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Static Output Feedback Variable Structure Control for a Class of Time-delay Systems

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Abstract

This paper considers the problem of static output feedback variable structure control for a class of time-delay systems. Attention is focused on the design of a static output feedback sliding mode surface and the variable structure controller. Based on Lyapunov stability theorem, in terms of linear matrix inequality (LMI), a sufficient condition for the solvability of this problem is presented which is dependent on the size of the time delay and can be solved by LMI toolbox in MATLAB. When the LMI is feasible, the explicit expression of the desired static output feedback sliding mode surface is also given. Then, a variable structure controller is obtained which make the systems states reach the sliding mode surface in finite time. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed approach.

Key words: Delay systems, static output feedback, variable structure control, linear matrix inequality (LMI)

1. Introduction

Time delay is frequently encountered in various engineering, communication, and biological systems. The characteristics of dynamic systems are significantly affected by the presence of time delays, even to the extent of instability in extreme situations. Therefore, the study of delay systems has received much attention, and various analysis and synthesis methods have been developed over the past years[1-6].

The variable structure control approach, based on using of discontinuous control laws, is known to be an efficient alternative way to tackle many challenging problems of robust stabilization. For instance, an appropriate sliding mode strategy can achieve stabilization by "dominating" nonlinear terms and additive disturbances, provided some appropriate "matching conditions" hold. However, the combination of delay phenomenon with relay actuators makes the situation much more complex: designing a sliding controller without taking delays into account may lead to unstable or chaotic behaviors or, at least, results in highly chattering behaviors. In [7], Chen considers the problem of sliding mode control for discrete-time multi-input multi-output systems. Fuzzy sliding mode controller design for uncertain time-delayed system has been studied in [8, 9,10]. Recently, the problems of variable structure control for uncertain discrete-time systems with delays have been studied[11–16].Sliding mode control for time-delay discrete systems with unmatched uncertainty and multi-input discrete systems has been present in [14, 15]. In [16], Zhang considers the quasi-sliding mode variable structure control for discrete linear constant system with time delay. But the results about static output feedback sliding mode control for delay systems have never been presented.

This paper presents the problem of static output feedback variable structure control for a class of nonlinear delay systems with norm-bounded uncertainties. Based on LMI approach, a new approach is given to design the static output feedback sliding mode surface. Then, a variable structure controller is obtained which make the systems states reach the sliding mode surface in finite time.

Notations: Throughout the paper, R^n denotes the n dimensional Eucliden space.

2. Problem Formulation

Consider the following nonlinear systems with delay

$$\begin{aligned} &\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) \\ &y(t) = Cx(t) \\ &x(t) = \psi(t) \qquad -d \le t \le 0 \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is systems state, $u(t) \in R^m$ is systems control input, $y(t) \in R^p$ is systems output. d is a systems state delay. $\psi(t)$ is the given initial state on $[-d, 0]$. $A \in R^{n\times n}$, $A_d \in R^{n\times n}$, $B \in R^{m\times n}$ and $C \in R^{p\times n}$ are known constant matrices. B has full column rank.

With Singular Value Decomposition of B,

$$B = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Omega \\ 0 \end{bmatrix} V^T,$$

a nonsingular transformation

$$T = \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix} \tag{2}$$

is constructed for systems (1) to make $TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$.

With the transformation $z(t) = Tx(t)$, the systems（1）can be rewritten as

$$\begin{aligned}\dot{z}(t) = \begin{bmatrix}\dot{z}_1(t)\\ \dot{z}_2(t)\end{bmatrix} &= TAx(t) + TA_d x(t-d) + TBu(t)\\ &= TAT^{-1}z(t) + TA_d T^{-1}z(t-d) + TBu(t)\end{aligned}$$

