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Static Output Feedback Sliding Mode Control for Nonlinear Systems with Delay

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Abstract: The problem of static output feedback sliding mode control for a class of nonlinear delay systems with norm-bounded uncertainties is considered in this paper. Based on Linear matrix inequality approach, a new approach is given to design the static output feedback sliding mode surface. Then, a sliding mode controller is obtained which make the systems states reach the sliding mode surface in finite time. All the conditions are expressed in terms of LMI. Finally, a numerical example is given to demonstrate the validity of the results.

Key words

Delay systems, static output feedback, sliding mode control

1. Introduction

Time delay is frequently encountered in various engineering, communication, and biological systems. The characteristics of dynamic systems are significantly affected by the presence of time delays, even to the extent of instability in extreme situations. Therefore, the study of delay systems has received much attention, and various analysis and synthesis methods have been developed over the past years[1−6].

As is known, based on using of discontinuous control laws, the sliding mode control approach is known to be an efficient alternative way to tackle many challenging problems of robust stabilization. Li considered the problem of adaptive fuzzy sliding mode control for a class of nonlinear time delay systems[7]. Kown gave an improved delay-dependent condition to design robust controller for uncertain time-delay systems[8]. Based on LMI approach, Chen considered the problem of exponential stability for uncertain stochastic systems with multiple delays[9]. Xia and Qu designed the robust sliding mode controller for uncertain systems with delays by using

LMI approach[10,11]. The problem of discrete-time output feedback sliding mode control for time-delay systems with uncertainty is researched in [12]. But the results about static output feedback sliding mode control for delay systems have never been presented.

This paper presents the problem of static output feedback sliding mode control for a class of nonlinear delay systems with norm-bounded uncertainties. Based on Linear matrix inequality approach, a new approach is given to design the static output feedback sliding mode surface. Then, a sliding mode controller is obtained which make the systems states reach the sliding mode surface in finite time.

Notations: Throughout the paper, R^n denotes the n dimensional Eucliden space.

2. Problem Formulation

Consider the following nonlinear systems with delay

$$\begin{aligned} &\dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-d) + Bu(t) \\ &y(t) = Cx(t) \\ &x(t) = \psi(t) \qquad -d \le t \le 0 \end{aligned} \tag{1}$$

Where $x(t) \in R^n$ is systems state, $u(t) \in R^m$ is systems control input, $y(t) \in R^p$ is systems output. d is a systems state delay. $\psi(t)$ is the given initial state on $[-d, 0]$. $A \in R^{n\times n}$, $A_d \in R^{n\times n}, B \in R^{m\times n}$ and $C \in R^{p\times n}$ are known constant matrices. B has full column rank. $\Delta A(t) \in R^{n\times n}$ and $\Delta A_d(t) \in R^{n\times n}$ are unknown matrices representing the uncertainties and satisfying

$$[\Delta A(t) \quad \Delta A_d(t)] = GD(t)[H \quad H_d] \tag{2}$$

where G, H and H_d are constant matrices with appropriate dimensions, $D(t)$ is unknown matrix satisfying

$$D^T(t)D(t) \le I$$

With Singular Value Decomposition of B,

$$B = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Omega \\ 0 \end{bmatrix} V^T,$$

a nonsingular transformation $T = \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix}$ is constructed for systems (1) to make $TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$.

With the transformation $z(t) = Tx(t)$, the systems（1）can be rewritten as

$$\begin{aligned} \dot{z}(t) = \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} &= T(A + \Delta A(t))x(t) + T(A_d + \Delta A_d(t))x(t-d) + TBu(t) \\ &= (TAT^{-1} + T\Delta A(t)T^{-1})z(t) + (TA_dT^{-1} + T\Delta A_d(t)T^{-1})z(t-d) + TBu(t) \end{aligned}$$

