AMSE JOURNALS –2014-Series: Advances D; Vol. 19; N° 2; pp 39-49
Submitted June 2014; Revised Dec. 26, 2014; Accepted Jan. 10, 2015

Multi-Attribute Error-Eliminating Decision-Making Method based on 2-tuple Linguistic

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Abstract

With respect to multiple attribute decision making problem with linguistic information of attribute values and state probability, a new decision making approach is proposed on the base of 2-tuple linguistic and error-eliminating. Firstly, the concepts of error-loss value, maximum error-loss value and their corresponding mathematical models are given on the base of Error-Eliminating theory. Secondly, after analyzing the one-to-one relationship between the linguistic information of attribute values and the linguistic information of error-loss, the way how decision matrix is converted into error-loss matrix is given. And then, the 2-tuple weighted averaging (T-WA) operator is used to calculate the comprehensive error-loss value, and the comprehensive error-loss matrix is built. At last, the expected error-loss 2-tuple weighted averaging (EET-WA) operator used to aggregate the comprehensive is defined, and the alternative strategies are prioritized and selected according to the aggregated result. The feasibility and effectiveness of this model has been proved by an example of application.

Key words

Multiple attribute decision making, Error-Eliminating theory, 2-tuple linguistic, linguistic information

1. Introduction

In recent years, multiple attribute decision making(MADM) problem with linguistic information of attribute values has attracted a lot of experts' attention[1-2]. The traditional/regular approaches for dealing with linguistic information of attribute values can be classified into two categories. (1) Converting linguistic information into fuzzy number, which is then operation processed. For example, linguistic information is converted into triangular fuzzy number or normal fuzzy number by Liu Pei-de and Liu Lin[3-4], which is then used for information aggregation. (2) Processing the linguistic information directly. For example, ILGDOWA operator, ILGDHWA operator and LTOPSIS model are used to aggregate the linguistic information of attribute value by Peide Liu and Elio Cables [5-6]. In both categories, loss of information and lack of precision are the main weaknesses, and the results are usually hard to be understood [7].

In order to these weaknesses, a 2-tuple linguistic representation model used to aggregate linguistic information is proposed by the Spaniard Herrera and Herrera-Viedma[8], who also propose the 2-tuple ordered weighted averaging (T-OWA) operator[9]. During the last decade, the 2-tuple linguistic representation model has been widely used, because of its usefulness, its accuracy, its interpretability, its simplicity in managing and so forth [10]. The generalized 2-tuple weighted average (G-2TWA) operator, generalized 2-tuple ordered weighted average (G-2TOWA) operator, induced generalized 2-tuple ordered weighted average (IG-2TOWA) operator, dependent 2-tuple ordered weighted averaging (D2TOWA) operator and dependent 2-tuple ordered weighted geometric (D2TOWG) operator are proposed by Wei, Wei and Zhao [11-12], on the base of 2-tuple linguistic representation model and T-OWA operator. The 2-tuple linguistic power average (2TLPA) operator and 2-Tuple linguistic power ordered weighted average (2TLPOWA) operator are proposed by Xu and Wang. And then, the approaches for multiple attribute group decision making under linguistic environment are built based on this two operators [13]. In order to deal with multi-attribute group decision making(MAGDM) problem with linguistic information, a 2-tuple MAGDM method based on VIKOR method is proposed by Zhang and Guo[7]. To deal with multi-criteria group decision making problem with linguistic information extended VIKOR method is given by Ju and Wang[14].

In terms of research methods, the attribute values are processed directly by 2-tuple linguistic model, and the error and error-loss are never considered in the above researches. In terms of research contents, the above researches mainly focuses on the situations where attribute values and(or) weights are linguistic information. However, in the real word, the size of error-loss is one of the most important factors which affect the alternative strategies selection. What's more, the situation of attribute values and state probability with linguistic information is overlooked. From what has been discussed, the purpose of this paper is to combine 2-tuple and error-eliminating theories to develop a new methodology for solving MADM problems with linguistic information of attribute values and state probability

2. Preliminaries

2.1 2-tuple linguistic

The 2-tuple linguistic represent model expresses linguistic information of attribute values by means of 2-tuples (s_k, α_k), the meanings of s_k and α_k are expressed as follows [7]:

(1) $S = \{s_0, s_1, \cdots, s_q\}$ is a linguistic term set, q is a even number, $s_k \in S$, and s_k must satisfy the following four characteristics[7,9]:

□ The set is ordered: $s_i \geq s_j$, if $i \geq j$;

□ There is a negation operator: $neg(s_i)=s_j$, where $j=q-i$;

□ Maximization operator: $max(s_i,s_j)=s_i$, if $s_i \geq s_j$;

□ Minimization operator: $min(s_i,s_j)=s_i$, if $s_i \leq s_j$.

