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Optimal Disturbance Rejection via Feedforward-PD for Bilinear Systems with External Sinusoidal Disturbances

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Abstract

The optimal control problem is considered for bilinear systems affected by sinusoidal disturbances. A feedforward-PD optimal disturbance rejection control law based on output feedback of LQR is developed. By analysis the optimal disturbances rejection algorithms of bilinear systems based on time-domain state-space representation and the PD-control algorithms based on frequency-domain output feedback, the optimal PD tuning parameters are derived to achieve the purpose of optimal disturbances rejection. Finally, simulation results show that the algorithm is high in efficiency, easy to implement and well robust with respect to external sinusoidal disturbances.

Key words

Bilinear System, Feedforward-PD Control, Disturbance Rejection, Dynamic Compensation

1. Introduction

Stability is an important performance for the control system, but there are various external disturbances affecting the performance of control system. Sinusoidal disturbance is one of the common disturbances widely existed in the engineering practices. How to rejection the external sinusoidal disturbance is a common problem in the system design. One method is feedforward-feedback optimal disturbance rejection control algorithms based on state-feedback. But, the state observer is often physically not realizable in practices. Although we can observe the system status by constructing the state observer using the methods of hardware or software sometimes, this may make the system complex, less stable, affect the dynamic performance of the original system and take into the interference noise [1]. All these have hampered the application of the optimal control based on state observer in practices. The PID controller has widely used yet the optimal turning parameters are hard to determine by using empirical methods. Actually, for the same controlled object, there must be some link. Based on this theory, Mao and Wu in Ref. [2] gave a PD dynamic compensation algorithm based on output feedback in order to reconstruct the system state information.

Zhang and Hoagg in Ref. [3] use a candidate-pool approach to identify the feedback and feedforward transfer function matrices, while guaranteeing asymptotic stability of the identified closed-loop transfer function matrix. Jajarmi et al. in Ref. [4] using a method of successive substitution is employed to convert the original time-delay optimal control problem into a sequence of linear time-invariant ordinary differential equations (ODEs) without delay and advance terms. Pan et al. in Ref. [5] presents an efficient hybrid feedback feedforward (HFF) adaptive approximation-based control (AAC) strategy for a class of uncertain Euler–Lagrange systems. Peng et al. in Ref. [6] provide a novel internal model based robust inversion feedforward and feedback 2DOF control approach was proposed for LPV system with disturbance and it was combines the internal model control and robust inversion based 2DOF control, it utilizes internal model based control to reject external disturbance, utilizes robust inversion 2DOF control to enhance the control resolution and guarantee the system control performance. This method is proofed to be feasible and can achieve the optimal control law in practices.

This paper gave a optimal disturbance rejection control algorithm based on feedforward-PD compensation [7], Proofed that this algorithm can get the optimal PD turning parameters of the optimal dynamic compensation network by a simple theoretical calculation, and achieved the objective of optimal control for the nonlinear system with sinusoidal disturbances [8]. This method has a good robustness with respect to sinusoidal disturbances, and solved the problem which was hard to obtain optimal PD turning parameters via empirical methods in practice.

2. Problem Statement

Consider bilinear systems with sinusoidal disturbances on $[0,\infty]$ described by

$$\dot{x}(t) = Ax(t) + \{Nx(t)\}u(t) + Bu(t) + Dv(t)$$
$$y = h(x) = Cx(t)$$
$$x(t_0) = x_0$$
$$\{Nx(t)\} = \sum_{j=1}^{n} N_j x_j(t) \qquad (1)$$

where $x \in R^n$, $u \in R^r$, $v \in R^p$,and $y \in R^m$ are the state vector, the control vector, the external disturbance vector, and the output vector, respectively. A, B, C and D are real constant matrices of appropriate dimensions. x_j is the j-th component of the vector x . $\{Nx\}u$ is bilinear term [9].

Transferred the bilinear system as a general expression of nonlinear systems

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + Dv(t)$$
$$y = h(x(t)) = Cx(t) \qquad (2)$$

where $f(x(t)) = Ax(t)$, $g(x(t)) = Nx(t) + B$, f, g is continuously differentiable function.

