AMSE JOURNALS –2014-Series: Advances D; Vol. 19; N° 2; pp 1-18
Submitted April 2014; Revised Sept. 16, 2014; Accepted Oct. 20, 2014

Search of Optimum Rotation Patterns in Two Occasions Successive Sampling

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Abstract

This paper considers an alternative estimation procedure of finite population mean on current occasion in two-occasion successive sampling. An exponential type estimator of current population mean has been proposed and its behavior is examined. Optimum replacement strategy for the proposed estimation procedure has been suggested. Empirical study is carried out to justify the proposition of the estimator and suitable recommendations have been made.

Keywords

Exponential, successive sampling, auxiliary variable, bias, mean square error, optimum replacement strategy.

AMS subject classification: 62D05

1. Introduction

There are many problems of practical interest in different fields of socio economic, agricultural and environmental sciences in which the various characters opt to change over time. For such situations, one time surveys are not sufficient, there is need of continuous monitoring. Surveys where sampling is done on successive occasions (over years or seasons or months) according to a specified rule, with partial replacement of units, is called successive (rotation) sampling. Successive (rotation) sampling provides a strong statistical tool for generating the reliable estimates at different occasions. A key issue, related to successive sampling, is the extent to which elements are sampled at the previous occasion should be retained in the sample selected at the current occasion; this is termed as optimum replacement policy.

The theory of rotation (successive) sampling appears to have started with the work of Jessen (1942). He pioneered using the entire information collected in the previous investigations (occasion). The theory of rotation (successive) sampling is due to Patterson (1950), Patterson's theory was examined and extended by Eckler (1955) Rao and Graham (1964), Chochran (1977), Gupta (1979), Das (1982), and Chaturvedi and Tripathi (1983), among others. Sen (1971) developed estimators for the population mean on the current occasion extracting the information from two auxiliary variables were available on previous occasion. Sen (1972, 1973) extended his work for several auxiliary variables. Singh and Singh (2001) used the auxiliary information on current occasion for estimating the current population mean in two occasions successive sampling. Singh (2003) extended their work for h-occasions successive sampling.

In many situations, information on an auxiliary variable may be readily available on the first as well as the second occasion; for example, tonnage (or seat capacity) of each vehicle or ship is known in the survey sampling of transportation, number of beds in different hospitals may be known in hospital surveys, number of polluting industries and vehicles are known in environmental survey, nature of employment status, educational status, food availability and medical aid of locality are well known in advance for estimating the various demographic parameters in demography surveys. Many other situations in biological (life) sciences could be explored to show the benefits of the present study. Utilizing the auxiliary information on both the occasions, Feng and Zou (1997) and Biradar and Singh (2001), Singh (2005), Singh and Priyanka (2006, 2007, 2008), Singh and Karna (2009) and Singh and Parsad (2010), Upadhaya *et. al* (2011), Singh *et. al* (2011) and Gupta *et. al* (2012), used the auxiliary information on the both occasions for estimating the current mean in successive sampling.

The objective of this paper is to propose some efficient and alternative type of estimators of population mean on current occasion in successive (rotation) sampling with two-occasion as follow up of the above work. Utilizing the information on two auxiliary variables which are positively and negatively correlated with study variable and readily available on both occasions besides the information on the study variable from the previous occasion, exponential type estimators have been proposed and their behavior are examined. A relative comparisons of efficiencies of the proposed estimators with sample mean estimator when there is no matching from previous occasion, and the natural successive sampling estimator when no auxiliary information is used at any occasion are made through empirical studies. Results are interpreted and consequently suitable recommendations have been made.

2. Sample structure and notations

Let $U=\left(U_1,U_2, ..., U_N\right)$ be the finite population of N units, which has been sampled over two occasions. The character under study is denoted by x(y) on the first (second) occasion, respectively. It is assumed that the information on two stable auxiliary variables z_1 and z_1 with known population means, which have positive and negative correlations respectively with the study variable x(y) on the first (second) occasion are available. A simple random sample (without replacement) of size n is drawn on the first occasion. A random subsample of size $m=n\lambda$ is retained (matched) for its use on the second occasion, while a fresh simple random sample (without replacement) of size $u=\left(n-m\right)=n\mu$ is drawn on the second occasion from the entire population so that the sample size on the second occasion is also n. Here λ and $\mu\left(\mu+\lambda=1\right)$ are the fractions of the matched and fresh samples, respectively, at the current (second) occasion. The values of λ or μ are required to be chosen optimally. In what follows we shall use the following notations:

$\bar{Y},\bar{X}$: Population means of the study variables x and y respectively.