Inserting （2）into the above formulation, we obtain

$$\begin{aligned}\dot{z}_1(t) &= U_2^T AU_2 z_1(t) + U_2^T AU_1 z_2(t) + U_2^T A_d U_2 z_1(t-d) + U_2^T A_d U_1 z_2(t-d)\\ \dot{z}_2(t) &= U_1^T AU_2 z_1(t) + U_1^T AU_1 z_2(t) + U_1^T A_d U_2 z_1(t-d) + U_1^T A_d U_1 z_2(t-d) + B_2 u(t)\end{aligned} \qquad (3)$$

For the systems（3）, selecting the static output feedback sliding mode surface as following

$$\sigma(t) = Sy(t) \qquad (4)$$

With $\sigma(t) = Sy(t) = SCT^{-1}z(t) = SC[U_2 \;\; U_1]z(t) = SCU_2 z_1(t) + SCU_1 z_2(t) = 0$, by the assumption that SCU_1 is nonsingular, we obtain

$$z_2(t) = -(SCU_1)^{-1} SCU_2 z_1(t) = -Fz_1(t)$$

where $F = (SCU_1)^{-1} SCU_2$.

Inserting the above formulation into the systems（3）, the sliding mode equation is obtained

$$\dot{z}_1(t) = U_2^T A(U_2 - U_1 F)z_1(t) + U_2^T A_d (U_2 - U_1 F)z_1(t-d) \qquad (5)$$

3. Main Results

Lemma1 [5] The LMI $\begin{bmatrix} Y(x) & W(x)\\ * & R(x)\end{bmatrix} > 0$ is equivalent to

$$R(x) > 0, Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where $Y(x) = Y^T(x), R(x) = R^T(x)$ depend on x.

Theorem1 The sliding mode equation (5) is stable， if there exist positive-definite matrices $\tilde{P}, \tilde{Q} \in R^{(n-m)\times(n-m)}$, matrices $X, \tilde{N}_1, \tilde{N}_2, \tilde{N}_3 \in R^{(n-m)\times(n-m)}$, constants ρ_1, ρ_2 and matrix $Z \in R^{m\times(n-m)}$ such that the following LMI holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & d\tilde{N}_1\\ * & \Sigma_{22} & \Sigma_{23} & d\tilde{N}_2\\ * & * & \Sigma_{33} & d\tilde{N}_3\\ * & * & * & -d\tilde{Q}\end{bmatrix} < 0 \qquad (6)$$

where

$$\Sigma_{11} = \tilde{N}_1 + \tilde{N}_1^T - U_2^T A(U_2 X^T - U_1 Z) - (U_2 X^T - U_1 Z)^T A^T U_2$$

$$\Sigma_{12} = \tilde{N}_2^T - \tilde{N}_1 - U_2^T A_d (U_2 X^T - U_1 Z) - \rho_1 (U_2 X^T - U_1 Z)^T A^T U_2$$

$$\Sigma_{13} = \tilde{P} + \tilde{N}_3^T + X^T - \rho_2 (U_2 X^T - U_1 Z)^T A^T U_2$$

$$\Sigma_{22} = -\tilde{N}_2 - \tilde{N}_2^T - \rho_1 U_2^T A_d (U_2 X^T - U_1 Z) - \rho_1 (U_2 X^T - U_1 Z)^T A_d^T U_2$$

$$\Sigma_{23} = -\tilde{N}_3^T + \rho_1 X^T - \rho_2 (U_2 X^T - U_1 Z)^T A_d^T U_2$$

$$\Sigma_{33} = d\tilde{Q} + \rho_2 X^T + \rho_2 X$$

We can design the sliding mode surface

$$\sigma(t) = Sy(t)$$

where matrix S satisfying

$$SC(U_1 F - U_2) = 0, F = ZX^{-T}$$

The proof is given in Appendix 1.

Remark1 Compared with traditional sliding mode surface controller design approach, three matrices $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3$ are introduced as slack variable in order to obtain less conservative results and more complicated condition. Meanwhile, we have proposed a new approach obtain the static output feedback sliding mode surface condition which can achieve the aim of further reducing conservatism.