3. Linear Design

To ensure that the control problem is well posed, let $F(x,u) = f(x(t)) + g(x(t))u(t)$, and assume nonlinear function $F(x,u), h(x)$ is fully smooth, satisfies the Lipschitz condition on $R^n \times R^r$. $v(t) \in R^m$ is an external disturbance vector [10]. Hypothesis origin is the equilibrium point, that is $F(0,0) \equiv 0$. With the Maclaurin expansion method, the system (1) may be transformed as:

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) + Dv(t) + p(x(t), u(t))$$
$$y(t) = \tilde{C}x(t) + q(x(t)) \quad t > 0$$
$$x(t_0) = x_0 \qquad (3)$$

where $p(\cdot,\cdot)$ and $q(\cdot)$ are nonlinear function vector and the order number is greater than (1). Jacobin matrixes $\tilde{A} = \partial F(0,0)/\partial x^T, \tilde{B} = \partial F(0,0)/\partial u^T, \tilde{C} = \partial F(0)/\partial x$. Then, we linearize the nonlinear function at the origin:

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) + Dv(t), \quad t > 0$$
$$y(t) = \tilde{C}x(t)$$
$$x(0) = x_0 \qquad (4)$$

where $x \in R^n$, $u \in R^r$, $v \in R^p$, and $y \in R^m$ are the state vector, the control vector, the external disturbance vector, and the output vector, respectively. $\tilde{A}, \tilde{B}, \tilde{C}$ and D are real constant matrices of appropriate dimensions.

Assumption 1: The sinusoidal disturbances can be expressed as [11]:

$$v(t)=\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_p \end{bmatrix}=\begin{bmatrix} a_1\sin(w_1t+\varphi_1) \\ a_2\sin(w_2t+\varphi_2) \\ \vdots \\ a_p\sin(w_pt+\varphi_p) \end{bmatrix} \quad (5)$$

where frequencies $w_i (i=1,2,\cdots,p)$ are known and $-\pi < w_1 \le w_2 \le \cdots \le w_p \le \pi,$ amplitudes a_i and phases $\varphi_i (i-1,2,\cdots,p)$ are measurable.

Because the system (1) subject to external sinusoidal disturbances, the steady-state values will not tend to constants, so we select the average infinite time quadratic performance index:

$$J=\lim_{T\to\infty}\frac{1}{T}\int_0^T (x^TQx+u^TRu)dt \quad (6)$$

where $Q\in R^{n\times n}$ and $R\in R^{n\times n}$ are positive semi-definite and positive definite matrix respectively, and satisfy the usual conditions for the optimal regulator.

4. Optimal Control Law

According to the optimality necessary conditions based on maximum principle, the optimal disturbance rejection control problems of system (4) and (5) with the quadratic performance (6) lead the following two-point boundary value (TPBV) problems

$$-\dot{\lambda}(t)=Qx(t)+\tilde{A}^T\lambda(t) \quad (7)$$

$$\dot{x}(t)=\tilde{A}x(t)+\tilde{B}u(t)+Dv(t) \quad (8)$$

$$\lambda(\infty)=0, x(0)=0 \quad (9)$$

And the optimal control law can be expressed as

$$u^*(t)=-R^{-1}\tilde{B}^T\lambda(t) \quad (10)$$

In order to solve the TPBV problems (7), (8) and (9), let

$$\lambda(t)=Px(t)+P_1v(t)+P_2v_m(t) \quad (11)$$

taking the derivatives to the sides in (11), we can get

$$\dot{\lambda}(t) = \dot{P}x(t) + P\dot{x}(t) + \dot{P}_1 v(t) + P_1\dot{v}(t) + \dot{P}_2 v_m(t) + P_2\dot{v}_m(t)$$
$$= (\dot{P} + P\tilde{A} - PSP)x(t) + (PD + \dot{P}_1 - P_2\Omega^2 - PSP_1)v(t) + (P_1 + \dot{P}_2 - PSP_2)v_m(t) \quad (12)$$

