Advances in Modelling and Analysis C
Vol. 73, No. 3, September, 2018, pp. 88-90
Journal homepage: http://iieta.org/Journals/AMA/AMA_C

Pareto optimal solutions for fractional multi-objective optimization problems using MATLAB

Tarek H.M. Abou-El-Enien

Department of Operations Research & Decision Support, Faculty of Computers & Information, Cairo University, 5 Dr. Ahmed Zoweil St.- Orman, Giza 12613, Egypt

Corresponding Author Email: t.hanafy@fci-cu.edu.eg

http//doi.org/10.18280/ama_c.730303

Received: 7 July 2018
Accepted: 18 August 2018

Keywords:
MATLAB®, Multi-objective Optimization problems, fractional programming, weighting method, Charens - Cooper transformation method

ABSTRACT

The purpose of this paper is to generate numerical Pareto optimal solutions for fractional multi-objective optimization (FMO) problems based on the Charens - Cooper transformation method and the weighting method using MATLAB® (R2014a). I introduce a MATLAB® code and a numerical hybrid algorithm for solving FMO problems. Also, i give an illustrative numerical example to clarify the main results developed in this paper. The hand solution of the numerical example and the solution by the code give the same results. The scientists and the engineers can apply the introduced code and the numerical hybrid algorithm to different practical FMO problems to obtain numerical solutions.

1. INTRODUCTION

Multi-objective optimization (MO) problems are a large class in optimization [1-8]. Fractional multi-objective optimization (FMO) problems are an important subclass of MO problems, [9-13]. The main objective of this paper is to present a numerical approach using MATLAB® [14-17], to solve fractional linear multi-objective optimization (FLMO) problems based on the Charens - Cooper transformation method [18], and the weighting method [2, 4, 7].

D. Bhati et al. present a review on FMO problems [11]. F. Pramy and Md. A. Islama present a modified method to find the Pareto optimal solutions of multi-objective linear fractional programming (MOLFP) problems [12]

In the following section, formulation of the FLMO problem and basic definitions are given. A computational hybrid algorithm based on the Charens - Cooper transformation method and the weighting method for solving the FLMO problems is given in section (3). A hand solution of an illustrative numerical example for the FLMO problem is introduced in section (4). A MATLAB® code and its run for the same numerical example (in section (4)) is presented in section (5). Finally, conclusions and future works are given in section (6).

2. FRACTIONAL LINEAR MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

Consider the following FLMO problem:

$$Maximize\ Q(X) = \left(Q_1(X), Q_2(X), \dots\dots, Q_k(X)\right) \qquad (1-1)$$

subject to

$$X \in \mathbb{X}=\{ X \in \mathbb{R}^n: AX \le b, X \ge 0\}, \qquad (1-2)$$

where the i^{th} objective function can be written as follows:

$$Q_i(X) = \frac{f_i(X)}{h(X)} = \frac{C_i X+\gamma_i}{DX+\beta}, i = 1,2, \dots., k, \qquad (2)$$

and
k : the number of objective functions,
m : the number of constraints,
n : the number of variables,
$\mathbb{R}$: the set of all real numbers,
X : an n-dimensional column vector of variables,
C_i : an n-dimensional row vector of constants, $i = 1,2, \dots., k$,
D : an n-dimensional row vector of constants,
A : an $(m \times n)$ coefficient matrix,
b : an m-dimensional row vector of constants,
γ_i and β : are constants , $i = 1,2, \dots., k$,
$\mathbb{N} = \{1,2, \dots., n\}$,
$\mathbb{R}^n = \{X = (x_1, x_2, \dots., x_n)^T: x_i \in \mathbb{R}, i \in \mathbb{N}\}$,

Furthermore, we assume that $DX + \beta$ is everywhere positive on the set of non-negative integers. The Pareto optimal solution for problem (1) can be defined as follows, [2, 4, 7, 12]:

Definition (1):
A point $X^* \in \mathbb{X}$ is said to be a Pareto optimal solution for the FLMO problem (1) , if and only if there does not exist another $X \in \mathbb{X}$, such that $Q_i(X) \ge Q_i(X^*)$, $(i = 1,2, \dots\dots, k)$, with strictly inequality holding for at least one i .

3. A COMPUTATIONAL HYBRID ALGORITHM FOR THE FLMO PROBLEMS

To find a Pareto optimal solution for the FLMO problem (1), a computational hybrid algorithm based on the Charens - Cooper transformation method and the weighting method is introduced as follows:

A computational hybrid algorithm:

Step (1): Use the nonnegative weighted sum approach to transform the FLMO problem (1) to the following problem:

$$Maximize \ \ \sum_{i=1}^{k} w_i Q_i(X)$$
Subject to (3)
$$X \in \mathbb{X}$$

where $w_i \geq 0, \quad i = 1,2,\dots,k, \quad \sum_{i=1}^{k} w_i = 1.$

Step (2): Use Charnes-Cooper transformation method by making the variable change:

$$\mu = \frac{1}{DX+\beta}, \tag{4-1}$$

with the additional variable changes:

$$Y = X\mu. \tag{4-2}$$

Step (3): Use equations (2, 4-1, 4-2) to transform problem (3) to the following equivalent problem:

$$Maximize \ \ \sum_{i=1}^{k} w_i \, Q_i(Y,\mu) = \sum_{i=1}^{k} w_i \, (C_i Y_i + \gamma_i \mu)$$
subject to (5)
$$X \in \mathbb{X} = \{ X \in \mathbb{R}^n : AY - b\mu \leq 0,$$
$$DY + \beta\mu = 1,$$
$$Y \geq 0, \mu > 0.\}$$

where $w_i \geq 0, \quad i = 1,2,\dots,k, \quad \sum_{i=1}^{k} w_i = 1$.

Step (4): Let $w_i = w_i^* \geq 0, \ i = 1,2,\dots,k,$ then use the simplex (or any other method) to obtain the optimal solution $X^* = \frac{Y^*}{\mu^*}$.

Step (5): Let X^* is the optimal solution to problem (5):

(i) If $w_i > 0$, for all i, thus X^* is a Pareto optimal solution, go to step (6)

(ii) If $w_i \geqq 0$, for all i , and X^* is a unique for problem (5), thus X^* is a Pareto optimal solution, go to step (6)

(iii) If $w_i \geqq 0$, for all i, and there are alternative solutions, thus, use the non-inferiority test.

Step (6): Stop.

4. ILLUSTRATIVE EXAMPLE FOR THE FLMO PROBLEMS:

Consider the following FLMO problem (let $w_1^* = w_2^* = 0.5$):

$$Maximize \ \ Q_1(x_1,x_2) = \frac{2x_1 + 2x_2}{5x_1 + 4x_2 + 4}$$
$$Minimize \ \ Q_2(x_1,x_2) = \frac{-2x_1 - 3x_2}{5x_1 + 4x_2 + 4}$$
subject to (6)
$$x_1 + 2x_2 \leq 7, \qquad 2x_1 \leq 5,$$
$$3x_2 \leq 13, \quad x_1, \ x_2 \geq 0,$$

Use the computational hybrid algorithm to solve the above problem.

Solution:

Let $$\mu = \frac{1}{5x_1+4x_2+4} \tag{7}$$

Use equation (6) and problem (2) to write the following problem:

$$Minimize \ Q(X) = w_1 Q_1 + w_2 Q_2$$
$$= [0.5(-2x_1 + 2x_2)\mu + 0.5(-2x_1 - 3x_2)\mu \,]$$
Subject to (8)
$$(5x_1 + 4x_2 + 4)\mu = 1 \ , \ \ (x_1 + 2x_2)\mu \leq 7\mu,$$
$$2x_1\mu \leq 5\mu, \qquad 3x_2\mu \leq 13\mu,$$
$$\mu > 0, \qquad x_1, \ x_2 \geq 0.$$

Let $y_1 = x_1\mu$ and $y_2 = x_2\mu$. Thus, problem (8) can be written as follows:

$$Minimize \ Q(y_1,y_2) = [-2y_1 - 2.5y_2 \,]$$
subject to
$$5y_1 + 4y_2 + 4\mu = 1, \quad y_1 + 2y_2 \leq 7\mu,$$
$$2y_1 \leq 5\mu, \qquad 3y_2 \leq 13\mu,$$
$$\mu > 0, \qquad y_1, \ y_2 \geq 0.$$

By solving the above linear programming problem, the optimal solution is:

$$Q^* = Q(y_1,y_2) \cong -0.49 \ \ , y_1^* \cong 0 \ \ , y_2^* \cong 19 \ \ , \ \mu^* \cong 0.06,$$
$$x_1^* \cong 0 \ \ , x_2^* \cong 3.16 \quad .$$

Also, to find a Pareto optimal solution to the above example, a MATLAB code based on the computational hybrid algorithm is introduced in the following section.

5. MATLAB CODE FOR THE FLMO PROBLEMS

In this section, a MATLAB program and its run for the given example in section (4) is presented as follows:

```
%weight of the first objective
w1=0.5;
%weight of the second objective
w2=0.5;
%numerator of the first objective, max
obj1_n=[2 , 2 ];
%numerator of the second objective, Min
obj2_n=[-2 , -3 ];
%the common denominator of the two objective functions
obj1_2_d=[5, 4, 4];
%LHS of the constraints
LHS_C=[1 2  ; 2 0  ; 0 3 ];
%RHS of the constraints
RHS_C=[7;5;13];
%add the coefficient of the new variable
%meu to the first objective
obj1_n(end+1)=0;
%add the coefficient of the new variable
%meu to the second objective
obj2_n(end+1)=0;
%add to the LHS of all the constraints
```

```
%the coefficient of the new variable meu
LHS_C(:,end+1)=0;
%multiply the objective functions by
%the weights and -1, since they are
%maximization, and add them together
%to have a single objective
obj=-1*w1*obj1_n +w2*obj2_n;
%get the RHS to the LHS by negative sign ,
%because it now contains variable meu
LHS_C(:,3)=-1*RHS_C;
%now the RHS is zero
RHS_C(:,1)=0;
%add the new equality constraint of the meu
%as 2 equations, one greater than and one less than
N_LHS_C=cat(1,LHS_C, obj1_2_d,-1*obj1_2_d);
% add the RHS of the two new constraints which is 1 & -1
N_RHS_C=cat(1,RHS_C,[1;-1]);
%solve the new problem using linear programming function
lb=zeros(3,1);
[x,fval]=linprog(obj,N_LHS_C,N_RHS_C,[],[],lb);
fprintf('The optimal objective function value :')
%The new equation is now min, no need to multiply by -1
z=fval
fprintf('The optimal values of the decision variables')
x
```