Theorem 2 For the nonlinear delay systems (1), with the controller

$$u(t) = -(SCB)^{-1}[SCAx(t) + SCA_d x(t-d) + k\sigma(t) + \varepsilon sign\sigma(t)] \tag{9}$$

where k, ε are constants satisfying $k > 0, \varepsilon > 0$, then the systems states will reach the sliding mode surface（4）in finite time.

Proof Along the solution of system (1) we have

$$\begin{aligned} \sigma^T(t)\dot{\sigma}(t) &= \sigma^T(t)SCAx(t) + \sigma^T(t)SCA_d x(t-d) \\ &\quad - \sigma^T(t)SCAx(t) - \sigma^T(t)SCA_d x(t-d) \\ &\quad - \sigma^T(t)k\sigma(t) - \sigma^T(t)\varepsilon sign\sigma(t) \\ &\le -\sigma^T(t)k\sigma(t) - \sigma^T(t)\varepsilon sign\sigma(t) < 0 \end{aligned} \tag{10}$$

With the controller（9）and the above equation（10）, we know that the reaching condition is satisfied.

Remark2 In the following, when the time delay $d = 0$, the system (1) will be simplified as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{11}$$

It is obvious that the proposed matrix transformation method can be still applied for the system (11). As a result, the static output feedback controller design schemes will be proposed as follows:

With the nonsingular transformation $T = \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix}$, the sliding mode equation will be written as

$$\dot{z}_1(t) = U_2^T A(U_2 - U_1 F) z_1(t) \tag{12}$$

Corollary1 For the given constant $\alpha > 0$, the sliding mode equation (12) is stable, if there exist positive-definite matrices $X \in R^{(n-m)\times(n-m)}$, matrix $Z \in R^{m\times(n-m)}$ such that the following LMI holds

$$U_2^T A(U_2 X - U_1 Z) + (U_2 X - U_1 Z)^T A^T U_2 < 0 \tag{13}$$

We can design the sliding mode surface

$$\sigma(t) = Sy(t) \tag{14}$$

where matrix S satisfying

$$SC(U_1 F - U_2) = 0,\ F = ZX$$

The proof is omitted.

Corollary2 For the systems (11), with the controller

$$u(t) = -(SCB)^{-1}[SCAx(t) + k\sigma(t) + \varepsilon sign\sigma(t)] \tag{15}$$

where k, ε are constants satisfying $k > 0, \varepsilon > 0$, then the systems states will reach the sliding mode surface（14）in finite time.

With the proofs of theorem1 and theorem2, Corollary1 and Corollary2 can be easily obtained.

4. Numerical Example

The temperature control system for polymerization reactor is a inertia link with time delay. The state space model of polymerization reactor is usually written as[6]

$$\begin{aligned} &\dot{x}_1(t) = x_2(t) \\ &\dot{x}_2(t) = -a_1 x_1(t) - a_2 x_2(t-d) + bu(t) \\ &y(t) = x_1(t) \end{aligned}$$

Where $a_1 = \dfrac{1}{\tau^2}$, $a_2 = \dfrac{2\xi}{\tau}$, $b = \dfrac{k}{\tau^2}$, $d > 0$ is time delay, ξ, τ, k are known constants.

It is impossible to avoid the external disturbance and time delay. We consider the following time-delay systems

$$\begin{aligned} &\dot{x}(t) = Ax(t) + A_d x(t-d) + Bu(t) \\ &y(t) = Cx(t) \\ &x(t) = \psi(t) \qquad -d \le t \le 0 \end{aligned}$$

where

$$A=\begin{bmatrix}0 & 1\\ -a_1 & 0\end{bmatrix}=\begin{bmatrix}0 & 1\\ -9 & 0\end{bmatrix}, A_d=\begin{bmatrix}0 & 0\\ 0 & -a_2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & -5\end{bmatrix}, B=\begin{bmatrix}1\\ 2\end{bmatrix}, C=\begin{bmatrix}1 & 0\end{bmatrix}, d=0.2$$

With Singular Value Decomposition of B, it is easy to obtain the nonsingular transformation

$$T=\begin{bmatrix}-\frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5}\\ \frac{1}{5}\sqrt{5} & \frac{2}{5}\sqrt{5}\end{bmatrix}.$$