where $S = \tilde{B}R^{-1}\tilde{B}^T$, and $v_m(t)$ can be described as

$$\dot{v}(t) = v_m(t) = \begin{bmatrix} w_1 a_1 \cos(w_1 t + \varphi_1) \\ w_2 a_2 \cos(w_2 t + \varphi_2) \\ \vdots \\ w_p a_p \cos(w_p t + \varphi_p) \end{bmatrix} \quad (13)$$

$$\ddot{v}(t) = \dot{v}_m(t) = \begin{bmatrix} -w_1 a_1 \sin(w_1 t + \varphi_1) \\ -w_2 a_2 \sin(w_2 t + \varphi_2) \\ \vdots \\ -w_p a_p \sin(w_p t + \varphi_p) \end{bmatrix} = -\Omega^2 v(t) \quad (14)$$

From the equations (7), (12) and (13), the following equation can be derived

$$(\dot{P} + P\tilde{A} - PSP + Q + \tilde{A}^T P)x(t) + (PD + \dot{P}_1 - P_2\Omega^2 - PSP_1 + \tilde{A}^T P_1)v(t) + (P_1 + \dot{P}_2 - PSP_2 + \tilde{A}^T P_2)v_m(t) = 0 \quad (15)$$

Because of selecting either $x(t), v(t)$ and $v_{m(t)}$ is all hold, so we can get matrix differential equations (16), (17), (18).

$$\dot{P} + P\tilde{A} - PSP + Q + \tilde{A}^T P = 0, \dot{P}(\infty) = 0 \quad (16)$$

$$PD + \dot{P}_1 - P_2\Omega^2 - PSP_1 + \tilde{A}^T P_1 = 0, \dot{P}_1(\infty) = 0 \quad (17)$$

$$P_1 + \dot{P}_2 - PSP_2 + \tilde{A}^T P_2 = 0, \dot{P}_2(\infty) = 0 \quad (18)$$

It is well known that P is the unique semi-definite solution of the *Riccati* matrix differential equation[12-13] (16). Substituting P into the matrix differential equations (17), (18), we can obtain the exclusive solutions of P_1 and P_2. Therefore, according to (10), the optimal disturbances rejection control law $u^*(t)$ can be expressed as

$$u^*(t) = -R^{-1}\tilde{B}^T\lambda(t) = -R^{-1}\tilde{B}^T(Px(t) + P_1 v(t) + P_2 v_m(t)) \quad (19)$$

To sum up, we can get the structure of the system as the Figure 1.

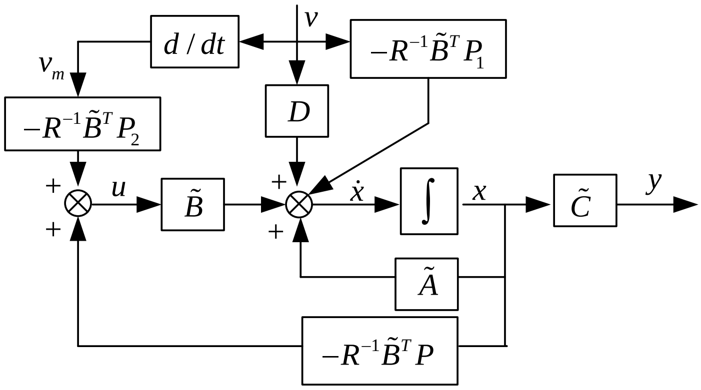

Fig.1. The structure of the optimal control system based on feedforward-feedback

5. PD Feedback Compensation Network

Because the output is measurable, the system closed-loop performance indicators can be directly or indirectly reflected in the frequency domain, and also in order to facilitate comparison of the two kinds of control mode, we discuss the question in the frequency domain. According to superposition principle of linear systems, when studying the closed-loop transfer function based on the input and output, we can ignore the external disturbances. All in all, assuming the system initial state is zero, we can get the following system structure, where $K=-R^{-1}\tilde{B}^T P$.