RUN

```
Optimization terminated.
The optimal objective function value:
z =
  -0.4861
The optimal values of the decision variables
x =
   0.0000
   0.1944
   0.0556
```

6. CONCLUSIONS AND FUTURE WORKS

In this paper, a computational hybrid algorithm and a MATLAB program based on the Charens - Cooper transformation method and the weighting method to generate Pareto optimal solutions for the FLMO problems are introduced. By comparing the results in sections 4 and 5: the hand solution of the numerical example by the computational hybrid algorithm and the solution by the software is the same.

The scientists and the engineers can apply the presented MATLAB program and the hybrid algorithm to different practical FLMO problems to obtain numerical Pareto optimal solutions.

Several algorithms and codes can be built to solve different kinds of FLMO problems using various methods to obtain numerical Pareto optimal solutions.

REFERENCES

[1] Abo–Sinna MA, Abou-El-Enien THM. (2005). An algorithm for solving large scale multiple objective decision making problems using TOPSIS approach. AMSE Journals, Advances in Modelling and Analysis A 42(6): 31-48.

[2] Chankong V, Haimes YY. (1983). Multiobjective decision making: Theory and methodology. North-Holand Series in Science and Engineering, North-Holand, USA,

[3] El-Sawy AA, El-Khouly NA, Abou-El-Enien THM. (2000). An algorithm for decomposing the parametric space in large scale linear vector optimization problems: A fuzzy approach. AMSE Journals, Advances in Modelling and Analysis C 55(2): 1-16.

[4] Hwang CL, Masud ASM. (1979). Multiple objective decision making methods and applications. Springer-Verlag, New York, USA. https://doi.org/10.1007/978-3-642-45511-7

[5] Osman MSA, Hasan SA, Abou-El-Enien THM, Mohamed SM. (2005). Multiple criteria decision making-theory, applications and softwares: A literature review. AMSE Journals, Advances in Modelling and Analysis B 48(2): 1-38.

[6] Sharma S, Bhattacharjee S, Bhattacharya A. (2015). Solution of multi-objective optimal DG placement problems using swine influenza model based optimization (SIMBO). AMSE Journals, Advances C 70(1): 46-70.

[7] Zeleny M. (1982). Multiple Criteria Decision Making. McGraw-Hill, New York.

[8] Sen GD, Sharma J, Goyal GR, Singh AK. (2017). A Multi-objective PSO (MOPSO) algorithm for optimal active power dispatch with pollution control. IIETA journals, Mathematical Modelling of Engineering Problems 4(3): 113-119. https://doi.org/10.18280/mmep.040301

[9] Abou-El-Enien THM, Saad OM. (2010). On the solution of a special type of large scale linear fractional multiple objective programming problems with uncertain data. Applied Mathematical Sciences 4(62): 3095–3105.

[10] Abou-El-Enien THM. (2014). Interactive TOPSIS algorithm for a special type of linear fractional vector optimization problems. International Journal of Information Technology and Business Management 31(1): 13-24.

[11] Bhati D, Singh P, Arya R. (2017). A taxonomy and review of the multi-objective fractional programming (MOFP) Problems. International Journal of Applied and Computational Mathematics, September 3(3): 2695–2717.

[12] Pramy FA, Islam MdA. (2017). Determining efficient solutions of multi-objective linear fractional programming problems and application. Open Journal of Optimization 6(4): 164-175. https://doi.org/10.4236/ojop.2017.64011

[13] Saad OM, Abou-El-Enien THM. (2008). On the solution of a special type of integer linear fractional multiple objective programming problems. AMSE Journals, Advances in Modelling and Analysis D 13(1): 42-55.

[14] Branch MA, Grace A. (1996). Optimization Toolbox, User's Guide. The MathWorks Inc.

[15] Lopez CP. (2014). MATLAB optimization techniques. Apress Academic, Springer, New York, USA,

[16] Venkataraman P. (2009). Applied Optimization with MATLAB Programming. 2ed, John Wiley & Sons, Inc, New Jersey, USA.

[17] Works M. (2010). MATLAB Reference Guide. The MathWorks Inc., Natick.

[18] Charnes A, Cooper WW. (1962). Programming with linear fractional criteria. Naval Res. Logist Quart. 9: 181–186.