Inserting (2) into the above formulation, we obtain

$$\begin{aligned}\dot{z}_1(t) &= (U_2^T AU_2 + U_2^T GDHU_2)z_1(t) + (U_2^T AU_1 + U_2^T GDHU_1)z_2(t) + (U_2^T A_d U_2 \\ &\quad + U_2^T GDH_d U_2)z_1(t-d) + (U_2^T A_d U_1 + U_2^T GDH_d U_1)z_2(t-d) \\ \dot{z}_2(t) &= (U_1^T AU_2 + U_1^T GDHU_2)z_1(t) + (U_1^T AU_1 + U_1^T GDHU_1)z_2(t) + (U_1^T A_d U_2 \\ &\quad + U_1^T GDH_d U_2)z_1(t-d) + (U_1^T A_d U_1 + U_2^T GDH_d U_1)z_2(t-d) + B_2 u(t)\end{aligned} \quad (3)$$

For the systems (3), selecting the static output feedback sliding mode surface as following

$$\sigma(t) = Sy(t) \quad (4)$$

With

$\sigma(t) = Sy(t) = SCT^{-1}z(t) = SC[U_2 \quad U_1]z(t) = SCU_2 z_1(t) + SCU_1 z_2(t) = 0$, by the assumption that SCU_1 is nonsingular, we obtain

$$z_2(t) = -(SCU_1)^{-1} SCU_2 z_1(t) = -Fz_1(t)$$

where $F = (SCU_1)^{-1} SCU_2$.

Inserting the above formulation into the systems (3), the sliding mode equation is obtained

$$\dot{z}_1(t) = \bar{A} z_1(t) + \bar{A}_d z_1(t-d) \quad (5)$$

where

$$\begin{aligned}\bar{A} &= U_2^T A(U_2 - U_1 F) + U_2^T GDH(U_2 - U_1 F) \\ \bar{A}_d &= U_2^T A_d (U_2 - U_1 F) + U_2^T GDH_d (U_2 - U_1 F)\end{aligned}$$

3. Main Results

Lemma 1 [3] For known constant $\varepsilon > 0$ and matrices D, E, F satisfying $F^T F \le I$, then the following matrix inequality is hold

$$DEF + E^T F^T D^T \le \varepsilon DD^T + \varepsilon^{-1} E^T E$$

Lemma 2 [5] The LMI $\begin{bmatrix} Y(x) & W(x) \\ * & R(x) \end{bmatrix} > 0$ is equivalent to

$$R(x) > 0, Y(x) - W(x)R^{-1}(x)W^T(x) > 0$$

where $Y(x) = Y^T(x), R(x) = R^T(x)$ depend on x.

Theorem1 For the given constant $\alpha > 0$, the sliding mode equation is stable, if there exist positive-definite matrices $\tilde{P}, \tilde{Q} \in R^{(n-m)\times(n-m)}$, matrices $X, \tilde{N}_1, \tilde{N}_2, \tilde{N}_3 \in R^{(n-m)\times(n-m)}$, constants ρ_2, ρ_3 and matrix $Z \in R^{m\times(n-m)}$ such that the following linear matrix inequality holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & d\tilde{N}_1 & \Sigma_{15} \\ * & \Sigma_{22} & \Sigma_{23} & d\tilde{N}_2 & \Sigma_{25} \\ * & * & \Sigma_{33} & d\tilde{N}_3 & 0 \\ * & * & * & -d\tilde{Q} & 0 \\ * & * & * & * & -\alpha I \end{bmatrix} < 0 \qquad (6)$$

where

$$\Sigma_{11} = \tilde{N}_1 + \tilde{N}_1^T - U_2^T A(U_2X^T - U_1Z) - (U_2X^T - U_1Z)^T A^T U_2 + \alpha U_2^T GG^T U_2$$
$$\Sigma_{12} = \tilde{N}_2^T - \tilde{N}_1 - U_2^T A_d(U_2X^T - U_1Z) - \rho_2(U_2X^T - U_1Z)^T A^T U_2 + \alpha\rho_2 U_2^T GG^T U_2$$
$$\Sigma_{13} = \tilde{P} + \tilde{N}_3^T + X^T - \rho_3(U_2X^T - U_1Z)^T A^T U_2 + \alpha\rho_3 U_2^T GG^T U_2$$
$$\Sigma_{15} = (U_2X^T - U_1Z)^T H^T$$
$$\Sigma_{22} = -\tilde{N}_2 - \tilde{N}_2^T - \rho_2 U_2^T A_d(U_2X^T - U_1Z) - \rho_2(U_2X^T - U_1Z)^T A_d^T U_2 + \alpha\rho_2^2 U_2^T GG^T U_2$$
$$\Sigma_{23} = -\tilde{N}_3^T + \rho_2 X^T - \rho_3(U_2X^T - U_1Z)^T A_d^T U_2 + \alpha\rho_2\rho_3 U_2^T GG^T U_2$$
$$\Sigma_{25} = (U_2X^T - U_1Z)^T H_d^T$$
$$\Sigma_{33} = d\tilde{Q} + \rho_3 X^T + \rho_3 X + \alpha\rho_3^2 U_2^T GG^T U_2$$