(2) α_k is the value of the symbolic translation expressing the value of deviation form the evaluation result to s_k, and $\alpha_k \in [-0.5, 0.5)$[7,9].

Definition 1 ([7,9,14]) Let $S = \{s_0, s_1, \cdots, s_q\}$ be a linguistic term set, $s_k \in S$ be a linguistic label. Then the corresponding 2-tuple linguistic form of s_k is expressed as follows:

$$\begin{aligned} &\theta : S \to S \times [-0.5, 0.5) \\ &\theta(s_k) = (s_k, 0), s_k \in S \end{aligned} \tag{1}$$

Definition 2([7,9,14]) Let β be the aggregation result of linguistic information symbolic, $\beta \in [0, q]$, and $q+1$ is the cardinality of linguistic term set S. Then the 2-tuple which expresses the same information to β can be expressed as follows:

$$\begin{aligned} &\Delta : [0, l] \to S \times [-0.5, 0.5), \\ &\Delta(\beta) = (s_k, \alpha_k) \end{aligned} \tag{2}$$

where, $k = round(\beta)$, $\alpha_k = \beta - k$, $\alpha_k \in [-0.5, 0.5)$。

Definition 3 ([7,9,14]) Let (s_k, α_k) be a 2-tuple and S be a linguistic term set, $s_k \in S$, $\alpha_k \in [-0.5, 0.5)$. Then the 2-tuople (s_k, α_k) can be returned to its corresponding numerical value $\beta \in [0, l] \subset R$ by the following function:

$$\begin{aligned} &\Delta^{-1} : S \times [-0.5, 0.5) \to [0, q] \\ &\Delta^{-1}(s_k, \alpha_k) = k + \alpha_k = \beta \end{aligned} \tag{3}$$

Definition 4 ([9]) Let $\{(s_1,\alpha_1),(s_2,\alpha_2),\cdots,(s_m,\alpha_m)\}$ be the set of 2-tuples and $(w_1,w_2,\ldots,w_m)^T$ be the weights vector. The 2-tuple weighted averaging (T-WA) operator is defined as

$$(s,\alpha)=\Delta\left(\sum_{i=1}^{m} w_i\Delta^{-1}(s_i,\alpha_i)\right) \tag{4}$$

When the weights vector is expressed in the form of 2-tuple $\left((w_1,\alpha_1'),(w_2,\alpha_2'),\cdots,(w_m,\alpha_m')\right)^T$, the T-WA operator is translate into the form of following function[7,9].

$$(s,\alpha)=\Delta\left(\frac{\sum_{i=1}^{m}\Delta^{-1}(s_i,\alpha_i)\Delta^{-1}(w_i,\alpha_i')}{\sum_{i=1}^{m}\Delta^{-1}(w_i,\alpha_i')}\right) \tag{5}$$

2.2 Error-eliminating theory

Error-eliminating theory takes error as the research object, the generation, transmission and transformation of error as the research contents, the decreasing or avoiding error-loss as the research purpose and the method of quantitative and qualitative as the research means [15-17]. The concepts of error, error function, and error-loss and so on are defined as follows according to Error-eliminating theory.

Definition 5 ([18-19]) Let *U* be the universe of discourse, $u\in U$, and *G* be a group of certain and qualified rules of *U*. If *u* cannot be derived by G (including: *u* cannot be derived completely, partly or unsure), then u is erroneous in *U* for *G*.

Definition 6 Let *U* be the universe of discourse, U' be a object set which is built on the base of *U*, $u'\in U'$, GY_j (j=1,2， ……*n*) be a intrinsic function of object set U'. When u' is erroneous completely, the maximization value of the possible loss of GY_j is called maximum error-loss value, which can be express as l_{max}.

Definition 7 Let *U* be the universe of discourse, U' be a object set which is built on the base of *U* , *G* be a group of rules of U' and $V=\{(u',G)|u'\in U'\}$， $f:V\rightarrow R$. Then *f* is called the error function based on *U* for *G* ($e=f(u')$ for short), where R is real number field, *e* is the error value of u' under the rules of *G*.