By solving the linear matrix inequality (6), we can obtain the sliding mode surface gain matrix $S=0.5792$ and the static output feedback sliding mode surface as following

$$\sigma(t)=Sy(t)$$

With the static output feedback variable structure controller (9) in Theorem 2, and choosing the initial conditions

$$\psi(t)=[-0.5\ \ 1]^T$$

the simulation results are shown in figs. 1-2

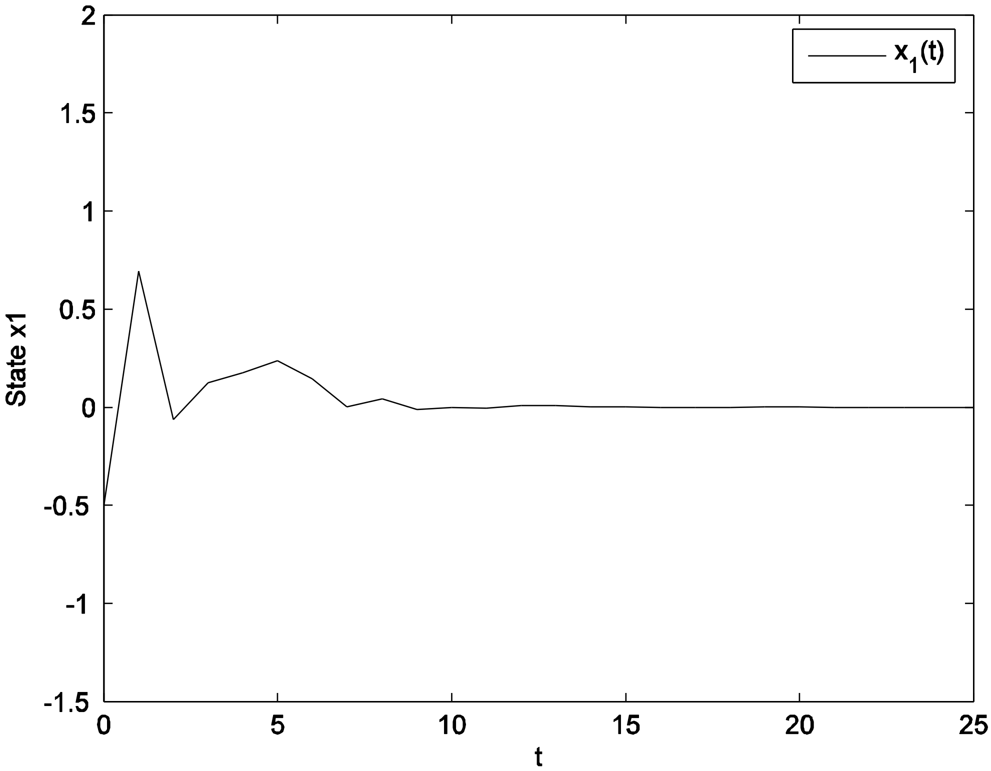

Fig.1. State $x_1(t)$ response of system

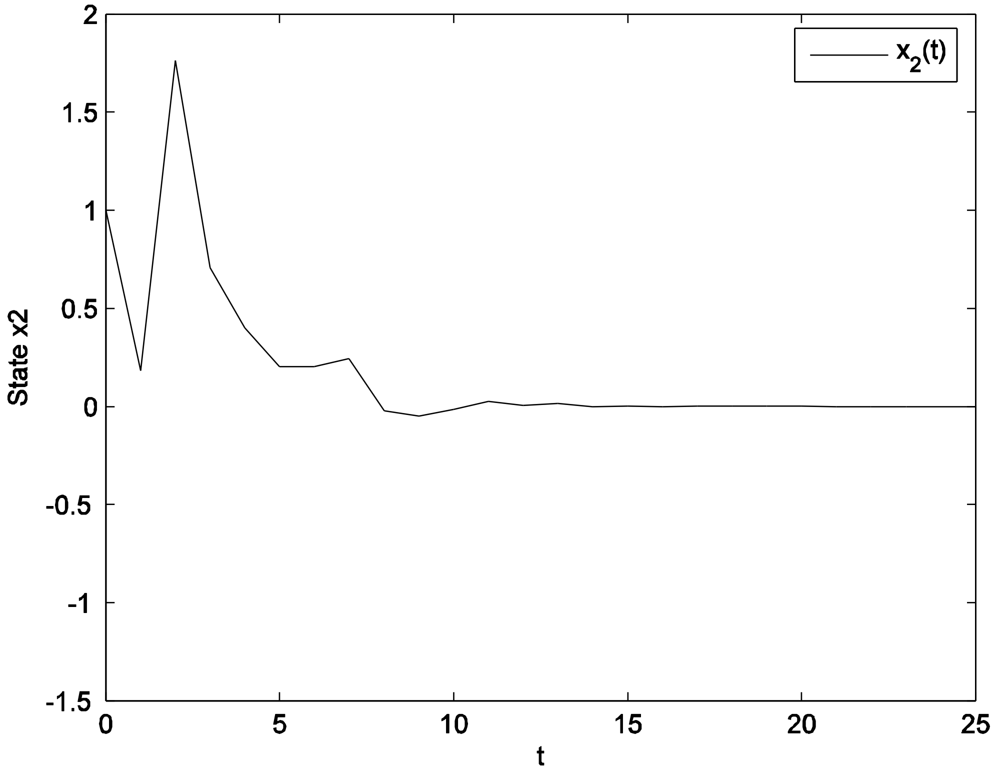

Fig.2. State $x_2(t)$ response of system

In the above figures, one can see that the system is well stabilized with the static output feedback variable structure control.

5. Conclusion

The problem of the static output feedback variable structure control for a class of time-delay systems is considered in this paper. Compared with traditional sliding mode surface design approach which needs known systems states, by using LMI technique, we obtain a new approach to design the sliding mode surface which only needs known systems output. And three slack matrices are introduced in order to obtain a more complicated sufficient condition which can achieve the aim of further reducing conservatism. Then the variable structure controller is designed to make the states reach sliding mode surface in finite time. A numerical example is provided to demonstrate the effectiveness of the proposed approach.