We can Design the sliding mode surface

$$\sigma(t) = Sy(t)$$

where matrix S satisfying

$$SC(U_1F - U_2) = 0, F = ZX^{-T}$$

Proof: Selecting Lyapunov functional such as

$$V(t) = z_1^T(t)Pz_1(t) + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)dsd\theta$$

where P, Q are two positive-definite matrices.

Then, along the solution of system (5) we have

$$\begin{aligned} \dot{V}(t) &= 2z_1^T(t)P\dot{z}_1(t) + d\dot{z}_1^T(t)Q\dot{z}_1(t) - \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds + 2(z_1^T(t)N_1 \\ &\quad + z_1^T(t-d)N_2 + z_1^T(t)N_3)(z_1(t) - z_1(t-d) - \int_{t-d}^{t} \dot{z}_1(s)ds) \\ &\quad + 2(z_1^T(t)M_1 + z_1^T(t-d)M_2 + \dot{z}_1^T(t)M_3)(-\bar{A}z_1(t) - \bar{A}_d z_1(t-d) + \dot{z}_1(t)) \\ &\le 2z_1^T(t)P\dot{z}_1(t) + d\dot{z}_1^T(t)Q\dot{z}_1(t) - \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds + 2(z_1^T(t)N_1 \\ &\quad + z_1^T(t-d)N_2 + z_1^T(t)N_3)(z_1(t) - z_1(t-d)) + 2(z_1^T(t)M_1 + z_1^T(t-d)M_2 \\ &\quad + \dot{z}_1^T(t)M_3)(-\bar{A}z_1(t) - \bar{A}_d z_1(t-d) + \dot{z}_1(t)) + d(z_1^T(t)N_1 \\ &\quad + z_1^T(t-d)N_2 + z_1^T(t)N_3)Q^{-1}(z_1^T(t)N_1 + z_1^T(t-d)N_2 + z_1^T(t)N_3)^T \\ &\quad + \int_{t-d}^{t} \dot{z}_1^T(s)Q\dot{z}_1(s)ds \\ &= \xi^T(t)\Xi\xi(t) \end{aligned}$$

where $N_1, N_2, N_3, M_1, M_2, M_3$ are constant matrices with appropriate dimensions to be confirmed.

$$\xi(t) = \begin{bmatrix} z_1^T(t) & z_1^T(t-d) & \dot{z}_1^T(t) \end{bmatrix}^T$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix}$$

$$\Xi_{11} = N_1 + N_1^T - M_1\bar{A} - \bar{A}^T M_1^T + dN_1Q^{-1}N_1^T$$

$$\Xi_{12} = N_2^T - N_1 - \bar{A}^T M_2^T - M_1\bar{A}_d + dN_1Q^{-1}N_2^T$$

$$\Xi_{13} = P + N_3^T - \bar{A}^T M_3^T + M_1 + dN_1Q^{-1}N_3^T$$

$$\Xi_{22} = -N_2 - N_2^T - M_2\bar{A}_d - \bar{A}_d^T M_2^T + dN_2Q^{-1}N_2^T$$

$$\Xi_{23} = -N_3^T - \bar{A}_d^T M_3^T + M_2 + dN_2Q^{-1}N_3^T$$

$$\Xi_{33} = dQ + M_3 + M_3^T + dN_3Q^{-1}N_3^T$$

The inequality

$$\Xi < 0 \qquad (7)$$

is equivalent to

$$\Xi = \Theta + d\begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ 0 \end{bmatrix} Q^{-1}\begin{bmatrix} N_1^T & N_2^T & N_3^T & 0 \end{bmatrix} - \begin{bmatrix} M_1U_2^TG \\ M_2U_2^TG \\ M_3U_2^TG \end{bmatrix} D\begin{bmatrix} H(U_2-U_1F) & H_d(U_2-U_1F) & 0 \end{bmatrix}$$