Definition 8 Let *U* be the universe of discourse, U' be a object set which is built on the base of *U*, $u'\in U'$, GY_j (j=1,2， ……*n*) be a intrinsic function of object set U'. When u' is erroneous, the loss value of GY_j is called error-loss value, expressing as *l*. *l* can be expressed as the product of error value and maximum error-loss value, namely $l=e\times l_{max}$.

3. Error-Eliminating decision-making method based on 2-tuple linguistic

3.1 The description of decision making problems

With respect to multiple attribute decision making problems with linguistic information of attribute values and state probability, the concerning sets, vectors and variables are described as follows. Let A be the set of alternative strategies, $a_i \in A(i = 1,2,\cdots,m)$, C be the set of attributes, $c_j \in C(j = 1,2,\cdots,n)$, T be the set of states, $t_k \in T(k = 1,2,\cdots,r)$. Let $x_{i,j}^{(k)}$ be the attribute value(in the form of linguistic information) of c_j of a_i in the state of t_k, and $x_{i,j}^{(k)} \in S$. Hence the decision matrix of the state t_k can be expressed as $X^{(k)} = \left(x_{i,j}^{(k)}\right)_{m\times n}$, and its 2-tuple form is $X^{(k)} = \left((x_{i,j}^{(k)},0)\right)_{m\times n}$. $W=(w_1,w_2,\ldots, w_n)$ be a weight vector of attribute, where $0 \le w_i \le 1$, $\sum_{i=1}^{n} w_i = 1$. The state probability just only can be described in the term of linguistic term set S^p. $P = \left((p_1,\alpha_1^p),(p_2,\alpha_2^p),\ldots,(p_r,\alpha_r^p)\right)$ be the 2-tuple probability vector , where $p_k \in S^p$, $\alpha_k^p \in [-0.5,0.5)$, and S^p and S have the same cardinality.

3.2 Constructing error-loss matrix

When discussing the problem of multiple attribute decision making under risk, by Definition 6, the maximum error-loss value of attribute c_j can be expressed as the difference between the maximum assessing value and minimum assessing value under condition t_k, shown as (6).

$$l_{\max} = \overline{x}_j^{(k)} - \underline{x}_j^{(k)} \tag{6}$$

Where $\overline{x}_j^{(k)} = \max\left(x_{1,j}^{(k)}, x_{2,j}^{(k)},\cdots, x_{m,j}^{(k)}\right)$, $\underline{x}_j^{(k)} = \min\left(x_{1,j}^{(k)}, x_{2,j}^{(k)},\cdots, x_{m,j}^{(k)}\right)$。

By Definition 7, the error function of attribute c_j in alternative strategies a_i under state t_k can be expressed as (7).

$$e = f\left(x_{i,j}^{(K)}\right) = \frac{\overline{x}_j^{(k)} - x_{i,j}^{(k)}}{\overline{x}_j^{(k)} - \underline{x}_j^{(k)}} \tag{7}$$

Where $\overline{x}_j^{(k)} = \max\left(x_{1,j}^{(k)}, x_{2,j}^{(k)},\cdots, x_{m,j}^{(k)}\right)$, $\underline{x}_j^{(k)} = \min\left(x_{1,j}^{(k)}, x_{2,j}^{(k)},\cdots, x_{m,j}^{(k)}\right)$.

By Definition 8 and (6) and (7), the error-loss value of attribute c_j in alternative strategies a_i can be expressed as (8).

$$l_{i,j}^{(k)} = \overline{x}_j^{(k)} - x_{i,j}^{(k)} \tag{8}$$

Assume that $\overline{x}_j^{(k)} = s_i$, $x_{i,j}^{(k)} = s_j$, $s_i \in S$, $s_j \in S$, by (5), then it must exist $i \geq j$, $s_i \geq s_j$. The greater the difference between s_i and s_j, the bigger the error-loss value will be. The degree of the error-loss can be described by new constructed ordered linguistic terms S^l, $l_{i,j}^{(k)} \in S^l$. Assuming $l_{i,j}^{(k)} = s_t^i, s_t^l \in S$, by (8), we can see,