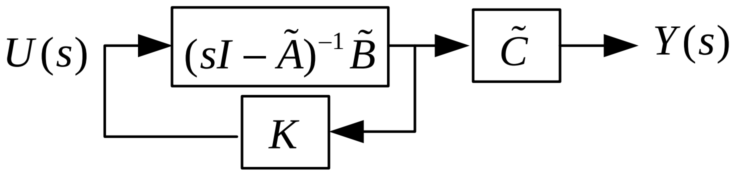

Fig.1. The structure of the system without disturbances

The closed-loop transfer function matrix of the system can be drawn from figure 2

$$We(s)=\frac{\tilde{C}(sI-\tilde{A})^{-1}\tilde{B}}{1-K(sI-\tilde{A})^{-1}\tilde{B}}$$

(2 0)

The figure 2 based on system state-feedback can be transformed into the following structure based on the output of the system where $G(s)$ is the PD-controller.

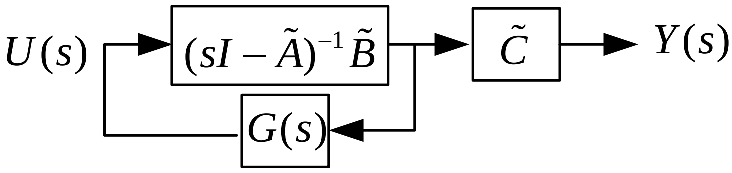

Fig.3 PD closed-loop control system

The system closed-loop function matrix is

$$We(s)=\frac{Y(s)}{U(s)}=\frac{\tilde{C}(sI-\tilde{A})^{-1}\tilde{B}}{1-G(s)\tilde{C}(sI-\tilde{A})^{-1}\tilde{B}}$$

(21)

So as to make the two kinds of control algorithms have same dynamic performances, let the matrix equation (20) equal to matrix equation (21), then the following PD-controller transfer function matrix can be obtained.

$$G(s)=\frac{K(sI-\tilde{A})^{-1}\tilde{B}}{\tilde{C}(sI-\tilde{A})^{-1}\tilde{B}}$$

(22)

Based on the above, for the time-domain state-space representation of determinate systems, the optimal state observer K can be obtained via Linear Quadratic Optimal Control Algorithm. Substituting K into (22), the $G(s)$ can be obtained. According to $G(s)$, the proportional parameters K_P and the differential time constant parameters T_D of the PD-controller can be determined. The structure of the feedforward-PD optimal disturbance rejection control system based on external sinusoidal disturbances is showed as

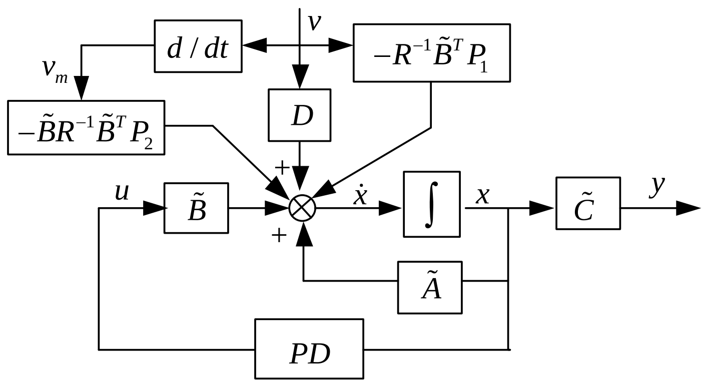

Fig.4 The structure of optimal control system based on feedforward-PD compensation

According to the Figure 4, the following equations can be derived

$$y(t) = \tilde{C}x(t) \tag{23}$$

$$K_P y(t) + K_P T_D \frac{dy(t)}{dt} = u(t) \tag{24}$$

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}u(t) + (D - SP_1)v(t) - SP_2 v_m(t) \tag{25}$$

Then we can obtain the optimal control law and the system status equation by calculating equations (23), (24), (25).

$$u^*(t) = Mx(t) + Nv(t) + Zv_m(t) \tag{26}$$

$$\dot{x}(t) = (\tilde{A} + \tilde{B}M)x(t) + (\tilde{B}N - SP_1)v(t) + (\tilde{B}Z - SP_2)v_m(t) \tag{27}$$

where $M = L*\left(K_P\tilde{C} + K_P T_D \tilde{C}\tilde{A}\right)$, $N = LK_P T_D *\left(\tilde{C}D - \tilde{C}SP_1\right)$, $Z = LK_P T_D \tilde{C}SP_2$, $L = \left(I - K_P T_D \tilde{C}\tilde{B}\right)^{-1}$.