$$-\begin{bmatrix} H(U_2-U_1F) & H_d(U_2-U_1F) & 0 \end{bmatrix}^T D^T \begin{bmatrix} M_1U_2^TG \\ M_2U_2^TG \\ M_3U_2^TG \end{bmatrix}^T < 0$$

where

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix}$$

$$\Theta_{11} = N_1 + N_1^T - M_1U_2^TA(U_2-U_1F) - (U_2-U_1F)^T A^TU_2M_1^T$$

$$\Theta_{12} = N_2^T - N_1 - M_1U_2^TA_d(U_2-U_1F) - (U_2-U_1F)^T A^TU_2M_2^T$$

$$\Theta_{13} = P + N_3^T + M_1 - (U_2-U_1F)^T A^TU_2M_3^T$$

$$\Theta_{22} = -N_2 - N_2^T - M_2U_2^TA_d(U_2-U_1F) - (U_2-U_1F)^T A_d^TU_2M_2^T$$

$$\Theta_{23} = -N_3^T + M_2 - (U_2-U_1F)^T A_d^TU_2M_3^T$$

$$\Theta_{33} = dQ + M_3 + M_3^T$$

With lemma1，we know that the following inequality holds for given constant $\alpha > 0$

$$
-\begin{bmatrix} M_1U_2^TG \\ M_2U_2^TG \\ M_3U_2^TG \end{bmatrix} D\begin{bmatrix} H(U_2-U_1F) & H_d(U_2-U_1F) & 0 \end{bmatrix}
$$

$$
-\begin{bmatrix} H(U_2-U_1F) & H_d(U_2-U_1F) & 0 \end{bmatrix}^T D^T \begin{bmatrix} M_1U_2^TG \\ M_2U_2^TG \\ M_3U_2^TG \end{bmatrix}^T
$$

$$
\le \alpha^{-1}\begin{bmatrix} H(U_2-U_1F) & H_d(U_2-U_1F) & 0 \end{bmatrix}^T \begin{bmatrix} H(U_2-U_1F) & H_d(U_2-U_1F) & 0 \end{bmatrix}
$$

$$
+\alpha \begin{bmatrix} M_1U_2^TG \\ M_2U_2^TG \\ M_3U_2^TG \end{bmatrix} \begin{bmatrix} M_1U_2^TG \\ M_2U_2^TG \\ M_3U_2^TG \end{bmatrix}^T
$$

With lemma2, we know that the inequality（7）is equivalent to

$$
\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & dN_1 & \Delta_{15} \\ * & \Delta_{22} & \Delta_{23} & dN_1 & \Delta_{25} \\ * & * & \Delta_{33} & dN_1 & 0 \\ * & * & * & -dQ & 0 \\ * & * & * & * & -\alpha I \end{bmatrix} < 0 \qquad (8)
$$

Where

$$\Delta_{11} = N_1 + N_1^T - M_1U_2^TA(U_2-U_1F) - (U_2-U_1F)^T A^TU_2M_1^T + \alpha M_1U_2^TGG^TU_2M_1^T$$
$$\Delta_{12} = N_2^T - N_1 - M_1U_2^TA_d(U_2-U_1F) - (U_2-U_1F)^T A^TU_2M_2^T + \alpha M_1U_2^TGG^TU_2M_2^T$$
$$\Delta_{13} = P + N_3^T + M_1 - (U_2-U_1F)^T A^TU_2M_3^T + \alpha M_1U_2^TGG^TU_2M_3^T$$
$$\Delta_{15} = (U_2-U_1F)^T H^T$$
$$\Delta_{22} = -N_2 - N_2^T - M_2U_2^TA_d(U_2-U_1F) - (U_2-U_1F)^T A_d^TU_2M_2^T + \alpha M_2U_2^TGG^TU_2M_2^T$$
$$\Delta_{23} = -N_3^T + M_2 - (U_2-U_1F)^T A_d^TU_2M_3^T + \alpha M_2U_2^TGG^TU_2M_3^T$$
$$\Delta_{25} = (U_2-U_1F)^T H_d^T$$
$$\Delta_{33} = dQ + M_3 + M_3^T + \alpha M_3U_2^TGG^TU_2M_3^T$$