$$s_t^l = s_i - s_j \tag{9}$$

For the difference between s_i and s_j is a integral number between 0 and q (here q+1 is the cardinality of S), S^l which has q+1 linguistic terms can be built to described the error-loss value and make one-to-one correspondence between $l_{i,j}^{(k)}$ and $\overline{x}_j^{(k)} - x_{i,j}^{(k)}$. For $S^l = (s_1^l, s_2^l, \cdots, s_q^l)$ when $s_i^l \geq s_j^l$, the error-loss of s_i^l is lesser than the error-loss of s_j^l. Obviously S^l has features of inverse operation and ordering of binary semantics. In (8), when s_i and s_j are in a same linguistic term, that is i-j=0, s_t^l is the biggest element s_q^l in S^l. When there is a grade difference between s_i and s_j, that is i-j=1, the error loss is lesser, s_t^l is the second largest element s_{q-1}^l in S^l. And so on, when the grade difference between s_i and s_j is q, that is i-j=q, the error loss is largest, s_t^l is the least element s_0^l in S^l. According to these rules, (9) can be expressed as (10).

$$s_{q-(i-j)}^l = s_i - s_j \tag{10}$$

By 2-tuple linguistic, we can transform (10) into (11).

$$\Delta^{-1}(s_{q-(i-j)}^l, \alpha_{q-(i-j)}) = q - \left(\Delta^{-1}(s_i, \alpha_i) - \Delta^{-1}(s_j, \alpha_j)\right) \tag{11}$$

Where, $\alpha_{q-(i-j)} \in [-0.5, 0.5)$, $\alpha_i \in [-0.5, 0.5)$, $\alpha_j \in [-0.5, 0.5)$, q is the linguistic granularity of ordered linguistic sets of S^l and S. By (11), (8) can be expressed as (12).

$$\left(l_{i,j}^{(k)}, \tilde{\alpha}_{i,j}^{(k)}\right) = \Delta\left(q - \left(\Delta^{-1}(\overline{x}_j^{(k)}, \alpha_j^{(k)}) - \Delta^{-1}(x_{i,j}^{(k)}, \alpha_{i,j}^{(k)})\right)\right) \tag{12}$$

Where, $l_{i,j}^{(k)} \in S^l$, $x_{i,j}^{(k)} \in S$, $\tilde{\alpha}_{i,j}^{(k)} \in [-0.5, 0.5)$, $\alpha_j^{(k)} \in [-0.5, 0.5)$, $\alpha_{i,j}^{(k)} \in [-0.5, 0.5)$, q+1 is the linguistic granularity of ordered linguistic sets of S^l and S.

By (12), decision matrix $X^{(k)} = \left((x_{i,j}^{(k)}, 0)\right)_{m \times n}$ can be transformed to error-loss matrix $L^{(k)} = \left((l_{i,j}^{(k)}, \tilde{\alpha}_{i,j}^{(k)})\right)_{m \times n}$.

3.3 Constructing comprehensive error-loss matrix

The 2-tuple linguistic information of row i in error-loss matrix $L^{(k)}$ using T-WA Operator can be integrated to obtain the comprehensive error-loss 2-tuple linguistic information $L_{i,k}$ of alternative strategies a_i of state t_k as follows:

$$L_{i,k} = \left(l_{i,k}, \alpha_{i,k}\right) = \Delta\left(\sum_{i=1}^{m} w_i \Delta^{-1}(l_{i,j}^{(k)}, \widetilde{\alpha}_{i,j})\right) \tag{13}$$

Where, $l_{i,k} \in S^l$, $\alpha_{i,k} \in [-0.5, 0.5)$, k=1,2,…,r。

By (13), the comprehensive error-loss matrix $L = \left((l_{i,k}, \alpha_{i,k})\right)_{m \times r}$ of different alternative strategies can be obtained under different conditions.

3.4 Calculating expected error loss value

By the expected utility theory, the expected error-loss value of an alternative strategy can be obtained by calculating the error loss value and its possibility under different conditions. The attribute error-loss value and state possibility are linguistic information. And the linguistic granularities of attribute error-loss value linguistic assessing sets S^l is the same as the linguistic granularity of state possibility linguistic assessing sets S^p. Where, the granularities of S^l, S^p and S are all q+1. Referring to weighted averaging operator of 2-tuple linguistic, by (5), expected error loss 2-tuple weighted averaging operator can be given as follows.