$\bar{Z}_1,\bar{Z}_2$: Population means of the auxiliary variables z_1 and z_2 respectively.

$\bar{y}_u,\bar{y}_m,\bar{x}_m,\bar{x}_n,\bar{z}_{ju},\bar{z}_{jm},\bar{z}_{jn}\left(j=1,2\right)$: The sample means of the respective variables based on the sample sizes shown in suffices.

$\rho_{yx}, \rho_{yz_1}, \rho_{yz_2}, \rho_{xz_1}, \rho_{xz_1}$: Population correlation coefficients between the variables as shown in subscripts.

C_x, C_y, C_z: Coefficients of variation of the variable given in the subscripts.

$S_x^2, S_y^2, S_{z_1}^2, S_{z_2}^2$: Population variances of the variables x, y, z_1 and z_2 respectively.

To estimate the population mean $\bar{Y}$ on second (current) occasion, two different estimators may be formulated. One estimator is based on the fresh sample of size u drawn on the current occasion and the other estimator is based on matched sample of size m common to both occasions. Formulation of estimators is described in the following sections.

3. Estimation procedure of population mean in presence of positively correlated auxiliary variable

To estimate the population mean $\bar{Y}$ based on the fresh sample of size u drawn on the second occasion and in presence of positively correlated variable z_1, an exponential type estimator is considered as

$$T_u = \bar{y}_u \exp\left(\frac{\bar{Z}_1 - \bar{z}_{1u}}{\bar{Z}_1 + \bar{z}_{1u}}\right) \tag{1}$$

Another estimator of population mean $\bar{Y}$ based on the sample size m common to both the occasions is again an exponential type estimator and is structured as

$$T_m = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right) \exp\left(\frac{\bar{Z}_1 - \bar{z}_{1m}}{\bar{Z}_1 + \bar{z}_{1m}}\right) \tag{2}$$

The final estimator of population mean $\bar{Y}$ is formulated as

$$T = \varphi T_u + (1-\varphi) T_m \tag{3}$$

where $\varphi \left(0 \leq \varphi \leq 1\right)$ is an unknown constant to be determined so as to minimize the mean square error of the estimator T.

Remark 3.1: For estimating the mean on each occasion the estimator T_u is suitable, which implies that more belief on T_u could be shown by choosing φ as 1(or close to 1), while for estimating the change from one occasion to the next, the estimator T_m could be more useful so

φ might be chosen as 0 (or close to 0). For asserting both the problems simultaneously, the suitable (optimum) choice of φ is required.

4. Bias and mean square error of proposed estimator T

Since, the estimators T_u and T_m are the exponential and exponential type estimators, respectively, they are biased estimators of $\bar{Y}$, therefore, the resulting estimator T defined in equation (3) is also biased for $\bar{Y}$. The bias B (.) and mean square error M (.) up to the first order of approximation of the estimator T is derived using the following transformations:

$\bar{y}_u=\bar{Y}(1+e_1), \bar{y}_m=\bar{Y}(1+e_2),\ \bar{x}_m=\bar{X}(1+e_3),\ \bar{x}_n=\bar{X}(1+e_4),\ \bar{z}_{1u}=\bar{Z}_1(1+e_5),\ \bar{z}_{1m}=\bar{Z}_1(1+e_6),$
$\bar{z}_{1n}=\bar{Z}_1(1+e_7)$ such that $E(e_i)=0$ and $|e_i|\le 1, \forall\ i=1, 2,...,7.$

Under the above transformations T_u and T_m take the following forms:

$$T_u=\bar{Y}(1+e_1)\exp\left[\frac{-e_5}{2}\left(1+\frac{e_5}{2}\right)^{-1}\right] \tag{4}$$

$$T_m=\bar{Y}(1+e_6)\exp\left[\frac{(e_4-e_3)}{2}\left(1+\frac{(e_4-e_3)}{2}\right)^{-1}\right]\exp\left[\frac{-e_6}{2}\left(1+\frac{e_6}{2}\right)^{-1}\right] \tag{5}$$