Pre- and Post-multiplying the inequality (8) by

$$diag\{M_0^{-1}, M_0^{-1}, M_0^{-1}, M_0^{-1}, I\} \text{ and } diag\{M_0^{-T}, M_0^{-T}, M_0^{-T}, M_0^{-T}, I\}$$

and giving some transformations

$$M_1 = M_0, M_2 = \rho_2 M_0, M_3 = \rho_3 M_0, X = M_0^{-1}, Z = FX^T, \tilde{P} = XPX^T, \tilde{Q} = XQX^T,$$

where ρ_2, ρ_3 are constants to be obtained, we know that the inequality (8) is equivalent to (6).

From the inequality（6）, we obtain

$$\dot{V}(t) \le -\xi^T(t)\Xi\xi(t) < 0$$

the sliding mode equation is stable.

Remark1 Compared with traditional sliding mode surface controller design approach, three matrices $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3$ are introduced as slack variable in order to obtain less conservative results and more complicated condition. Meanwhile, we have proposed a new approach obtain the static output feedback sliding mode surface condition which can achieve the aim of further reducing conservatism.

Theorem 2 For the nonlinear delay systems (1), with the controller

$$\begin{aligned} u(t) = -(SCB)^{-1}[SCAx(t) + SCA_d x(t-d) + \frac{\| SC \| \sigma(t)}{\| \sigma(t) \|}(\| GH \| + \| GH_d \| \\ + \| B_\omega \| \rho(t)) + k\sigma(t) + \varepsilon sign\sigma(t)] \end{aligned} \tag{9}$$

where k, ε are constants satisfying $k > 0, \varepsilon > 0$, then the systems states will reach the sliding mode surface（4）in finite time.

Proof: Along the solution of system (1) we have

$$\begin{aligned} \sigma^T(t)\dot{\sigma}(t) &= \sigma^T(t)SC(A + \Delta A)x(t) + \sigma^T(t)SC(A_d + \Delta A_d)x(t-d) \\ &+ \sigma^T(t)SCB_\omega \omega(t) - \sigma^T(t)SCAx(t) - \sigma^T(t)SCA_d x(t-d) \\ &- \frac{\sigma^T(t)\| SC \| \sigma(t)}{\| \sigma(t) \|}(\| GH \| + \| GH_d \| + \| B_\omega \| \rho(t)) - \sigma^T(t)k\sigma(t) \\ &- \sigma^T(t)\varepsilon sign\sigma(t) \\ &\le -\sigma^T(t)k\sigma(t) - \sigma^T(t)\varepsilon sign\sigma(t) < 0 \end{aligned} \tag{10}$$

With the controller（9）and the above equation（10）, we know that the reaching condition is satisfied.

Remark2 In the following, when the time delay $d = 0$, the system (1) will be simplified as

$$\dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t) \tag{11}$$

It is obvious that the proposed matrix transformation method can be still applied for the system (11). As a result, the static output feedback controller design schemes will be proposed as follows:

With the nonsingular transformation $T = \begin{bmatrix} U_2^T \\ U_1^T \end{bmatrix}$, the sliding mode equation will be written as

$$\dot{z}_1(t) = \bar{A}z_1(t) \tag{12}$$

where

$$\bar{A} = U_2^T A(U_2 - U_1 F) + U_2^T GDH(U_2 - U_1 F)$$

Corollary1 For the given constant $\alpha > 0$, the sliding mode equation (12) is stable， if there exist positive-definite matrices $X \in R^{(n-m)\times(n-m)}$, matrix $Z \in R^{m\times(n-m)}$ such that the following linear matrix inequality holds

$$\begin{bmatrix} U_2^T A(U_2X-U_1Z)+(U_2X-U_1Z)^T A^T U_2+\alpha U_2^T GG^T U_2 & (U_2X-U_1Z)^T H^T \\ * & -\alpha I \end{bmatrix} < 0 \quad (13)$$

We can Design the sliding mode surface

$$\sigma(t) = Sy(t) \quad (14)$$

where matrix S satisfying

$$SC(U_1F-U_2)=0,\ F=ZX$$

The proof is omitted.