Definition 9 Let $\left((l_{i,1}, \alpha_{i,1}), (l_{i,2}, \alpha_{i,2}), \cdots, (l_{i,r}, \alpha_{i,r})\right)$ be the error loss vector of alternative strategy a_i under different states, the corresponding possibility is $\left((p_1, \alpha_1^p), (p_2, \alpha_2^p), \ldots, (p_r, \alpha_r^p)\right)^T$, and

$$L_i^e = (l_i, \alpha_i) = \Delta\left(\frac{\sum_{j=1}^{r} \Delta^{-1}(l_{i,j}, \alpha_{i,j},) \Delta^{-1}(p_i, \alpha_r^p)}{\sum_{j=1}^{r} \Delta^{-1}(p_j, \alpha_j^p)}\right), l_{i,k} \in S^l,\ \alpha_i \in [-0.5, 0.5) \tag{14}$$

is called the expected error-loss 2-tuple weighted averaging operator(EET-WA).

By Definition 9, the 2-tuple linguistic information L_i^e of expected error loss in alternative strategy a_i can be obtained by aggregating the 2-tuple linguistic information of row i in comprehensive error loss matrix L.

By the rule of 2-tuple linguistic comparison, when L_i^e of strategy a_i is larger the loss of expected error is smaller and strategy a_i is more optimal.

3.5 Procedures of 2-tuple linguistic error eliminating decision making

Step 1: Obtain decision matrix $X^{(k)}$, attribute weight vector W, condition possibility vector P by analyzing and investigating different conditions.

Step 2: Calculate error loss matrix $L^{(k)}$ under different states by (12).

Step 3: Calculate comprehensive error loss matrix L using T-WA operator by (13).

Step 4: Calculate expected error loss 2-tuple linguistic information L_i^e of each alternative strategies using EET-WA operator by (14).

Step 5: By the rule of 2-tuple linguistic comparison, sort alternative strategies according to the values of L_i^e and choose the best strategy.

4. Application of the proposed method

In the process of agricultural informationization construction, a town plans to choose a village that meets requirements and then, with additional support, turn it into an agricultural informationization model village. After a preliminary assessment, 4 villages stand out as candidates, marked as a_1、a_2、a_3、a_4. The different geographical locations, industrial structures and population qualities of the 4 villages leads to great difference of demonstrative effect c_1, promotional and directional effect c_2, long-term instructional effect c_3. Based on previous experience, the forecasting result of model village construction may be successful t_1, qualified t_2, or unsuccessful t_3. For the lack of statistical data and the ambiguity of understanding, the attributes and possibilities of different situations can only be described in natural linguistic terms. Assume that a linguistic assessment set is composed of 7 elements, the value of attribute is the element of linguistic assessment set S={s_0=extremely poor, s_1=very poor, s_2=poor，s_3=medium, s_4=large, s_5=very large, s_6=extremely large}, the error loss situation of attribute is the element of linguistic assessment set S^l={ s_0^l=extremely large，s_1^l=very large，s_2^l=large，s_3^l=medium，s_4^l=little，s_5^l=very little，s_6^l=nothing}, the possibility of different situation is the element of linguistic assessment set S^P={ s_0^p=extremely low，s_1^p=very low，s_2^p=low，s_3^p=medium，s_4^p=high，s_5^p=very high，s_6^p=extremely high}. After assessing the decision information table under risk of four candidates, shown in table 1, the weights of index c_1~c_3 are w_1=0.2、w_2=0.3、w_3=0.5. The possibility of different situation is t_1 low（s_2^p）, t_2 very high（s_5^p）, t_3 medium（s_3^p）. Which village is the best choice?

Table 1 the decision information under risk of four candidates

candidates	t_1			t_2			t_3		
	c_1	c_2	c_3	c_1	c_2	c_3	c_1	c_2	c_3
a_1	s_6	s_4	s_4	s_5	s_3	s_3	s_5	s_3	s_1
a_2	s_3	s_5	s_3	s_2	s_5	s_1	s_0	s_5	s_0
a_3	s_4	s_3	s_5	s_2	s_1	s_2	s_1	s_0	s_2
a_4	s_5	s_4	s_5	s_4	s_2	s_4	s_2	s_1	s_4

By (12), the decision information of table 1 can be transformed into error loss matrix of different situation, shown as (15), (16), (17) (The detail derivation is given in appendix).