The bias B (.) and mean square error M (.) up to the first order of approximation of the estimator T is derived and presented in the following theorems:

Theorem 4.1: Bias of the estimator T to the first order approximation is obtained as

$$B(T)=\varphi B(T_u)+(1-\varphi)B(T_m) \tag{6}$$

where

$$B(T_u)=\bar{Y}\left[\left(\frac{1}{u}-\frac{1}{N}\right)\left(\frac{3}{8}-\frac{1}{2}\rho_{yz_1}\right)\right]C_y^2 \tag{7}$$

$$B\left(T_m\right)=\bar{Y}\left[\frac{1}{m}\left(\frac{3}{4}-\frac{1}{4}\rho_{yz_1}-\frac{1}{2}\rho_{yx}\right)-\frac{1}{n}\left(\frac{3}{8}+\frac{1}{4}\rho_{yz_1}-\frac{1}{2}\rho_{yx}\right)-\frac{1}{N}\left(\frac{3}{8}-\rho_{yz_1}\right)\right]C_y^2 \tag{8}$$

Proof: The bias of the estimator T is given by

$$B\left(T\right)=E\left[T-\bar{Y}\right]=\varphi E\left[T_u-\bar{Y}\right]+\left(1-\varphi\right)E\left[T_m-\bar{Y}\right] \tag{9}$$

$$=\varphi B\left(T_u\right)+\left(1-\varphi\right)B\left(T_m\right) \tag{10}$$

where $B\left(T_u\right)=E\left[T_u-\bar{Y}\right]$ and $B\left(T_m\right)=E\left[T_m-\bar{Y}\right]$.

Substituting the expressions of T_u and T_m from equations (4) and (5) into equation (10), expending binomially, taking expectations, and retaining the terms up to the first order approximations, we have the expression of the bias of the estimator T as shown in equation (6)

Theorem 4.2: Mean square error of the estimator T to the first order approximations is obtained as

$$M(T)=\varphi^2 M(T_u)+(1-\varphi)^2 M(T_m)+2\varphi(1-\varphi)C(T_u,T_m) \tag{11}$$

where

$$M\left(T_u\right)=\left[\left(\frac{1}{u}-\frac{1}{N}\right)\left(\frac{5}{4}-\rho_{yz_1}\right)\right]S_y^2 \tag{12}$$

$$M\left(T_m\right)=\left[\frac{1}{m}\left(\frac{3}{2}-\frac{1}{2}\rho_{yz_1}-\rho_{yx}\right)-\frac{1}{n}\left(\frac{1}{4}+\frac{1}{2}\rho_{yz_1}-\rho_{yx}\right)-\frac{1}{N}\left(\frac{5}{4}-\rho_{yz_1}\right)\right]S_y^2 \tag{13}$$

$$C\left(T_u,T_m\right)=\left[-\frac{1}{N}\left(\frac{5}{4}-\rho_{yz_1}\right)\right]S_y^2 \tag{14}$$

Proof: The mean square error of the estimator T is given by

$$M\left(T\right)=E\left[T-\bar{Y}\right]^2=E\left[\varphi\left(T_u-\bar{Y}\right)+\left(1-\varphi\right)\left(T_m-\bar{Y}\right)\right]^2$$

$$=\varphi^2 M(T_u)+(1-\varphi^2)M(T_m)+2\varphi(1-\varphi)C(T_u,T_m) \tag{15}$$

where $M(T_u)=E[T_u-\bar{Y}]^2$, $M(T_m)=E[T_m-\bar{Y}]^2$ and $C(T_u,T_m)=E[(T_u-\bar{Y})(T_m-\bar{Y})]$.

Substituting the expressions of T_u and T_m from equations (4) and (5) into equation (15), expanding binomially, taking expectations, and retaining the terms up to the first order approximations, we have the expression of the mean square error of the estimator T as shown in equation (11)

Remark 4.1: Since, x and y are same study variable over two occasions and z_1 is an auxiliary variable positively correlated to x and y, therefore, ensuring on the stability nature (Reddy 1978) of the coefficient of variation and following Cochran (1977) and Feng and Zou (1997), the coefficients of variation of x, y, and z_1 are considered to be approximately equal.