Corollary2 For the nonlinear systems (11), with the controller

$$u(t) = -(SCB)^{-1}[SCAx(t) + \frac{\| SC \| \| GH \| \sigma(t)}{\| \sigma(t) \|} + k\sigma(t) + \varepsilon sign\sigma(t)] \quad (15)$$

where k,ε are constants satisfying $k>0,\varepsilon>0$, then the systems states will reach the sliding mode surface（14）in finite time.

With the proofs of theorem1 and theorem2, Corollary1 and Corollary2 can be easily obtained.

4. Numerical Example

The temperature control system for polymerization reactor is a inertia link with time delay. The state space model of polymerization reactor is usually written as [6]

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= -a_1x_1(t) - a_2x_2(t) + bu(t) \\ y(t) &= x_1(t) \end{aligned}$$

It is impossible to avoid the external disturbance and time delay. We consider the nonlinear delay system with norm-bounded uncertainties as following

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A(t))x(t) + (A_d+\Delta A_d(t))x(t-d) + Bu(t) \\ y(t) &= Cx(t) \\ x(t) &= \psi(t) \qquad -d \le t \le 0 \end{aligned}$$

where

$$A=\begin{bmatrix} -30 & 0 \\ 0 & -20 \end{bmatrix}, A_d=\begin{bmatrix} -2 & 0.5 \\ -0.5 & -2 \end{bmatrix}, B=\begin{bmatrix} 1 \\ 2 \end{bmatrix}, C=\begin{bmatrix} 1 & 9.5 \end{bmatrix}, \omega(t)=0.1\sin t, G=\begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$H=\begin{bmatrix} 0.01 & 0 \end{bmatrix}, H_d=\begin{bmatrix} 0 & 0.01 \end{bmatrix}, \psi(t)=\begin{bmatrix} 1 \\ -1 \end{bmatrix}, d=0.2, D(t)=\cos t$$

With Singular Value Decomposition of B, It is easy to obtain the nonsingular transformation

$$T = \begin{bmatrix} -\frac{2}{5}\sqrt{5} & \frac{1}{5}\sqrt{5} \\ \frac{1}{5}\sqrt{5} & \frac{2}{5}\sqrt{5} \end{bmatrix}.$$

By solving the linear matrix inequality (6), we can obtain the sliding mode surface gain matrix $S = 0.8$ and the static output feedback sliding mode surface as following

$$\sigma(t) = Sy(t)$$

With the static output feedback variable structure controller (9) in Theorem 2, and choosing the initial conditions