$$L^{(1)} = \begin{pmatrix} (s_6^l,0) & (s_5^l,0) & (s_5^l,0) \\ (s_3^l,0) & (s_6^l,0) & (s_4^l,0) \\ (s_4^l,0) & (s_4^l,0) & (s_6^l,0) \\ (s_5^l,0) & (s_5^l,0) & (s_6^l,0) \end{pmatrix} \qquad (15)$$

$$L^{(2)} = \begin{pmatrix} (s_6^l,0) & (s_4^l,0) & (s_5^l,0) \\ (s_3^l,0) & (s_6^l,0) & (s_3^l,0) \\ (s_3^l,0) & (s_2^l,0) & (s_4^l,0) \\ (s_5^l,0) & (s_3^l,0) & (s_6^l,0) \end{pmatrix} \qquad (16)$$

$$L^{(3)} = \begin{pmatrix} (s_6^l,0) & (s_4^l,0) & (s_3^l,0) \\ (s_1^l,0) & (s_6^l,0) & (s_2^l,0) \\ (s_2^l,0) & (s_1^l,0) & (s_4^l,0) \\ (s_3^l,0) & (s_2^l,0) & (s_6^l,0) \end{pmatrix} \qquad (17)$$

By (13), the comprehensive error loss matrix can be obtained as (18).

$$L = \begin{pmatrix} (s_5^l,0.2) & (s_5^l,-0.1) & (s_4^l,-0.1) \\ (s_4^l,0.4) & (s_4^l,-0.1) & (s_3^l,0) \\ (s_5^l,0) & (s_3^l,0.2) & (s_3^l,0.7) \\ (s_6^l,-0.5.) & (s_5^l,-0..1) & (s_4^l,0.2) \end{pmatrix} \qquad (18)$$

By (14), the expected error loss values of alternative strategies a_1~a_4 are $L_1^e = (s_5^l,-0.34)$, $L_2^e = (s_4^l,-0.27)$, $L_3^e = (s_4^l,0.41)$, $L_4^e = (s_5^l,-0.19)$, obviously $a_4 \succ a_1 \succ a_2 \succ a_3$.

5. Conclusion

For the complexity of the physical world and the ambiguity of understanding, decision information can hardly be described quantitatively. Moreover, the size of the error-loss is a essential element affecting the decision making. Hence, in this paper, in order to reducing and avoiding the error loss, considering the attribute value and the state possibility is a multi-attribute decision making problem, a method that transforms the linguistic assessing information of attribute into error-loss linguistic information is proposed and the EET-WA Operator is proposed based on expected utility theory and T-WA Operator. Finally information integrated by 2-tuple linguistic represents the model. The research expands the application area of error eliminating and provides a new idea and method to look at similar problems.

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Appendix

Here，how the attribute value (s_4) of attribute c_2 in alternative strategies a_1 under state t_1 be change into error loss value is used as an example to explain the equations (15) to (17).

Form the above application example, we can know that because the cardinality of linguistic assessment set S is q+1=7，so q=6; the maximum assessing value of attribute c_2 under state t_1 is $\overline{x}_2^{(1)} = \max\left(x_{1,2}^{(1)}, x_{2,2}^{(1)}, x_{3,2}^{(1)}, x_{4,2}^{(1)}\right) = \max(s_4, s_5, s_3, s_4) = s_5$; the attribute value of attribute c_2 in alternative strategies a_1 under state t_1 is $x_{1,2}^{(1)} = s_4$. By Equation (1), the 2-tuple linguistic form of $\overline{x}_2^{(1)}$ and $x_{1,2}^{(1)}$ can be expressed as $(\overline{x}_2^{(1)}, \alpha_2^{(1)}) = (s_5, 0)$ and $(x_2^{(1)}, \alpha_{1,2}^{(1)}) = (s_4, 0)$. After q=6, $(\overline{x}_2^{(1)}, \alpha_2^{(1)}) = (s_5, 0)$, $(x_2^{(1)}, \alpha_{1,2}^{(1)}) = (s_4, 0)$ are substituted into Equation(12), the $\left(l_{1,2}^{(1)}, \widetilde{\alpha}_{1,2}^{(1)}\right) = \Delta\left(7 - \left(\Delta^{-1}(s_5, 0) - \Delta^{-1}(s_4, 0)\right)\right)$ is determined. Combining (2) and (3), it can easy know that $\left(l_{1,2}^{(1)}, \widetilde{\alpha}_{1,2}^{(1)}\right) = (s_5^l, 0)$. Similarly, the error loss matrix $L^{(1)}$ of state t_1, the error loss matrix $L^{(2)}$ of state t_2 and the error loss matrix $L^{(3)}$ of state t_3 can easily be computed.