5. Minimum mean square error of the estimator T

Since the mean square error of the estimator T in equation (11) is a function of unknown constant φ, therefore, it is minimized with respect to φ and subsequently the optimum value of φ, say φ_{opt} is obtained as

$$\varphi_{opt}=\frac{M(T_m)-C(T_u,T_m)}{M(T_u)+M(T_m)-2C(T_u,T_m)} \tag{16}$$

Now substituting the value of φ_{opt} from equation (16) into equation (11), we get the optimum mean square error of the estimator T as

$$M(T)_{opt}=\frac{M(T_u).M(T_m)-[C(T_u,T_m)]^2}{M(T_u)+M(T_m)-2C(T_u,T_m)} \tag{17}$$

Further, substituting the values of $M(T_u)$, $M(T_m)$ and $C(T_u, T_m)$ from equations (12)-(14) in equations (16) and (17), we get the simplified values of φ_{opt} and $M(T)_{opt}$, which are shown below:

$$\varphi_{opt} = \left[\frac{\mu(A_1 + A_2\mu)}{A_1 + A_2\mu^2}\right] \tag{18}$$

and

$$M(T)_{opt} = \left[\frac{A_3 + A_4\mu - A_5\mu^2}{A_1 + A_2\mu^2}\right]\frac{S_y^2}{n} \tag{19}$$

where $A_1 = \frac{5}{4} - \rho_{yz_1}$, $A_2 = \frac{1}{4} + \frac{1}{2}\rho_{yz_1} - \rho_{yx}$, $A_3 = (1-f)A_1^2$, $A_4 = A_1A_2$, $A_5 = fA_4$, $f = \frac{n}{N}$ and $\mu = \frac{u}{n}$ is the fractions of fresh sample drawn at the current (second) occasion.

6. Optimum replacement policy for estimator T

The optimum mean square error $M(T)_{opt}$ in equation (19) is a function of μ (fraction of sample to be drawn afresh at the second occasion). It is an important factor in reducing the cost of the survey, therefore, to determine the optimum value of μ so that $\bar{Y}$ may be estimated with maximum precision and minimum cost, we minimize $M(T)_{opt}$ with respect to μ which results in a quadratic equation in μ, which is shown as

$$\mu^2 D_1 - 2\mu D_2 + D_3 = 0 \tag{20}$$

where, $D_1 = A_2A_4$, $D_2 = (A_1A_5 + A_2A_3)$ and $D_3 = A_1A_4$.

solving the equation (20), the solutions of μ (say $\hat{\mu}$) are obtained as

$$\hat{\mu} = \frac{-D_2 \pm \sqrt{D_2^2 - D_1D_3}}{D_1} \tag{21}$$

From equation (21) it is clear that the real values of $\hat{\mu}$ exist, iff, the quantities under square root is greater than or equal to zero. For any combinations of correlations, which satisfy the condition of real solutions, two real values of $\hat{\mu}$ are possible. Hence, while choosing the values of $\hat{\mu}$, it should be remembered that $0 \le \hat{\mu} \le 1$, and all other values of $\hat{\mu}$ are said to be inadmissible. If both the values of $\hat{\mu}$ are admissible, the lowest one is the best choice as it reduces the cost of the survey. From equation (21), substituting the admissible value of $\hat{\mu}$ (say μ_0) in equation (19), we have the optimum value of mean square error of the estimator T, which is shown below:

$$M\left(T_0\right)_{opt} = \left[\frac{A_3 + A_4\mu_0 - A_5\mu_0^2}{A_1 + A_2\mu_0^2}\right]\frac{S_y^2}{n} \tag{22}$$