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Appendix 1 Proof of Theorem 1

Proof: Selecting Lyapunov functional such as

$$V(t) = z_1^T(t)Pz_1(t) + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)dsd\theta$$

where P, Q are two positive-definite matrices.

Then, along the solution of system (5) we have

$$\begin{aligned}
\dot{V}(t) &= 2z_1^T(t)P\dot{z}_1(t) + d\dot{z}_1^T(t)Q\dot{z}_1(t) - \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds + 2(z_1^T(t)N_1 + z_1^T(t-d)N_2 \\
&\quad + z_1^T(t)N_3)(z_1(t) - z_1(t-d) - \int_{t-d}^{t} \dot{z}_1(s)ds) + 2(z_1^T(t)M_1 + z_1^T(t-d)M_2 \\
&\quad + \dot{z}_1^T(t)M_3)(-Az_1(t) - A_d z_1(t-d) + \dot{z}_1(t)) \\
&\le 2z_1^T(t)P\dot{z}_1(t) + d\dot{z}_1^T(t)Q\dot{z}_1(t) - \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds + 2(z_1^T(t)N_1 + z_1^T(t-d)N_2 \\
&\quad + z_1^T(t)N_3)(z_1(t) - z_1(t-d)) + 2(z_1^T(t)M_1 + z_1^T(t-d)M_2 + \dot{z}_1^T(t)M_3)(-Az_1(t) \\
&\quad - A_d z_1(t-d) + \dot{z}_1(t)) + d(z_1^T(t)N_1 + z_1^T(t-d)N_2 + z_1^T(t)N_3)Q^{-1}(z_1^T(t)N_1 \\
&\quad + z_1^T(t-d)N_2 + z_1^T(t)N_3)^T + \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds \\
&= \xi^T(t)\Xi\xi(t)
\end{aligned}$$

where $N_1, N_2, N_3, M_1, M_2, M_3$ are constant matrices with appropriate dimensions to be confirmed.