In conclusion, there are several advantages of the algorithm:

It is effective to reject the external sinusoidal disturbances.

Transforming the time-domain result into the frequency-domain's is in line with the traditional engineering design methods.

The whole design process is very convenient by using the special LQR functions of the MATLAB.

6. Example and Simulation

Consider the bilinear system, its parameters as follows [14]:

$$A = \begin{bmatrix} 0 & 1 \\ -0.5 & 1.2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [2 \quad 0],\ D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix},\ N1 = \begin{bmatrix} 0.6 \\ 0.2 \end{bmatrix},\ N2 = \begin{bmatrix} 0.5 \\ 0.8 \end{bmatrix} \tag{28}$$

The system linearization at the origin, we get

$$\tilde{A} = \begin{bmatrix} 0 & 1 \\ -0.5 & 1.2 \end{bmatrix},\ \tilde{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \tilde{C} = [2 \quad 0],\ D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{29}$$

The sinusoidal disturbances can be expressed as

$$v(t) = \begin{bmatrix} 2\sin 0.5t \\ \sin(t + 0.25\pi) \end{bmatrix} \tag{30}$$

Select the average infinite time quadratic performance index as (5), its parameters as follows,

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, R = 1$$

(3 1)

The optimal PD compensation network via optimal adapting is $G(s) = -0.309 - 1.56s$. In the simulations, select the discrete time interval $t = 0.1$, time span $T = 30$ to approximate infinite time. With respect to performance index (6), the simulation comparative curves of x , u and J as follows: the feedforward-feedback optimal control algorithm; the feedforward-PD optimal control algorithm.