7. Estimation procedure of population mean in presence of negatively correlated auxiliary variable

In section 3, we have proposed the estimator T of population mean $\bar{Y}$ at current occasion under the assumption that the auxiliary variable z_1 is positively correlated with x(y) at first (second) occasion. Sometimes, one may desire to develop the estimator of $\bar{Y}$ at current occasion when the auxiliary variable is negatively correlated with x(y) at the first (second) occasion. Motivated with this argument we propose the estimator T^* which is suitable for the negatively correlation situation. Proceeding as discussed in the section 3, the estimators of population mean $\bar{Y}$ based on the fresh and matched samples respectively for the negatively correlated auxiliary variable z_2 are defined as:

$$T_u^* = \bar{y}_u \exp\left(\frac{\bar{z}_{2u} - \bar{Z}_2}{\bar{z}_{2u} + \bar{Z}_2}\right) \tag{23}$$

$$T_m^* = \bar{y}_m \exp\left(\frac{\bar{x}_n - \bar{x}_m}{\bar{x}_n + \bar{x}_m}\right) \exp\left(\frac{\bar{z}_{2m} - \bar{Z}_2}{\bar{z}_{2m} + \bar{Z}_2}\right) \tag{24}$$

Finally, considering the convex linear combination of the estimators T_u^* and T_m^*, we have the final estimator T^* of $\bar{Y}$ as

$$T^* = \psi T_u^* + (1-\psi) T_m^* \tag{25}$$

where $\psi (0 \le \psi \le 1)$ is an unknown constant to be determined so as to minimize the mean square error of the estimator T^*.

8. Bias and Mean Square Error of the Estimator T^*

Following the properties of the estimator T as discussed in section 4, similarly the bias and mean square error of the estimator T^* are derived as

$$B(T^*) = \psi B(T_u^*) + (1-\psi) B(T_m^*) \tag{26}$$

where

$$B(T_u^*) = \bar{Y}\left[\left(\frac{1}{u} - \frac{1}{N}\right)\left(-\frac{1}{8} + \frac{1}{2}\rho_{yz_2}\right)\right] C_y^2 \tag{27}$$

and

$$B(T_m^*) = \bar{Y}\left[\frac{1}{m}\left(\frac{1}{4} + \frac{1}{4}\rho_{yz_2} - \frac{1}{2}\rho_{yx}\right) - \frac{1}{n}\left(\frac{3}{8} - \frac{1}{4}\rho_{yz_2} - \frac{1}{2}\rho_{yx}\right) - \frac{1}{N}\left(-\frac{1}{8} - \rho_{yz_2}\right)\right] C_y^2 \tag{28}$$

$$M(T^*) = \psi^2 M(T_u^*) + (1-\psi)^2 M(T_m^*) + 2\psi(1-\psi) C(T_u^*, T_m^*) \tag{29}$$

where

$$M(T_u^*) = \left[\left(\frac{1}{u} - \frac{1}{N}\right)\left(\frac{5}{4} + \rho_{yz_2}\right)\right] S_y^2 \tag{30}$$

$$M(T_m^*) = \left[\frac{1}{m}\left(\frac{3}{2} + \frac{3}{2}\rho_{yz_2} - \rho_{yx}\right) - \frac{1}{n}\left(\frac{1}{4} - \frac{1}{2}\rho_{yz_2} - \rho_{yx}\right) - \frac{1}{N}\left(\frac{5}{4} + \rho_{yz_2}\right)\right] S_y^2 \tag{31}$$

$$C\left(T_u^*, T_m^*\right) = \left[-\frac{1}{N}\left(\frac{5}{4} - \rho_{yz_2}\right)\right] S_y^2 \tag{32}$$

9. Minimum mean square error and optimum replacement policy of the estimator T^*

Proceeding as discussed in sections (5), the optimum mean square error of estimator T^* is obtained as

$$\psi_{opt} = \left[\frac{\mu\left(B_1 + B_2\mu\right)}{B_1 + B_2\mu^2}\right] \tag{33}$$

and

$$M\left(T^*\right)_{opt} = \left[\frac{B_3 + B_4\mu - B_5\mu^2}{B_1 + B_2\mu^2}\right]\frac{S_y^2}{n} \tag{34}$$

where $B_1 = \frac{5}{4} + \rho_{yz_2}$, $B_2 = \frac{1}{4} - \frac{1}{2}\rho_{yz_2} - \rho_{yx}$, $B_3 = \left(1-f\right)B_1^2$, $B_4 = B_1B_2$, and $B_5 = fB_4$.

Following the discussion given in section (6), the solution of μ say $\hat{\mu}$ is obtained as

$$\hat{\mu} = \frac{-Q_2 \pm \sqrt{Q_2^2 - Q_1Q_3}}{Q_1} \tag{35}$$

where, $Q_1 = B_2B_4$, $Q_2 = \left(B_1B_5 + B_2B_3\right)$ and $Q_3 = B_1B_4$.