$$\psi(t) = [2 \quad -0.5]^T$$

the simulation results are shown in figs. 1-2

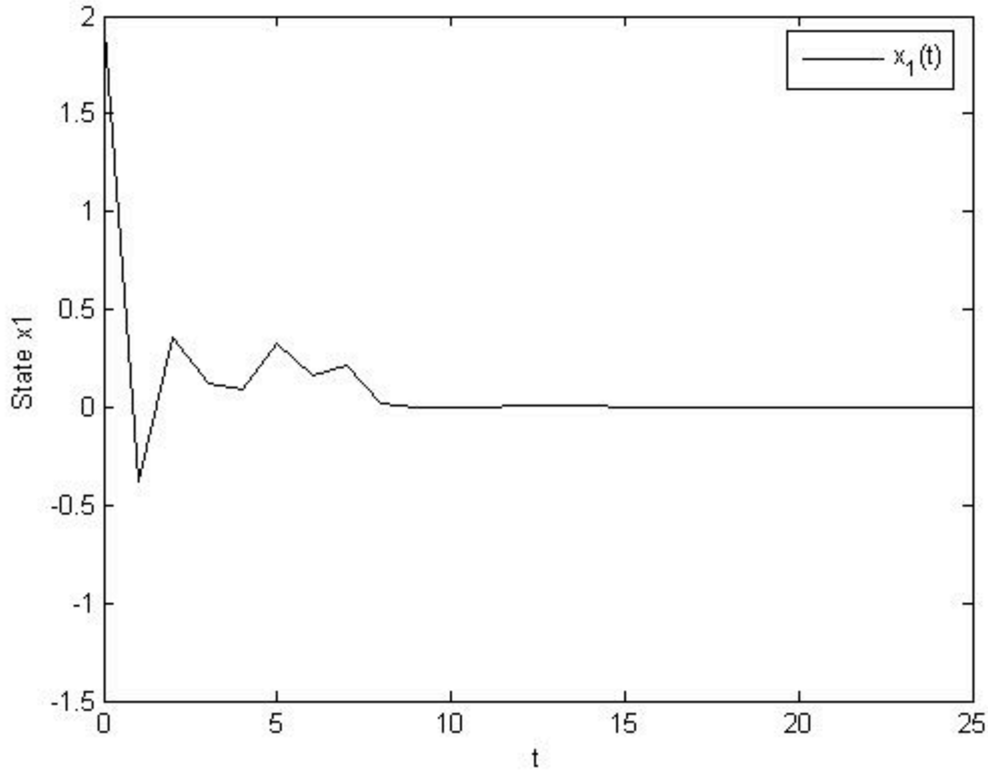

Fig.1. State $x_1(t)$ response of system

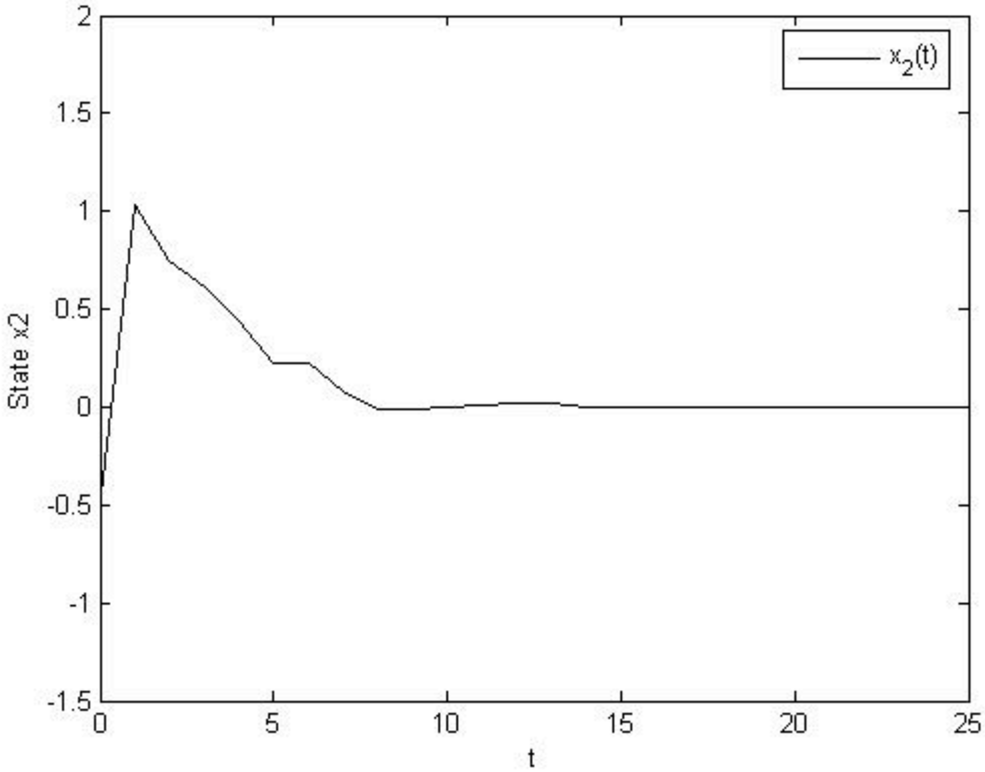

Fig.2. State $x_2(t)$ response of system

In the above figures, one can see that the system is well stabilized with the static output feedback sliding mode controller.

5. Conclusion

This paper considers the problem of the static output feedback sliding mode control for a class of nonlinear delay systems with norm-bounded uncertainties. A static output feedback sliding mode surface is designed by using linear matrix inequality approach. Then the sliding mode controller is designed to make the states reach sliding mode surface in finite time.

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