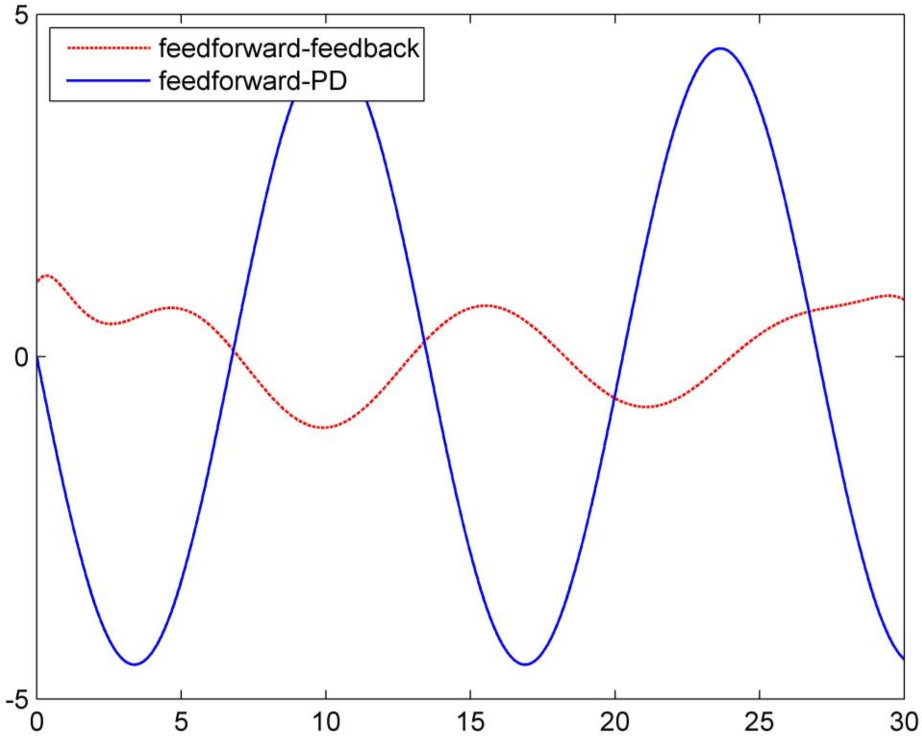

Fig.5 Comparative curves of system state x1

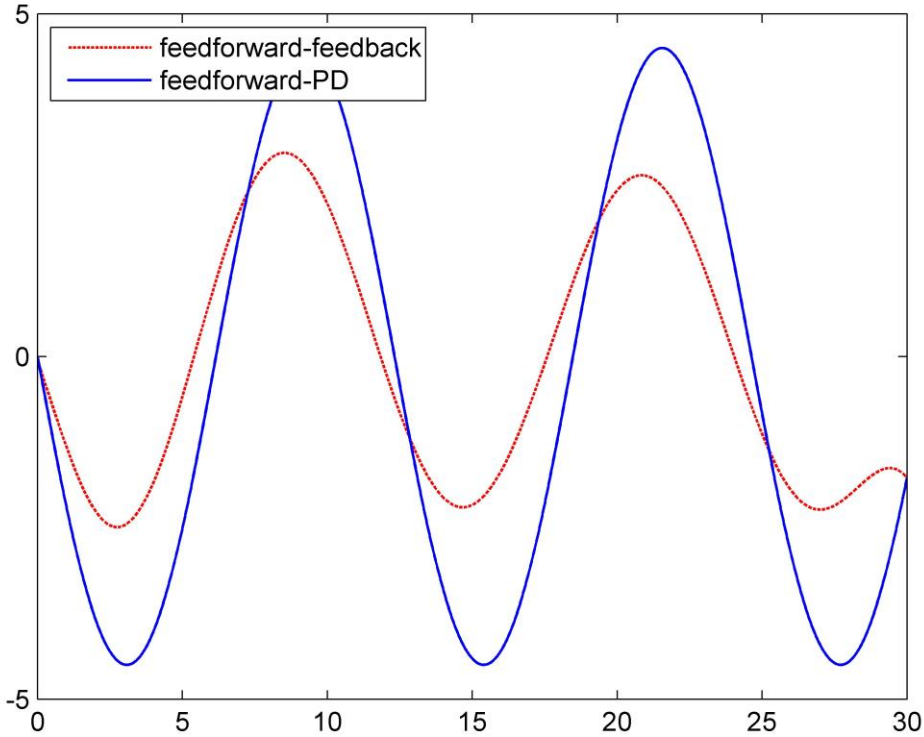

Fig.6 Comparative curves of system state x2

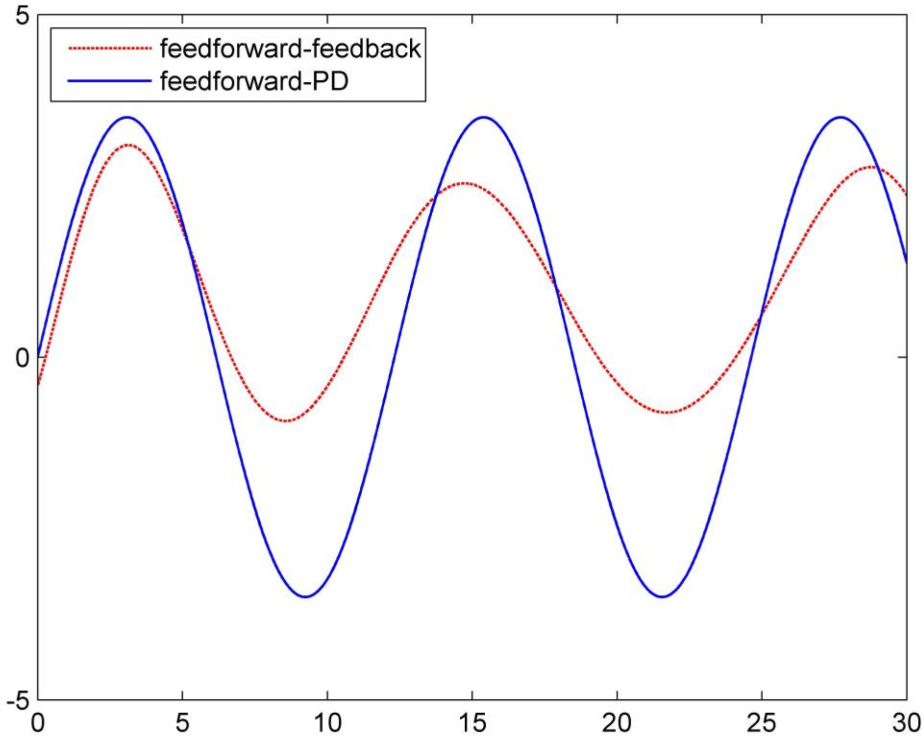

Fig .7 Comparative curves of optimal control law

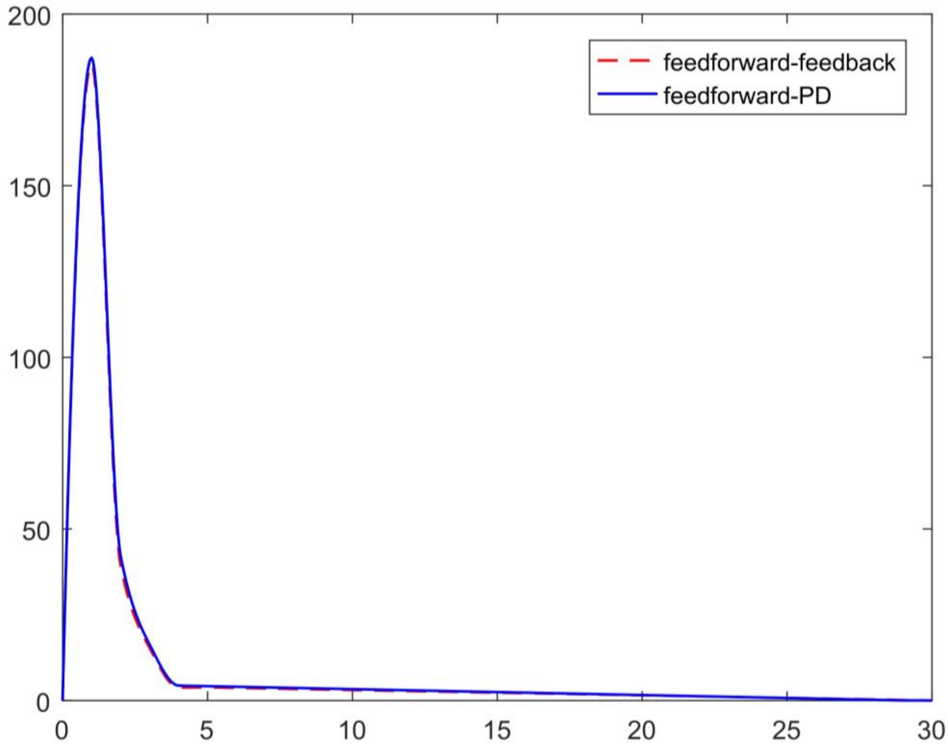

Fig .8 Comparative curves of optimal performance index

From the simulation curves, it can be seen that the presented feedforward-PD optimal control algorithm is effective to the external sinusoidal disturbances, and its robustness can be comparable to the classic feedforward-feedback optimal control algorithm. It also proofed that the whole design is convenient to realize.

7. Conclusion

In this paper, we have presented a design scheme of the disturbance rejection control for bilinear systems affected by external sinusoidal disturbances. Design the optimal PD compensation network to realize the purpose of external sinusoidal disturbance rejection. Finally, simulation results show that the feedforward-PD optimal control is high in efficiency, easy to implement and well robust with respect to external sinusoidal disturbances. Future efforts should be directed towards proving kinds of nonlinear systems and towards proving the controllers leads to table and robust closed-loop systems.

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