Let the admissible value of $\hat{\mu}$ be μ_0^* and subsequently the optimum mean error of the estimator T is given as

$$M\left(T_0^*\right)_{opt} = \left[\frac{B_3 + B_4\mu_0^* - B_5\mu_0^{*2}}{B_1 + B_2\mu_0^{*2}}\right]\frac{S_y^2}{n} \tag{36}$$

10. Efficiency Comparison

The percent relative efficiencies of the estimator T and T^* with respect to (i) sample mean estimator $\bar{y}_n$, when there is no matching and (ii) natural successive sampling estimator $\hat{\bar{Y}} = \delta \bar{y}_u + (1-\delta)\bar{y}'_m$ when no auxiliary information is used at any occasion, where $\bar{y}'_m = \bar{y}_m + b_{yx}\left(\bar{x}_n - \bar{x}_m\right)$, have been computed for different choices of correlations ρ_{yx}, ρ_{yz_1}, ρ_{yz_2}, ρ_{xz_1} and ρ_{xz_2} are presented in Tables 1-2. Since, $\bar{y}_n$ and $\hat{\bar{Y}}$ are unbiased estimators of $\bar{Y}$, therefore, following Sukhatme et.al.(1984) the variance of $\bar{y}_n$ and optimum variance of $\hat{\bar{Y}}$ are given by

$$V(\bar{y}_n) = \left(\frac{1}{n} - \frac{1}{N}\right) S_y^2 \tag{37}$$

$$V(\hat{\bar{Y}})_{opt} = \left[1 + \sqrt{1-\rho_{yx}^2}\,\right] \frac{S_y^2}{2n} - \frac{S_y^2}{N} \tag{38}$$

For N=5000, n=500 and different choices of correlations, Tables-1-2 present the optimum values of μ_0 and μ_0^*, and percent relative efficiencies E_1, E_2, E_1^* and E_2^* of the estimator T and T^* with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$ respectively, where

$$E_1 = \frac{V(\bar{y}_n)}{M(T_0)_{opt}} X\,100\ ,\ E_2 = \frac{V(\hat{\bar{Y}})_{opt}}{M(T_0)_{opt}} X100,\ E_1^* = \frac{V(\bar{y}_n)}{M(T_0^*)_{opt}} X\,100 \text{ and } E_2^* = \frac{V(\hat{\bar{Y}})_{opt}}{M(T_0^*)_{opt}} X100$$

Table 1: Optimum values of μ_0 and percent relative efficiencies of the estimator T with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$.

$\rho_{yx} \downarrow$	$\rho_{yz_1} \rightarrow$	0.4	0.5	0.6	0.7	0.8	0.9
	μ_0	0.4797	0.4705	0.4594	0.4458	0.4286	**0.4059**
0.3	E_1	112.36	124.65	140.10	160.18	187.50	227.17
	E_2	109.49	121.46	136.51	156.08	182.70	221.36
	μ_0	0.4929	0.4844	0.4741	0.4613	0.4450	0.4232
0.4	E_1	115.78	128.72	145.03	166.32	195.39	237.79
	E_2	110.41	122.74	138.30	158.60	186.31	226.76
	μ_0	0.5076	*	0.4907	0.4791	0.4641	0.4437
0.5	E_1	119.63	-	150.67	173.42	204.64	250.44
	E_2	110.73	-	139.47	160.52	189.40	231.80
	μ_0	0.5243	0.5179	0.5100	0.9091	0.4868	0.4686
0.6	E_1	124.02	138.65	157.27	181.82	215.74	265.93
	E_2	110.24	123.25	139.80	161.62	191.77	236.39
	μ_0	0.5434	0.5387	0.5327	0.5251	0.5147	0.7143
0.7	E_1	129.11	144.90	165.12	192.00	229.52	285.71
	E_2	108.61	121.89	138.90	161.51	193.07	240.34
	μ_0	0.5659	0.5635	0.5604	0.5563	0.5505	0.5420
0.8	E_1	135.14	152.42	174.77	204.84	247.45	312.61
	E_2	105.11	118.55	135.94	159.32	192.46	243.14
	μ_0	0.5931	0.5941	0.5955	0.5973	0.6000	0.6044
0.9	E_1	142.51	161.82	187.19	222.00	272.73	353.55
	E_2	-	111.11	128.53	152.43	187.26	242.75

(*) indicate μ_0 does not exist and (-) denote no gain.

