 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^9 \Big[0$$

$$+3 \times \frac{3!}{1!}(x_k - x_0)^4 \times (-1)^{2n+k} \times \left| D(3n+1)_{2n+1+k,1} \right| + 3(x_k - x_0)^5 \times (-1)^{n+1+k} \left| D(3n+1)_{n+1+k,1} \right|$$

$$+\frac{3!}{1!}(x_k - x_0)^4 \times (-1)^{2n+1+k} \left|D(3n+1)_{2n+1+k,1}\right| + 0 + (x_k - x_6)^6 \times (-1)^{1+k} \left|D(3n+1)_{k+11}\right| \Big].$$

Notice

$$(-1)^{2n+k} = (-1)^{2n+1+k+1}$$

$$(-1)^{n+1+k} = -(-1)^{n+1+k+1}$$

$$(-1)^{2n+1+k} = -(-1)^{2n+1+k+1}$$

The determinants $D(3n+4)_{1+k,1}$ **is:**

$$D(3n+4)_{1+k} = (-1)^{3n+1} \times 2(x - x_0)^3 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^9 \Big[0$$

$$+3 \times \frac{3!}{1!}(x_k - x_0)^4 \times (-1)^{2n+1+k+1} \times \left|D(3n+1)_{2n+1+k,1}\right| - 3(x_k - x_0)^5 \times (-1)^{n+1+k+1} \left|D(3n+1)_{n+1+k,1}\right|$$

$$-\frac{3!}{1!}(x_k - x_0)^4 \times (-1)^{2n+1+k+1} \left|D(3n+1)_{2n+1+k,1}\right| - (x_k - x_6)^6 \times (-1)^{1+k} \left|D(3n+1)_{k,1}\right| \Big].$$

$$= (-1)^{3n+1} 2(x - x_0)^3 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^9 \left[\frac{2 \times 3!}{1!}(x_k - x_0)^4 D(3n+1)_{2n+1+k,1} - 3(x_k - x_0)^5 D(3n+1)_{n+1+k,1}\right.$$

$$\left. + (x_k - x_0)^6 D(3n+1)_{k,1}\right].$$

$$= (-1)^{3n+1} 2(x - x_0)^3 (x_k - x_0)^6 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^9 \Big[D(3n+1)_{k,1} - \frac{3}{(x_k - x_0)} D(3n+1)_{n+1+k,1}$$

$$+ \frac{2 \times 3!}{(x_k - x_0)^2} D(3n+1)_{2n+1+k,1}\Big]$$

That is form (2.1).

Similarly following theorem can be get;

Theorem 2.12

$$D(3n+4)_{n+3+k,\,1} = (-1)^{3n+1} 2(x - x_0)^3 (x_k - x_0)^6 \prod_{\substack{i=1 \\ i \neq k}}^{n} (x_i - x_0)^9 \Big[D(3n+1)_{n+1+k,1}$$

$$-\frac{3!}{x_k - x_0} D(3n+1)_{2n+1+k,1}\Bigg].$$

Theorem 2.13.

$$D\left(3n+4\right)_{2n+4+k,1} = \left(-1\right)^{3n+5} 2(x-x_0)^3 (x_k - x_0)^6 \prod_{\substack{i=1\\ i\neq k}}^{n} (x_i - x_0)^9 D(3n+1)_{2n+1+k,1}.$$

Theorem 2.14.

$$D\left(3n+3\right)_{1.1} = (-1)^{3n+6} 2\left(x-x_1\right)^3 (x_0 - x_1) \prod_{i=2}^{n} (x_i - x_1)^9 [D(3n)_{11} - \frac{3}{x_0 - x_1} D(3n)_{n+2,1}].,$$

(2.12)

where $D(3n+3)_{11}$ is the cofactor of D(3n+3)in paper [], and $D(3n)_{11}$ $D(3n)_{n+2,1}$ are the cofactor of following determinant D(3n):

$$D(3n) = \begin{vmatrix} 0 & 1 & x_0 & x_0^2 & x_0^3 & \cdots & x_0^{3n-2} \\ 1 & 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^{3n-2} \\ & & & & & \cdots & \\ 1 & 1 & x_n & x_n^2 & x_n^3 & \cdots & x_n^{3n-2} \\ 1 & 1 & x & x^2 & x^3 & \cdots & x^{3n-2} \\ 0 & 0 & 1 & 2x_0 & 3x_0^2 & \cdots & (3n-2)x_0^{3n-3} \\ 0 & 0 & 1 & 2x_1 & 3x_1^2 & \cdots & (3n-2)x_1^{3n-3} \\ & & & & & \cdots & \\ 0 & 0 & 1 & 2x_n & 3x_n^2 & \cdots & (3n-2)x_n^{3n-3} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_2 & \cdots & \frac{(3n-2)!}{(3n-4)!}x_2^{3n-4} \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-2)!}{(3n-4)!}x_n^{3n-4} \\ & & & & & \cdots & \\ 0 & 0 & 0 & \frac{2!}{0!} & \frac{3!}{1!}x_n & \cdots & \frac{(3n-2)!}{(3n-4)!}x_n^{3n} \end{vmatrix}.$$

Where each item of x_1 has disappeared comparing with D(3n+3) in form( )

3 Conclusion.

From the compute the special determinants of high-order. the relationships of determinants and their minor cofactor have been obtained namely theorem 2.1 theorem

2.2, theorem 2.3, theorem 2.4 theorem 2.5. As same as paper [1-4] the relationships will be used in simplifying more complex addition of fraction, and solve more complex matrix equation. This is significant of simulation technology, for example, to design outline in engineering, In interpolations the method is very useful to solve the problems of mathematical modeling , Subject the limit of papers, the results, which have not been seen in papers [5-16] will be public in other paper .

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