$$\xi(t) = \begin{bmatrix} z_1^T(t) & z_1^T(t-d) & \dot{z}_1^T(t) \end{bmatrix}^T$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix}$$

$$\begin{aligned}
\Xi_{11} &= N_1 + N_1^T - M_1A - A^TM_1^T + dN_1Q^{-1}N_1^T \\
\Xi_{12} &= N_2^T - N_1 - A^TM_2^T - M_1A_d + dN_1Q^{-1}N_2^T \\
\Xi_{13} &= P + N_3^T - A^TM_3^T + M_1 + dN_1Q^{-1}N_3^T \\
\Xi_{22} &= -N_2 - N_2^T - M_2A_d - A_d^TM_2^T + dN_2Q^{-1}N_2^T \\
\Xi_{23} &= -N_3^T - A_d^TM_3^T + M_2 + dN_2Q^{-1}N_3^T \\
\Xi_{33} &= dQ + M_3 + M_3^T + dN_3Q^{-1}N_3^T
\end{aligned}$$

The inequality

$$\Xi < 0 \tag{7}$$

is equivalent to

$$\Xi = \Theta + d \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ 0 \end{bmatrix} Q^{-1} \begin{bmatrix} N_1^T & N_2^T & N_3^T & 0 \end{bmatrix} < 0$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix}$$

$$\Theta_{11} = N_1 + N_1^T - M_1 U_2^T A(U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_1^T$$

$$\Theta_{12} = N_2^T - N_1 - M_1 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_2^T$$

$$\Theta_{13} = P + N_3^T + M_1 - (U_2 - U_1 F)^T A^T U_2 M_3^T$$

$$\Theta_{22} = -N_2 - N_2^T - M_2 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A_d^T U_2 M_2^T$$

$$\Theta_{23} = -N_3^T + M_2 - (U_2 - U_1 F)^T A_d^T U_2 M_3^T$$

$$\Theta_{33} = dQ + M_3 + M_3^T$$

With lemma1， we know that the inequality（7）is equivalent to

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & dN_1 \\ & \Delta_{22} & \Delta_{23} & dN_1 \\ & & \Delta_{33} & dN_1 \\ & & & -dQ \end{bmatrix} < 0 \qquad (8)$$

Where

$$\Delta_{11} = N_1 + N_1^T - M_1 U_2^T A(U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_1^T$$

$$\Delta_{12} = N_2^T - N_1 - M_1 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A^T U_2 M_2^T$$

$$\Delta_{13} = P + N_3^T + M_1 - (U_2 - U_1 F)^T A^T U_2 M_3^T$$

$$\Delta_{22} = -N_2 - N_2^T - M_2 U_2^T A_d (U_2 - U_1 F) - (U_2 - U_1 F)^T A_d^T U_2 M_2^T$$

$$\Delta_{23} = -N_3^T + M_2 - (U_2 - U_1 F)^T A_d^T U_2 M_3^T$$

$$\Delta_{33} = dQ + M_3 + M_3^T$$

Pre- and Post-multiplying the inequality (8) by

$$diag\{M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, I\} \text{ and } diag\{M_0^{-T}, M_0^{-T}, M_0^{-T}, M_0^{-T}, I\}$$

and giving some transformations

$$M_1 = M_0, M_2 = \rho_1 M_0, M_3 = \rho_2 M_0, X = M_0^{-1}, Z = FX^T, \tilde{P} = XPX^T, \tilde{Q} = XQX^T,$$

where ρ_1, ρ_2 are constants to be obtained, we know that the inequality (8) is equivalent to (6).

From the inequality（6），we obtain

$$\dot{V}(t) \le -\xi^T(t) \Xi \xi(t) < 0$$

the sliding mode equation is stable.

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