Table 2: Optimum values of μ_0^*, and percent relative efficiencies of the estimator T^* with respect to $\bar{y}_n$ and $\hat{\bar{Y}}$.

ρ_{yx} ↓	ρ_{yz_1} →	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9
0.3	μ_0^*	0.4797	0.4705	0.4594	0.4458	0.4286	**0.4059**
	E_1^*	112.36	124.65	140.10	160.18	187.50	227.17
	E_2^*	109.49	121.46	136.51	156.08	182.70	221.36
0.4	μ_0^*	0.4929	0.4844	0.4741	0.4613	0.4450	0.4232
	E_1^*	115.78	128.72	145.03	166.32	195.39	237.79
	E_2^*	110.41	122.74	138.30	158.60	186.31	226.76
0.5	μ_0^*	0.5076	*	0.4907	0.4791	0.4641	0.4437
	E_1^*	119.63	-	150.67	173.42	204.64	250.44
	E_2^*	110.73	-	139.47	160.52	189.40	231.80
0.6	μ_0^*	0.5243	0.5179	0.5100	0.9091	0.4868	0.4686
	E_1^*	124.02	138.65	157.27	181.82	215.74	265.93
	E_2^*	110.24	123.25	139.80	161.62	191.77	236.39
0.7	μ_0^*	0.5434	0.5387	0.5327	0.5251	0.5147	0.7143
	E_1^*	129.11	144.90	165.12	192.00	229.52	285.71
	E_2^*	108.61	121.89	138.90	161.51	193.07	240.34
0.8	μ_0^*	0.5659	0.5635	0.5604	0.5563	0.5505	0.5420
	E_1^*	135.14	152.42	174.77	204.84	247.45	312.61
	E_2^*	105.11	118.55	135.94	159.32	192.46	243.14
0.9	μ_0^*	0.5931	0.5941	0.5955	0.5973	0.6000	0.6044
	E_1^*	142.51	161.82	187.19	222.00	272.73	353.55
	E_2^*	-	111.11	128.53	152.43	187.26	242.75

(*) indicate μ_0^* does not exist and (-) denote no gain.

11. Interpretation of Results

(1) The following interpretations may be read out from Table 1:

(a) For fixed value of ρ_{yz_1} the value of E_1 and μ_0 are increasing while E_2 is increasing for initial values of ρ_{yx} and then decreasing for higher values of ρ_{yx}. This behavior is in agreement with Sukhatme et.al (1984) results, which explains that less the value of ρ_{yx}, more the fractions of fresh sample required at the current occasion.

(b) For fixed value of ρ_{yx}, the values of E_1 and E_2 are increasing while values of μ_0 is decreasing with increasing values of ρ_{yz_1} which is highly desirable pattern. This behavior indicates that if the information on auxiliary variable is available on both the occasions, their use at estimation stage reduces the cost of the survey as well as makes the estimates more precise.

(2) From Table 2, it may be noticed that the values obtained in this table is exactly similar to that of Table-1. Hence, the interpretations of the results of Table-2 are similar to that of Table-1.

12. Conclusions

It is clear from the interpretations of above results, that the use of information on auxiliary variables at estimation stage is highly fruitful in terms of the proposed estimators T and T^*. It may be seen from the Tables 1-2 that the proposed estimators are preferable and reliable over sample mean estimator and the natural successive sampling estimator in estimation of population mean at current occasion in two-occasion successive sampling. Moreover, the use of exponential type estimators in two-occasion successive sampling is perhaps in its beginning phase in sample survey. The proposed estimators are most suited in the estimation of (i) agriculture production (ii) pollution level (iii) industrial production etc. at different points of time. Therefore, the estimators T and T^* may be recommended to survey statisticians for its practical applications.

Acknowledgements

Authors are thankful to the referee for his valuable suggestions. Authors are also thankful to UGC, New Delhi and Indian School of Mines, Dhanbad for providing financial assistance and necessary infrastructure to complete the present research paper.

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